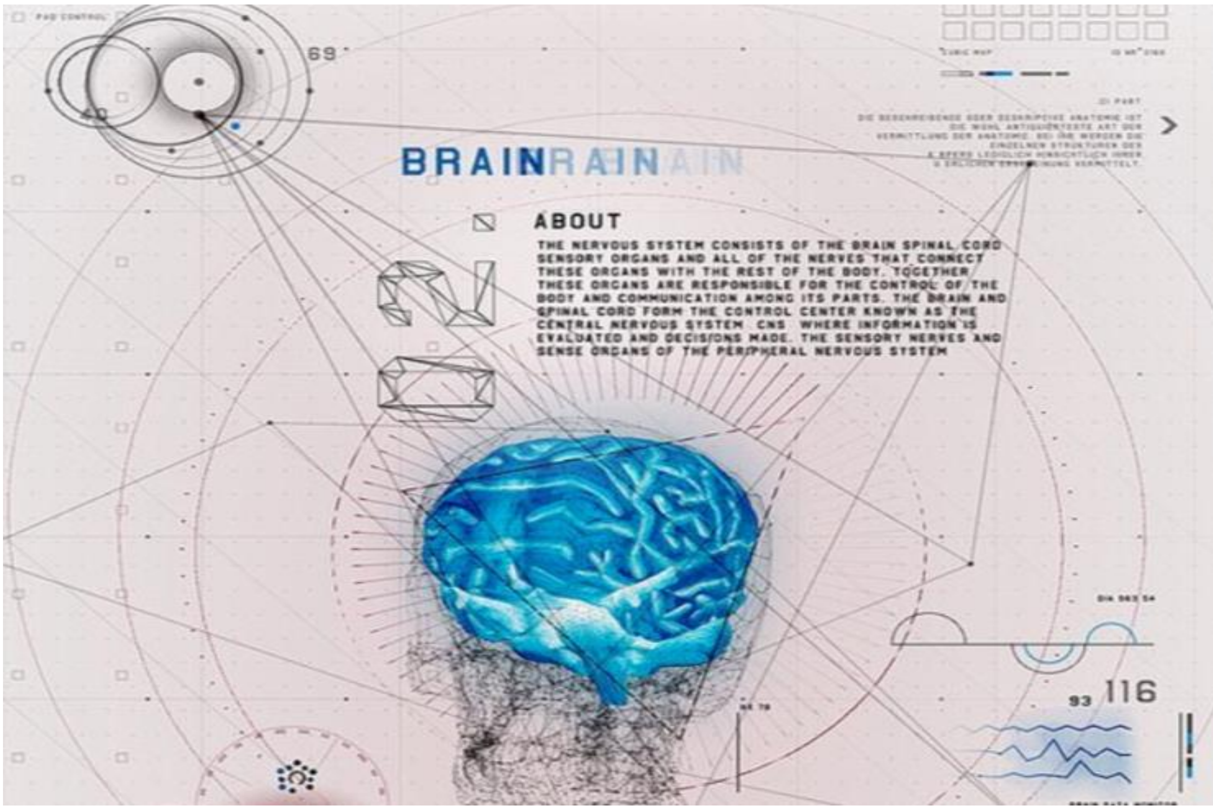
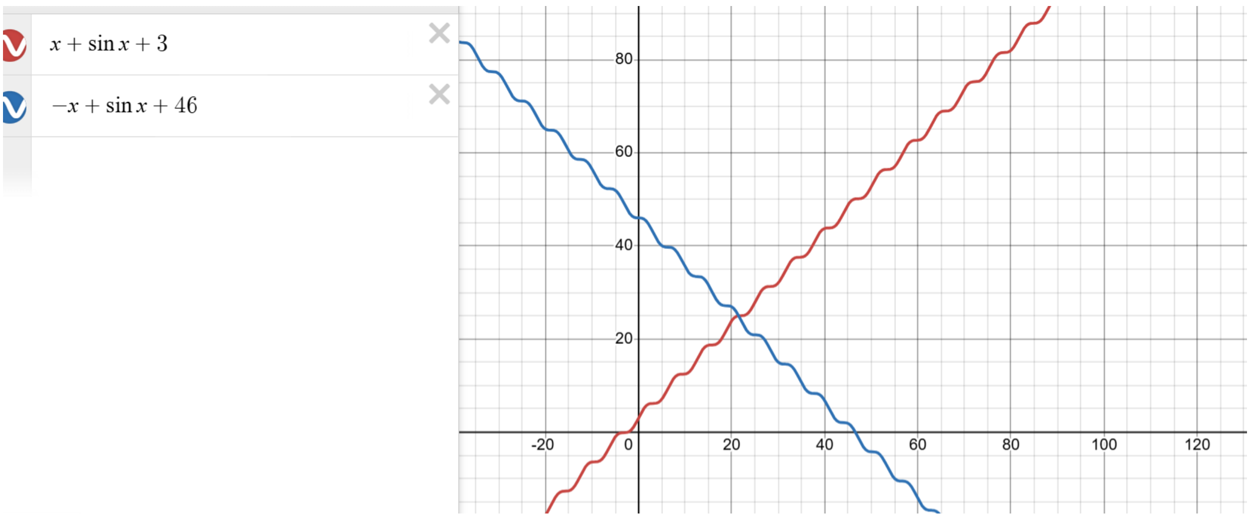


