AH National Qualifications 2024

X847/77/11

Mathematics
Paper 1 (Non-calculator)

MONDAY, 13 MAY

9:00 AM – 10:00 AM

Total marks — 35

Attempt ALL questions.

You must NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

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FORMULAE LIST

Standard derivatives

$f(x)$	$f'(x)$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x$	$-\frac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
$\sec x$	$\sec x \tan x$
$\text{cosec}\, x$	$-\text{cosec}\, x \cot x$
$\ln x$	$\frac{1}{x}$
e^x	e^x

Standard integrals

$f(x)$	$\int f(x)\,dx$
$\sec^2(ax)$	$\frac{1}{a}\tan(ax)+c$
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n=\frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n=\frac{a\left(1-r^n\right)}{1-r},\ r\neq 1$$

$$\sum_{r=1}^{n} r=\frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2=\frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3=\frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n=\sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r}={}^nC_r=\frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x)=f(0)+f'(0)x+\frac{f''(0)x^2}{2!}+\frac{f'''(0)x^3}{3!}+\frac{f^{iv}(0)x^4}{4!}+\ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$\left[r(\cos\theta + i\sin\theta)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 35

Attempt ALL questions

1. Differentiate the following with respect to x:

(a) $y = \cot 3x$ **2**

(b) $f(x) = 5x(4x-7)^{\frac{1}{2}}$. **2**

2. A complex number is defined by $z = 1 + i$.

(a) Express z in polar form. **2**

(b) Use de Moivre's theorem to evaluate z^8. **2**

3. A geometric sequence of positive terms has third term 36 and fifth term 16.

(a) Calculate the value of the common ratio. **2**

(b) Calculate the value of the first term. **1**

(c) State why the associated geometric series has a sum to infinity. **1**

(d) Find the value of this sum to infinity. **1**

4. Matrix A is defined by $A = \begin{pmatrix} 6 & 1 \\ 11 & 3 \end{pmatrix}$.

(a) Find A^{-1}, the inverse of matrix A. **2**

Matrix B is defined by $B = \begin{pmatrix} -4 & 3 \\ -5 & 2 \end{pmatrix}$.

(b) Find the matrix M such that $AM = B$. **2**

MARKS

5. The function $f(x)$ is defined by $f(x) = x^3 - x$, $x \in \mathbb{R}$.

(a) Determine whether $f(x)$ is even, odd or neither. **2**

(b) Show that the graph of $y = f(x)$ has a point of inflection. **2**

6. (a) Find the 2×2 matrix, A, associated with a reflection in the x-axis. **1**

(b) Describe the transformation associated with the matrix $B = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. **1**

(c) Find the 2×2 matrix, C, associated with a reflection in the x-axis followed by the transformation associated with $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. **2**

7. A curve is defined by the equation $x^2 y + 4xy^2 = -32$, $y > 0$.

(a) Use implicit differentiation to find an expression for $\frac{dy}{dx}$. **3**

The curve has only one stationary point.

(b) Find the coordinates of the stationary point. **3**

8. Use the substitution $u = \tan 2x$ to evaluate $\int_0^{\frac{\pi}{8}} \frac{\sqrt{\tan 2x}}{\cos^2 2x}\,dx$. **4**

[END OF QUESTION PAPER]

AH National Qualifications 2024

X847/77/12

Mathematics Paper 2

MONDAY, 13 MAY

10:30 AM – 1:00 PM

Total marks — 80

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

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FORMULAE LIST

Standard derivatives	
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$\tan^{-1} x$	$\frac{1}{1+x^2}$
$\tan x$	$\sec^2 x$
$\cot x$	$-\text{cosec}^2 x$
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Standard integrals	
$f(x)$	$\int f(x)\,dx$
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$\frac{1}{a^2+x^2}$	$\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c$
e^{ax}	$\frac{1}{a}e^{ax}+c$

Summations

(Arithmetic series) $$S_n = \frac{1}{2}n\left[2a+(n-1)d\right]$$

(Geometric series) $$S_n = \frac{a(1-r^n)}{1-r},\ r \neq 1$$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2},\quad \sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6},\quad \sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

Binomial theorem

$$(a+b)^n = \sum_{r=0}^{n}\binom{n}{r}a^{n-r}b^r \quad \text{where} \quad \binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Maclaurin expansion

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \frac{f^{iv}(0)x^4}{4!} + \ldots$$

FORMULAE LIST (continued)

De Moivre's theorem

$$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$$

Vector product

$$\mathbf{a}\times\mathbf{b} = |\mathbf{a}||\mathbf{b}|\sin\theta\,\hat{\mathbf{n}}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \mathbf{i}\begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} - \mathbf{j}\begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} + \mathbf{k}\begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}$$

