A Lyrical Loop

It starts with a single line: in class, whilst shopping and even in the shower, a melody begins to surface. Perhaps a chorus you heard earlier, or a tune hidden within background noise. Hours later it's still there – repeating, looping, refusing to leave. These persistent fragments of music are often referred to as earworms. They are a common experience, with 90% of people reporting having an earworm on a weekly basis. But what is seemingly random and uncontrollable is actually far more structured. The persistence of these musical loops may not just be a quirk of memory, but a consequence of how the brain responds to patterns, prediction, and repetition – ideas that lie at the heart of mathematics.

Detecting patterns:

Our society is unknowingly built upon patterns and structures much like the rigid rules of mathematics taught in school. From the earliest stages of civilisation, humans have embedded patterns within daily life through architecture, art, timekeeping and culture. Ancient pyramids display early uses of geometric ratios, whilst Andean textiles utilise binary structures within woven cloth. Order and periodic cycles are echoed within music. Together, rhythm and pitch build a foundation of beats, bars and counts; perfectly paired with numerical ratios of frequency – simple yet harmonious. These relationships are meticulously developed into scales and melodies comprised of structured sequences of notes, revealing how music is fundamentally a system of mathematical patterns expressed through sound. In fact, it's this assurance of repetition and familiarity that makes music comforting to listeners. This idea is explored through Gestalt psychology – founded by Max Wertheimer, Wolfgang Köhler, and Kurt Koffka in early 20th century Germany. Gestalt psychology is a school of thought that emphasises perceiving and understanding psychological phenomena as organised wholes rather than the sum of individual parts. They observed that the human brain tends to automatically organise data, such as sound we receive, which enables us to recognise patterns with ease. Unfortunately, this innate skill isn't consciously translated into everyday life – an ability many desire when trying to find the car keys! We find order in disorder. Neurocognitive models of music perception address the formation of auditory patterns as a stage in the formation of meaning; the human mind grouping any object not in part but as a "unified" whole. Music is generally polyphonic (a combination of pure tones or frequencies mixed together in a harmonious manner), you don't hear notes individually but as groups or patterns, melodies or chords. Therefore, listening to music is like recognising a pattern rather than individual data points or notes. As a result, music becomes easier to recognise, remember and mentally replay. Earworms arise when these structured patterns are sufficiently simple for the brain to store, yet incomplete enough that the mind continues attempting to 'finish' them. So let's put your brain's ability to the test: below are some mathematical and musical patterns, just finish the sequence.

2, 4, 6, 8, 10, __

4, 7, 10, 13, __

1, 1, 2, 3, 5, __

C, D, E, F, G, A, __

clap, clap, tap, clap, __

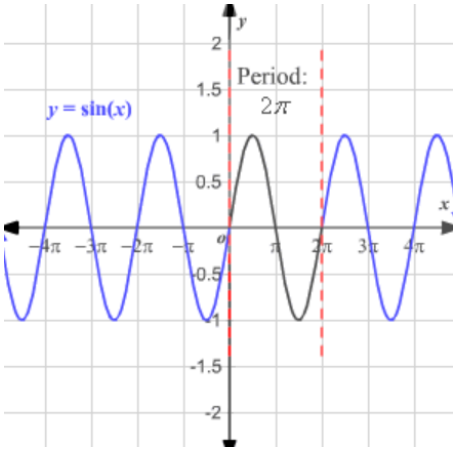
do, re, mi, fa, so, __

1, 3, 5, 7, 2, 4, __

Due to the wonders of Gestalt psychology, you should have been able to recognise those patterns fairly quickly – and may have even found them familiar. The strength of grouping data points as a unified whole is truly seen when you are able to unconsciously continue the pattern in your head, even when it's not necessary for you to do so. Earworms are a prime example of this, a melody persistently playing within the unconscious part of your mind. Whilst infuriating, earworms are a sign of impeccable intellectual health – proved by Gestalt psychology.