A window to the Brain

The eyes are the window to the soul.

By: Anushka Verma

April 2026



INTRODUCTION:

Surgeon: *The tumour came back. I removed everything I could see.*

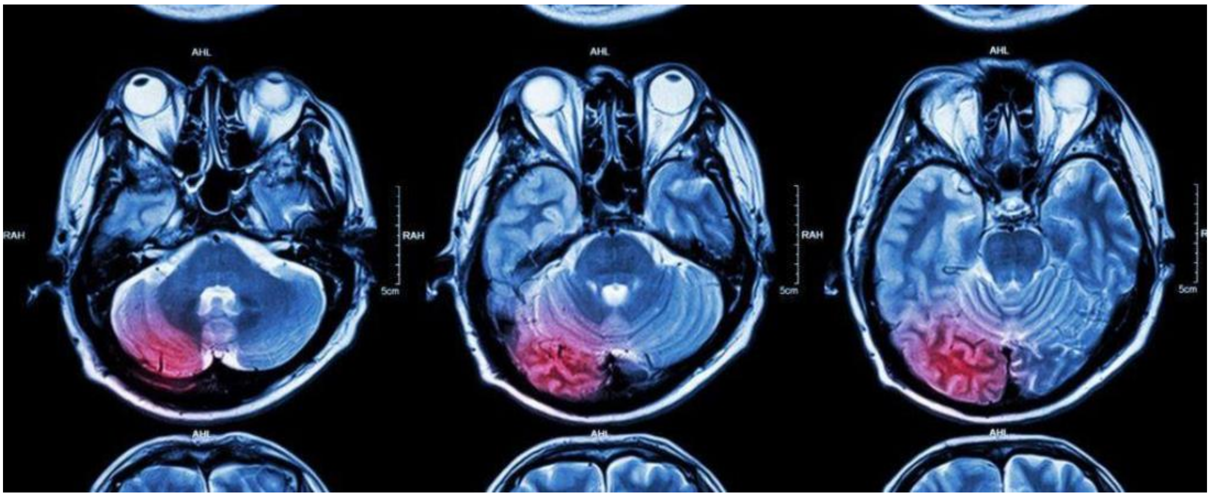
Mathematician: *You removed everything visible. The diffusion equation says cells were already 4cm beyond the boundary.*

Surgeon: *So surgery is pointless?*

Mathematician: *Surgery is blind. Mathematics isn't.*

Glioblastoma has a 15-month median survival. Because by the time the tumour is large enough to see, it has already moved on diffusing silently through brain tissue, invisible on every scan, unreachable by any scalpel.

The skull is a closed box. And we have been trying to open it with the wrong tools. That is what this essay is about.



