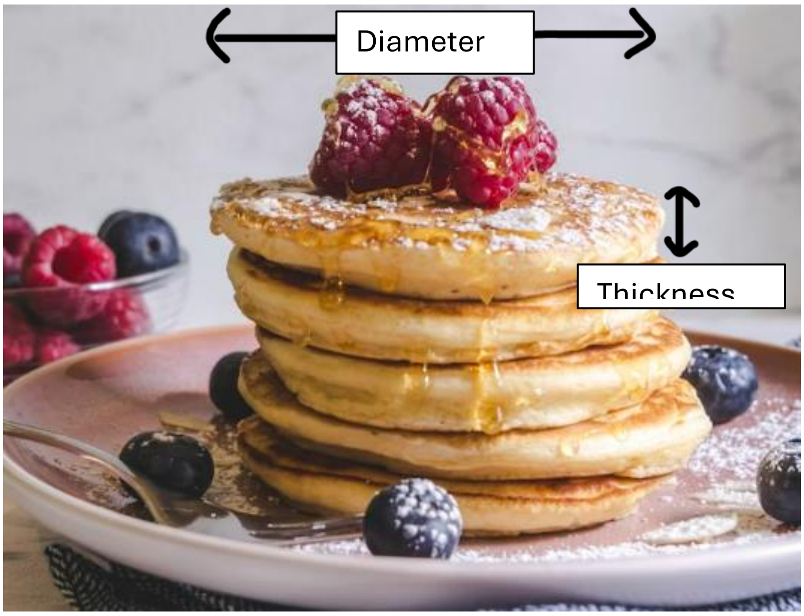
A flipping essay on pancakes

By Sanaa Umarji

1 Introduction

I love pancakes, but I'm not exactly a great chef- however I do enjoy maths. Even something as simple as a pancake turns out to involve more algebra and combinatorics than you'd expect, from getting the right proportions, to the order they are served.

2 The perfect pancake



The perfect pancake is extremely subjective; however choice of preference can be narrowed down to 3 variables: thickness of the pancake (fluffiness), diameter of the pancake and the size of the pancake. Personally, i find the amount of maple syrup on my pancake also very important, and for this case the maple syrup coverage is dependent on the pancake dimensions.

Another assumption is the pancake is the standard flat circular shape, modelled by a cylinder. Furthermore, it is given we can achieve any desired thickness and size, even much larger thicknesses like 5cm or more seen in Japanese pancakes.

The volume of a cylinder is $(1)\ V = \pi r^2 h$, here we will be using r as the radius (hald the diameter of pnacake) and h as the thickness of the pancake

The surface area exposed to maple syrup is the top face (πr^2) and edges ($2\pi r \cdot h$), forming a total contact area of $(2)\ A = \pi r^2 + 2\pi r h$.

Treat V as a given constant as we have a fixed amount of mixture.

Rearrange equation (1) to get:

$$h = \frac{V}{\pi r^2}$$

Sub into (2):

$$A = \pi r^2 + 2\pi r\left(\frac{V}{\pi r^2}\right)$$

$$(3)\ A = \pi r^2 + \frac{2V}{r}$$

We want to minimise the surface area of the given volume, so we want the minimum stationary point of equation (3).

$$\frac{dA}{dr} = 2\pi r - \frac{2V}{r^2}$$

At local minima, $\frac{d^2A}{dr^2} > 0$

$$\frac{d^2A}{dr^2} = 2\pi + \frac{4V}{r^4}$$

V is a volume and r is a length, so both are positive, hence $2\pi + \frac{4V}{r^4} > 0$

So $\frac{d^2A}{dr^2} > 0$, thus any stationary points are local minima

At stationary points $\frac{dA}{dr} = 0$:

$$2\pi r - \frac{2V}{r^2} = 0$$

$$\frac{2\pi r^3 - 2V}{r^2} = 0$$

Note: r is a length so $r > 0$

$$2\pi r^3 - 2V = 0$$

$$(4)\ V = \pi r^3$$

We now have a formula for the radius size that gives the minimum surface area for a given volume.

Equate (4) and (1) as they both equal V:

$$\pi r^2 h = \pi r^3$$

$$h = r$$

Thus, the optimal shape for a pancake is one such that the thickness equals the radius. For a stack of pancakes, it can also be modelled by a cylinder in the same way, and it tells you that the radius of a pancake should equal the height of all the pancakes combined for maximum coverage, and hency you know the optimum number of pancakes for your stack (h/thickness of each pancake).