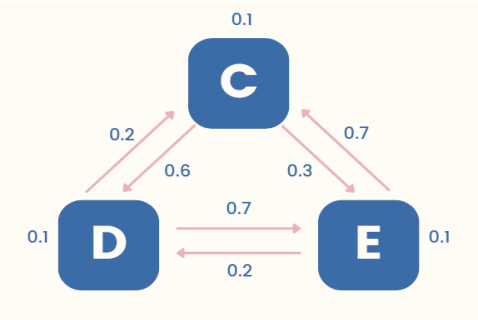
Repetition and loops:

Repetition is the basis of any good song and it's not just so that you know the words. The chorus provides a captivating hook, reeling listeners in with a catchy 4 bar phrase. Memorable sections are often repeated throughout the song to fulfil our expectations, rewarding listeners as they correctly anticipate the upcoming tune. David Huron explored the deep enjoyment familiarity and nostalgia brings listeners in his book 'Sweet Anticipation: Music and the Psychology and Expectation', and stated that "music is typically replete with repetition which permits expectation to operate". Fulfilled expectations are rewarding and Huron argues that repeated patterns can help to create those expectations. Repetition used widely within songs such as familiar structures or chord progressions can 'teach' your brain to look for similar patterns – even in unfamiliar music. When we hear something repeatedly, it's easier to replay and predict. This can eventually become internal, where your brain can 'hear' the next phrase before it happens. The loop can even continue without the music as your brain fights to complete the pattern – an earworm. Cognitive loops are triggered by ongoing cycles of prediction and recall. In mathematics, repetition is formalised through periodic functions, where a function repeats its values at regular intervals. Periodic functions, such as the sin graph, repeat after a certain time frame, or period. This can make it easier to understand and interpret the behaviour of familiar functions. The repetition of the sin graph (see below) is particularly comforting as it is predictable, pattern based and easy for the brain to process. Recognising repeating structures alludes to a sense of control whilst reducing mental effort and creates a feeling of mastery and understanding. When we become 'masters' of sequences, lyrics or tunes, the brain likes to revisit this knowledge. Understanding them becomes the foundation of earworms.



Expectation and surprise:

At the mathematical crux of what makes a song remarkable, stand out, and a true earworm, is probability. Catchy songs usually follow set patterns enough to feel familiar and meet our expectations, with just enough breaks to remain interesting. Breaking the chain of a sequence help to pique the listener's interest, ensuring they pay attention and leading to a stronger, more adept memory. Markov chains are a mathematical system, created by Russian mathematician Andrei Markov in the early 1900s. They use the probability of certain events to predict the transition from one state to another. They decide what comes next based on the state you're in now. The Markov property states that the future state of the process depends only on the present state and not on the past. This idea of 'memorylessness' helps the mind to focus on only the information it needs rather than getting bogged down by all past events and probabilities – we don't need your life story thanks! Listening to a song is similar to training a simple predictive model: after enough repetition, the brain learns which note comes next and begins to generate the sequence itself. Imagine a simple melody comprised of three notes: C, D and E. We as a listener are consistently attempting to recognise the sequence and predict what comes next. Markov chains can help us to achieve this by looking at the state we are in currently (the note C) and finding it's probable succession (D). For example, if the melody started with C, there is a 60% chance the next note will be D. If it started with D, there is a 70% chance the next note will be E, and if it started with E, there is a 70% chance the next note will be C. This is represented in the diagram below.



Whilst this can clearly display our data, if a melody contains all 7 notes or additional 'states', it can be harder to understand. To combat this, mathematicians use a transition matrix to organise these probabilities. With each row showing the current state and the columns representing the probability of each new phase.

$$T = \begin{array}{c} \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccc} C & D & E \end{array} \\ \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.2 & 0.1 & 0.7 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} \end{array}$$

As a result, even in silence, hearing a C may trigger the expectation of D, followed by E. The melody can continue internally, creating a self-sustaining loop – an earworm. However, Andrei didn't stop there. By multiplying transformation matrices we are able to ascertain an approximation of how probable each event is within the next 'state'. Below is a calculation showing how we can map the state (the note C being played) onto the next probable state by using the transition matrix.