Surgeon: *So what do you propose? You can't operate through an eye.*

Mathematician: *No. But you can measure through one. You just need the right mathematics to extract it.*

Surgeon: *And you have that mathematics?*

Mathematician: *We have had parts of it for two hundred years. We just didn't know we were building a brain scanner.*

What follows is the story of those. The conversation between surgeon and mathematician is fictional. The mathematics is not.

SECTION 2: THE EYE IS THE BRAIN TISSUE:

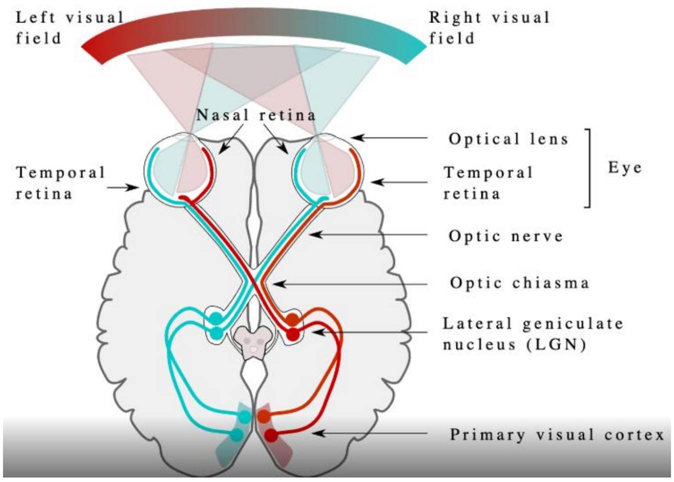
Surgeon: *You keep saying the retina is brain tissue. That sounds like a metaphor.*

Mathematician: *It isn't. Look at an embryology textbook. Week three of human development. The forebrain pushes two small pouches outward toward the surface. Those pouches become the eyes. The retina doesn't connect to the brain, it is brain, displaced forward.*

Surgeon: *That still doesn't explain how I read pressure from it.*

Mathematician: *Because the connection isn't just cellular. It's hydraulic.*

The optic nerve, the cable of roughly one million nerve fibres carrying visual information from the retina to the brain, is surrounded by a sleeve of tissue called the optic nerve sheath. This sheath is not empty. It is filled with cerebrospinal fluid, the same fluid that bathes and cushions the brain inside the skull.



This is the key fact. The CSF is continuous. The fluid around your brain and the fluid around your optic nerve behind your eye are the same body of liquid, connected directly. When pressure rises inside the skull, as it does when a tumour grows and occupies space, that pressure does not stay neatly contained behind the bone. It travels. Down the optic nerve sheath. All the way to the back of the eye.

Surgeon: *So the eye is pressurised by the brain, And that's visible?*

Mathematician: *Not directly. But it deforms structures that are visible. That's where the mathematics begins.*

*The structure that deforms is called the **lamina cribrosa**. It is roughly 1.5mm in diameter and less than 0.5mm thick, stretched across the back of the.*

Surgeon: *So you're telling me the entire diagnostic chain starts with a 0.5mm collagen plate bending by a few micrometres.*

Mathematician: *Yes.*

Surgeon: *That's impossibly small to measure.*

Mathematician: *That's what Joseph Fourier is for.*

SECTION 3: THE MATHEMATICS OF SEEING IN DEPTH: FOURIER AND THE LIGHT THAT KNOWS WHERE IT HAS BEEN:

Surgeon: *You want to measure a deflection of a few micrometres inside a living eye. Without touching it. Without surgery. With light. But light reflects off surfaces. It can't see through tissue.*

Mathematician: *Ordinary light can't. But ordinary light is a single frequency, a pure note. What we use is a chord. And Fourier is how we separate the notes.*