Here, any thickness can be chosen, so no matter what style of pancake, whether crepe thin or Japanese pancake, you can minimise the surface area and thus maximise the maple syrup coverage.

Some further assumptions that can be improved in future are:

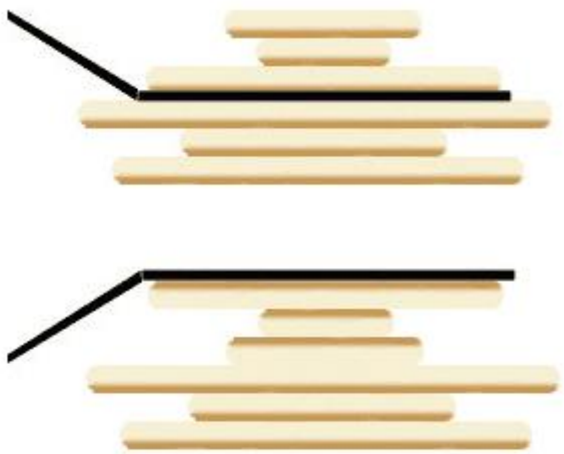
-Maple syrup does not flow evenly across the pancake, so the spread could be modelled by another equation

-The shape of a pancake could be modelled more as a cylinder with a curved top and sides.

3 The pancake flipping problem

I am a pretty inconsistent baker, so when i make pancakes they all end up different sizes. Once made I place them on top of each other, so they end up in a random order. To make it look a bit more aesthetic, I want the largest pancake at the bottom with them decreasing in size going up.

The problem is that I only have a spatula, so I can only flip sections of the stack at a time. What is the minimum number of flips needed to sort the pancakes?



Assume that every pancake has one good side and one bad side and let n be the number of pancakes in the stack.

The first route to solving a problem is often to just test it and get a feel for it, perhaps to spot patterns or just better understand the problem.

For 1 pancake no flips are required as it is already sorted.

For 2 pancakes there are 3 cases, one where it's all sorted and no flips are needed and one where 1 flip is needed to flip the whole stack.

For 3 pancakes we can see there are 3*2*1=6 combinations, and by testing each case we find the minimum number of flips needed is 3.

We can continue this for more and more pancakes, and we see by 10 pancakes, the minimum number of flips is 11. So, it's not a clear linear pattern.

Also, as we increase the number of pancakes the number of stack combinations increases by a factor of n. And it will become more time consuming to not only check each case but ensure our solution is the fewest for flips for a case. Thus, a better method is needed. In fact, the number of flips for a given number of pancakes is called a pancake number, and the search for pancake numbers is an ongoing problem in mathematics.

Some bounds for the minimum number of flips for n pancakes have been found, but for as small as 20 pancakes, the pancake number is unknown!

In fact, this is a popular problem which has links to many fields, particularly computer science. And in 2011, Laurent Bulteau, Guillaume Fertin, and Irena Rusu proved that the problem of finding the shortest sequence of flips for a given stack of pancakes is NP-hard [1]. This means in terms of time complexity (a measure of how long an algorithm would take) it would take very long, especially as n increases, though to check a given pancake number is correct, it may be easier.

Furthermore, before being improved even Bill gates' had his share of the cake with his and Christos Papadimitriou lower bound of $\frac{17}{16}n$.

The latest bounds for a pancake number is $\frac{15}{14}n$ and $\frac{18}{11}n$.

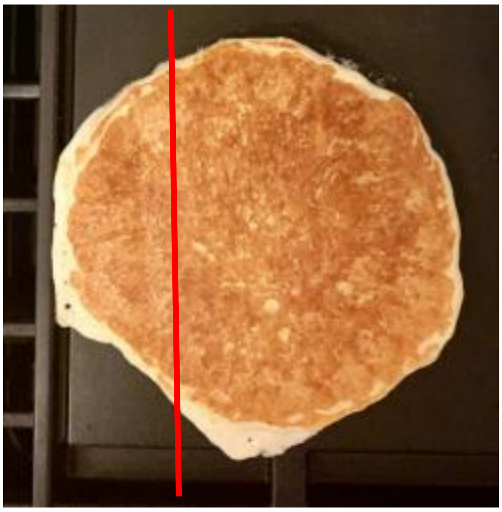
Being the bad chef, I am, I also usually end up making pancakes with one good side and one slightly dodgy (or even burnt) side. When I stack them on my plate, their orientation is completely random as usual. However, if I want them to look even remotely presentable, I need all the good sides facing upwards.

The same conditions as the previous problem apply as this is an extension of the flipping. problem seen before. Here research is especially active as it has application to biology, particular the research of E-coli.