$$v_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0.1 & 0.6 & 0.3 \\ 0.2 & 0.1 & 0.7 \\ 0.7 & 0.2 & 0.1 \end{bmatrix} = \begin{bmatrix} 1(0.1)+0(0.2)+0(0.7) \\ 1(0.6)+0(0.1)+0(0.2) \\ 1(0.3)+0(0.7)+0(0.1) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0.6 \\ 0.3 \end{bmatrix}$$

$$v_1 = \begin{bmatrix} 0.1 \\ 0.6 \\ 0.3 \end{bmatrix}$$

Markov chains are useful throughout the world of mathematics and have even been used to generate musical melodies by analysing what the average person predicts to come next in the sequence. Utilising this allows artists to choose the level of entropy their song has. Entropy is a mathematical measure of predictability, measured in bits. Whilst nursery rhymes sit on the lower-end of the scale, popular music generally has 'medium' entropy. This creates the perfect harmony of predictability, repetition and surprise; enticing listeners, making their song unforgettable.

Simple ratios:

Simple ratios play an important role in music, especially when it comes to harmony and how our brains perceive sound. Musical notes are essentially soundwaves with specific frequencies, and when these frequencies form simple ratios - such as 2:1, 3:2, or 4:3 - they tend to sound more pleasant together. For example, in songs like *"Let It Be"* by The Beatles, the use of harmonious chords based on simple frequency ratios creates a calm and balanced feeling. Similarly, *"Someone Like You"* by Adele uses clear, structured chord progressions that rely on these simple mathematical relationships, helping the melody feel emotionally powerful yet easy to follow. In an octave, one note has exactly double the frequency of another, which creates a sense of stability that most people find naturally satisfying. Our brains are drawn to these simple mathematical relationships because they are easier to process and recognise. When patterns are clear and predictable, like in simple ratios, they become more memorable, which helps catchy melodies stick in our minds. This connection between mathematics and music shows how our appreciation of sound is closely linked to our ability to recognise and enjoy simple patterns.

The sweet spot:

To have a successful song you need the correct balance of predictability and surprise. If music is too simple, listeners get bored however complexity can make a score hard to follow. Claude Shannon published "A Mathematical Theory of Communication" in 1948, which established the

basic results of information theory. Information theory is the mathematical study of quantifying, storing and transmitting information. Crucially, Shannon realised that in order to have a theory, communication signals must be treated in isolation from the meaning of the messages they transmit. This is illustrated within a correspondence between Victor Hugo, a French novelist, and his publisher following the publication of Les Misérables in 1862. Hugo sent his publisher a card simply reading "?" and received a card with just the symbol "!". Without context of Hugo's relationship with his publisher and the public, these messages are meaningless – to Hugo, they were everything. Similarly, a long, complete message in perfect French would convey little useful knowledge to someone who could understand only English. The amount of knowledge conveyed by a signal isn't directly related to the size of the message. Claude Shannon's information theory offers a surprisingly elegant way to understand why some music captivates us while others fall flat. Shannon described information as a measurable quantity using entropy. In information theory, entropy is defined as

$$H = -\sum p_i \log_2 p_i,$$

where p_i represents the probability of each event. In this context, each musical note can be treated as an 'event'. A melody that repeats the same note has zero entropy - it contains no new information - and quickly becomes dull. At the opposite extreme, a sequence in which every note is equally likely maximises entropy, producing something that feels random and almost impossible to follow. Most music lies between these extremes, balancing repetition with variation. This balance allows patterns to be recognised whist still introducing enough novelty to sustain interest – precisely the conditions under which earworms are most likely to occur. In principle, Shannon's entropy could be calculated for a piece of music by modelling notes as probabilistic events. A highly repetitive pop song would produce a lower entropy value, while a more complex or improvisational piece would yield higher entropy. However, real music involves dependencies between notes, meaning more advanced models using conditional probability would be required for accuracy. We can use the mathematical model for entropy to find the predictability of Twinkle, Twinkle, Little Star:

The melody mainly uses these notes (in C major):

- C, G, A, F, E, D

If we roughly count frequencies, we might get something like:

- C ≈ 0.30
- G ≈ 0.25
- A ≈ 0.15
- F ≈ 0.10
- E ≈ 0.10
- D ≈ 0.10
- Now calculate:
- $H = -(0.3\log_2 0.3 + 0.25\log_2 0.25 + 0.15\log_2 0.15 + 0.1\log_2 0.1 + 0.1\log_2 0.1 + 0.1\log_2 0.1)$

$$= -(0.3 \cdot (-1.737) + 0.25 \cdot (-2) + 0.15 \cdot (-2.737) + 0.1 \cdot (-3.322) + 0.1 \cdot (-3.322)) + 0.1 \cdot (-3.322))$$
$$= 0.521 + 0.5 + 0.411 + 0.332 + 0.332 + 0.332 = 2.428 \text{ bits}$$

Maximum entropy for 6 notes: $\log_2 6 \approx 2.58$

By finding the entropy of Twinkle, Twinkle, Little Star, we find that the entropy is relatively close to the maximum. This reflects a balance between predictability and variation, helping to explain why it's both memorable and easy to mentally replay.

Conclusion:

Earworms reveal that your brain is consistently engaged in pattern recognition, prediction and probabilistic reasoning. Music, far from just artistic, is deeply mathematical in its structure. Even your most annoying and repetitive thoughts are shaped by probability and numbers. An earworm isn't just a song – it's a perfectly engineered pattern, small enough to stay in your mind, strong enough to refuse to leave.

References:

1. BBC Bitesize (2023) *Why do songs get stuck in your head?* [Internet], Available from: <https://www.bbc.co.uk/bitesize/articles/zmbr8p3> [Accessed 01 March 2026].
2. Bustamante, N. (2025) *What is Gestalt Psychology? Theory, Principles, & Examples*. [Internet], Available from: <https://www.simplypsychology.org/what-is-gestalt-psychology.html> [Accessed 01 March 2026]
3. Geeks for Geeks (2025) *Markov Chain*. [Internet], Available from: <https://www.geeksforgeeks.org/machine-learning/markov-chain/> [Accessed 01 March 2026].
4. Ghosh, D., Sengupta, R., Sanyal, S. and Banerjee, A. (2017) Gestalt Phenomenon in Music: Which Frequencies Do We Really Hear?. In: Ghosh, D., Sengupta, R., Sanyal, S. and Banerjee, A. *Musicality of Human Brain through Fractal Analytics*. Springer Singapore, pp.145-163.
5. Huron, D. (2006) *Sweet Anticipation: Music and the Psychology of Expectation*. [Internet], Cambridge, MIT Press. Available from: ResearchGate <https://www.researchgate.net/publication/209436188_Sweet_Anticipation_Music_and_the_Psychology_of_Expectation> [Accessed 01 March 2026].
6. Klotz, S. (2023) *Musical Affordances and the Gestalt Legacy: enriching music perception*. [Internet], Available from: <https://www.researchgate.net/publication/376469225_Musical_Affordances_and_the_Gestalt_Legacy_enriching_music_perception> [Accessed 01 March 2026].
7. Markowsky, G. (2026) *Claude Shannon*. [Internet], Available from: <https://www.britannica.com/science/computer-science/Networking-and-communication> [Accessed 01 March 2026].
8. Schubert, E. (2025) *There's a Scientific Reason Why Some Songs Get Stuck in Your Head*. [Internet], Available from: <https://www.sciencealert.com/theres-a-scientific-reason-why-some-songs-get-stuck-in-your-head> [Accessed 01 March 2026].
9. Sekhon, R. and Bloom, R. (n.d.) *Introduction to Markov Chains*. [Inernet] Available from: <https://math.libretexts.org/Bookshelves/Applied_Mathematics/Applied_Finite_Mathematics_(Sekhon_and_Bloom)/10%3A_Markov_Chains/10.01%3A_Introduction_to_Markov_Chains> [Accessed 01 March 2026].