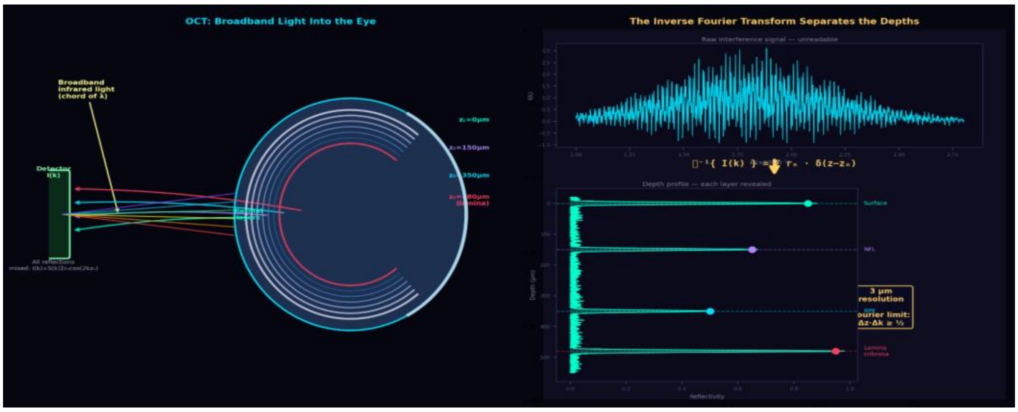
Joseph Fourier and the Signal Hidden in Everything

In 1822, the French mathematician Joseph Fourier published *Théorie Analytique de la Chaleur.* His central claim: any function no matter how jagged, complex, or irregular, can be written as an infinite sum of simple sine and cosine waves of different frequencies:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos\left(\frac{2\pi n x}{T}\right) + b_n \sin\left(\frac{2\pi n x}{T}\right) \right]$$

Fourier was solving heat flow. He could not have known he was also writing the operating principle of a brain scanner.

How OCT Turns Light Into Depth:



The detector measures the **interference pattern** between the returning light and a reference beam - a signal $I(k)$as a function of wavenumber $k = 2\pi/\lambda$:

$$I(k) = S(k) \sum_n r_n \cos(2k \cdot z_n)$$

Surgeon: *So the depth information is there. Just encoded.*

Mathematician: *Encoded as frequencies. Which means to decode it, you do exactly one thing.*

Surgeon: *Fourier transform.*

Mathematician: *Inverse Fourier transform. You run Fourier backwards.*

$$\tilde{I}(z) = \mathcal{F}^{-1}\{I(k)\} = \sum_{n} r_n \cdot \delta(z - z_n)$$

OCT sees finer, through a living eye, using only mathematics applied to light.

The Uncertainty Principle,Why 3 Micrometres and Not Finer

The resolution of an OCT scan is governed by the **Fourier uncertainty principle:**

$$\Delta z \cdot \Delta k \geq \frac{1}{2}$$

To achieve $\Delta z = 3\mu m$, you need a light source with bandwidth:

$$\Delta k \geq \frac{1}{2 \times 3 \times 10^{-6}} \approx 1.67 \times 10^{5}\ \mathrm{m}^{-1}$$

Surgeon: *That inequality. I've seen something like it before.*

Mathematician: *Heisenberg's uncertainty principle. $\Delta x \cdot \Delta p \geq \hbar/2$. The universe has one set of rules,we keep finding them in different disciplines.*

Surgeon: *So, the limit on seeing inside an eye is set by the same equation as the limit on knowing where an electron is.*

Mathematician: *Yes. Fourier wrote it in 1822. Heisenberg found it again in 1927. They were describing the same thing.*

What the Scan Actually Shows

An OCT scan of a healthy eye shows the retinal layers as a series of distinct horizontal bands - each one a different cell type, cleanly resolved. In a patient with raised intracranial pressure, something changes. The lamina cribrosa -that 1.5mm collagen plate is displaced posteriorly. Its position in the depth map shifts by 50 to 200 micrometres compared to a normal eye.