4 Pancake cutting

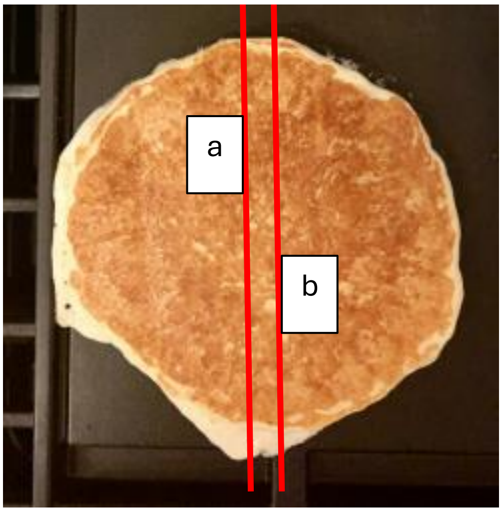
As you should have figure by now, i am not a very good baker, so again I have a problem with my pancakes. These pancakes are all different shapes, instead of neat circles and in a stack. Despite this, I want to be able to cut my pancakes perfectly in half to share, but I only have a knife. Can I cut the oddly shaped stack perfectly in half?

For now, let's look at one oddly shaped pancake. Here we are looking at particularly thin pancakes, so we model this as a 2D shape.



Assume the area of the pancake is 1. Let the area to the left of the red knife be defined by some $f(x)$, hence the area to the right of it is $1 - f(x)$.

If there is no line where the $(right\ area)\ f(x) = 1 - f(x)\ (left\ area)$, then there must be a jump in area somewhere in the pancake. Assume we can have a jump in the area and draw 2 lines on the pancake.

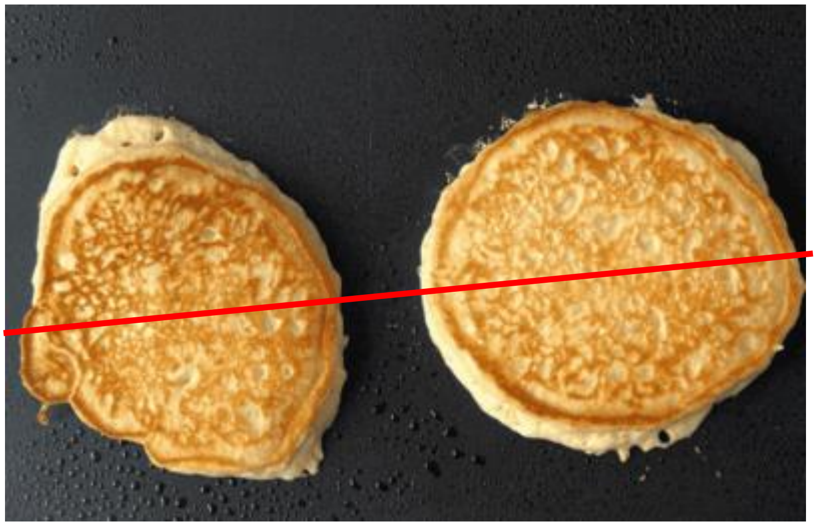


Look at line A and B. The area between them is some positive number as a pancake is bounded and the area between is bounded by a rectangle. As the 2 lines move closer, this are between decreases to 0. Thus, there is no jump in area as it was always bounded. This shows that $f(x)$ is a continuous function using a visual demonstration of the intermediate value theorem.

Since f(x) is continuous, there must be a point where the left area equals the right area.

This proves no matter the shape of pancake we can cut it exactly in half with one clean cut.

What if we have 2 pancakes flat on the same countertop, instead of stacked. We can prove we can cut these both in half with one straight line through both too.



If you imagine the area between them to be removed and for them to touch, it's the same as 1 pancake, allowing the theorem to be extended

With multiple pancakes in a stack, the problem reflects that of the ham sandwich theorem.

5 Bibliography

https://www.sciencedirect.com/science/article/pii/S0304397508003575?via%3Dihub

https://www.youtube.com/watch?v=m3drS_8BpU0

https://web.archive.org/web/20120214070902/http://www.utdallas.edu/news/2008/09/17-002.php?WT.mc_id=NewsEmails&WT.mc_ev=EmailOpen

https://mathcircles.org/wp-content/uploads/2026/01/2016-Winter-_-Spring-MTCircular.pdf#page=16

https://www.sciencedirect.com/science/article/pii/0166218X94000093

https://plus.maths.org/maths-minut-flipping-pancakes

https://arxiv.org/pdf/2511.15864