Matrix transformation

Anti-clockwise rotation through an angle, θ, about the origin, $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$

[Turn over

MARKS

Total marks — 80

Attempt ALL questions

1. Given $y = \frac{\sin 7x}{1+x^2}$, find $\frac{dy}{dx}$. **2**

2. Use the Euclidean algorithm to find integers a and b such that $533a + 455b = 13$. **3**

3. (a) Use Gaussian elimination to express z in terms of λ for the system of equations:

$$\begin{aligned} x - y - 3z &= 1 \\ 2x - 3y - 5z &= 8 \\ x + 2y + \lambda z &= -7. \end{aligned}$$

4

(b) State the value of λ for which this system is inconsistent. **1**

(c) Determine the solution of this system when $\lambda = -1$. **1**

4. Solve the differential equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 8y = 0$$

given that $y = -2$ and $\frac{dy}{dx} = 22$ when $x = 0$. **5**

5. (a) State and simplify the general term in the binomial expansion of $\left(2x^2 - \frac{1}{x^3}\right)^{16}$. **3**

(b) Hence, or otherwise, find the coefficient of $\frac{1}{x^{18}}$ in the expansion of $\left(2x^2 - \frac{1}{x^3}\right)^{16}$. **2**

MARKS

6. A curve is defined parametrically by $x = t^2$ and $y = 4t\ln t$ where $t > 0$.

Find fully simplified expressions for:

(a) $\frac{dy}{dx}$ **3**

(b) $\frac{d^2y}{dx^2}$. **3**

7. (a) Find and simplify the Maclaurin expansion, up to and including the term in x^3, for:

(i) e^{2x} **2**

(ii) $\sin 3x$. **2**

(b) Hence find the Maclaurin expansion for $e^{2\sin 3x}$ up to and including the term in x^3. **2**

8. A solid is formed by rotating part of the curve with equation $y = \frac{1}{\sqrt{1+x^2}}$ about the x-axis through 2π radians, from $x = 0$ to $x = a$.

The value of the volume of the solid is $\frac{\pi^2}{3}$.

Determine the value of a. **5**

9. An arithmetic sequence has first term −3 and common difference d.

(a) State an expression for the third term. **1**

The eighth term is five times the third term.

(b) Find the value of d. **1**

(c) Determine algebraically the least number of terms required so that the sum of the associated series is greater than 500. **3**

[Turn over

MARKS

10. A metal rod is heated such that its volume increases at a constant rate of 12 mm^3 per minute.

The volume of the rod is modelled, throughout the process, by $V = 5\pi r^3$, where r is measured in millimetres.

Find the rate at which r is increasing when $r = 10$. **4**

11. Consider statements A and B below.

For each statement: if true, provide a proof; if false, provide a counterexample. **3**

A: The sum of the squares of any two consecutive integers is always prime.

B: The sum of the squares of any two consecutive integers is always odd.

12. Given $z = x + iy,\ y \neq 0$, solve the equation

$$z^2 + 20\bar{z} - 156 = 0$$

where $\bar{z}$ is the complex conjugate of z. **5**

13. (a) Express $\dfrac{-2}{x(x+1)}$ in partial fractions. **2**

(b) Use integration by parts to find $\int xe^{3x}dx$. **3**

(c) Using your answers to (a) and (b), solve

$$\frac{dy}{dx} - \frac{2y}{x(x+1)} = \frac{x^3e^{3x}}{(x+1)^2}.$$ **5**

14. A plane passes through A(2, –1, 8), B(1, 1, –1) and C(4, –2, 11).

(a) (i) Determine $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **1**

(ii) Hence find the Cartesian equation of the plane. **3**

A line is defined by the equations $\dfrac{x-1}{1} = \dfrac{y+1}{-1} = \dfrac{z+1}{4}$.

(b) Show that the line and the plane do not intersect. **3**

MARKS

15. A storage tank contains a mixture of salt and water. An additional amount of salt and water pours in while, at the same time, some of the existing mixture pours out.

The process can be modelled by the differential equation

$$\frac{dW}{dt} = \frac{36 - W}{120}, \; W < 36$$

where W is the amount of salt in kilograms at time t minutes.

Initially, the storage tank contains 8 kilograms of salt.

(a) Express W in terms of t. **5**

(b) Find the rate at which the amount of salt is increasing after 67 minutes. **2**

As the process continues, the amount of salt approaches a limit L kilograms.

(c) Find the value of L, justifying your answer. **1**

[END OF QUESTION PAPER]