SECTION 4,THE INVERSE PROBLEM: RUNNING MATHEMATICS BACKWARDS:

Surgeon: *The lamina cribrosa has moved 120 micrometres posteriorly. Now what?*

Mathematician: *Now we ask: what pressure caused that deflection?*

Surgeon: *Can't you just read it off a graph?*

Mathematician: *You could, if the relationship were simple. It isn't. The optic nerve sheath is not a rigid tube. It's a pressurised elastic cylinder filled with fluid, attached to a flexible plate. To go from deflection to pressure, you need a physical model. And then you need to run it backwards.*

Surgeon: *Backwards.*

Mathematician: *That's what makes it hard.*

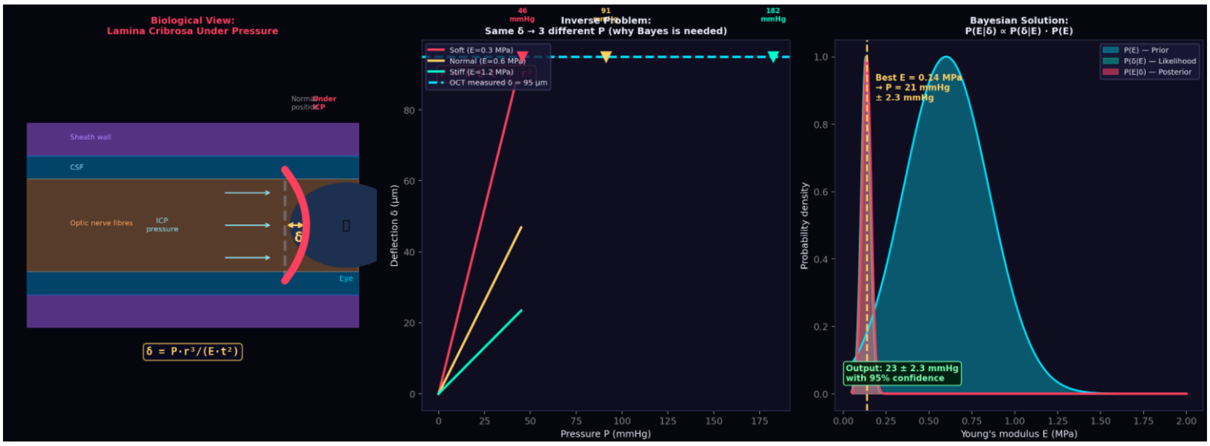
The Forward Problem, Easy

In mathematics, a **forward problem** is easy.

Given intracranial pressure P → predict lamina cribrosa deflection δ

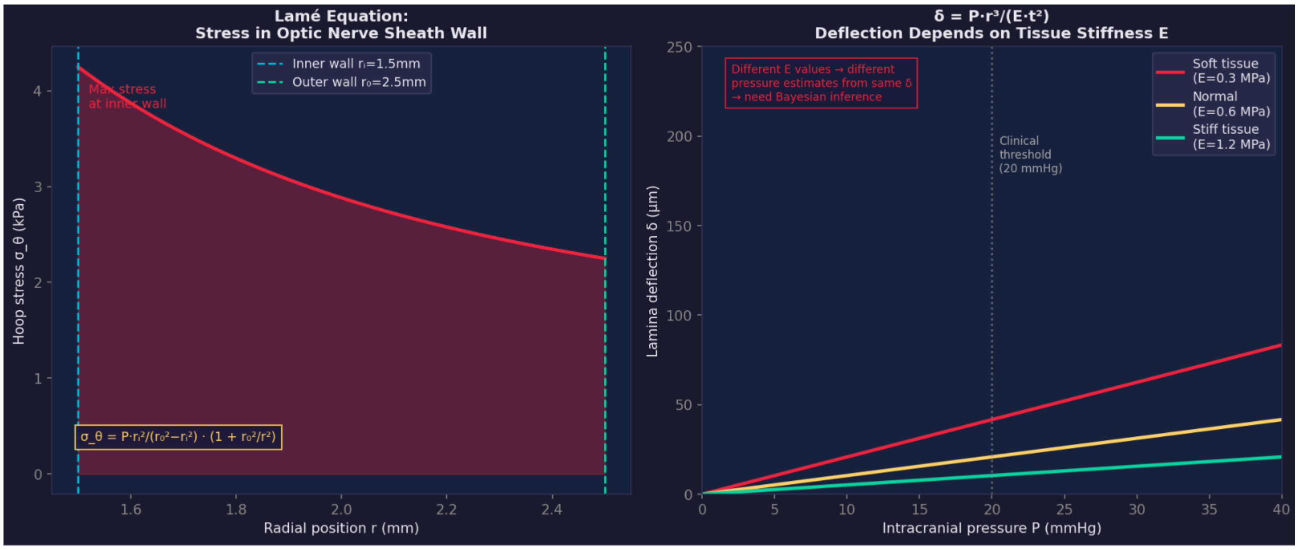
The physical model treats the optic nerve sheath as a thick-walled elastic cylinder under internal pressure. The stress in the wall is governed by the **Lamé equation** for a pressurised cylinder:

$$\sigma_\theta = \frac{P \cdot r_i^2}{r_o^2 - r_i^2}\left(1 + \frac{r_o^2}{r^2}\right)$$



For small deflections, this relationship is approximately linear:

$$\delta = \frac{P \cdot r^3}{E \cdot t^2}$$



Surgeon: *So, flip it. Solve for P.*

Mathematician: *That's the inverse problem. And flipping it is where everything gets dangerous.*

The Inverse Problem, Hard

The **inverse problem** is: given the measured deflection δ from the OCT scan, recover the pressure P that caused it.

Algebraically, rearranging:

$$\boxed{P = \frac{\delta \cdot E \cdot t^2}{r^3}}$$

This looks straightforward. But there is a critical problem hiding in it: **E, the Young's modulus, is not the same for every patient.**

The lamina cribrosa stiffens with age, with glaucoma, with diabetes. Its elastic properties vary between individuals by a factor of up to three. If you use the wrong value of E, your pressure estimate is wrong, and wrong in a way that is hard to detect because the deflection measurement itself was accurate.

Surgeon: *So, the mathematics works, but the biology breaks it.*

Mathematician: *The biology introduces uncertainty. Which is why the inverse problem requires more than algebra. It requires a framework for handling uncertainty in the model itself.*

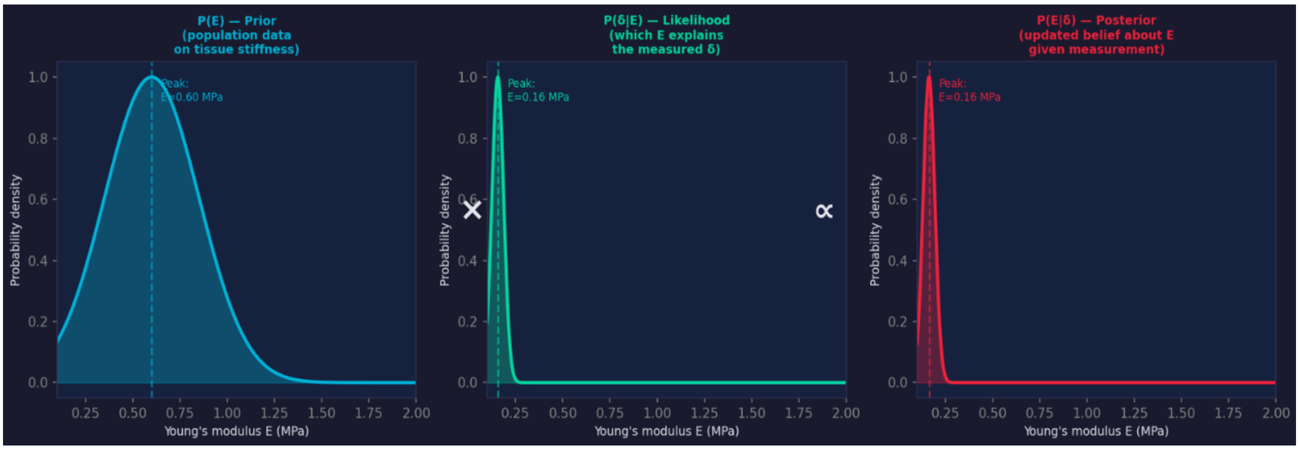
This is where **Bayesian inference** enters.

BAYESIAN INTERFERENCE:

Rather than treating Eas a single unknown value, the model treats it as a **probability distribution**,a range of plausible values weighted by how likely each one is given everything known about the patient. The pressure estimate becomes not a single number but a distribution:

$$P(E \mid \delta) \propto P(\delta \mid E) \cdot P(E)$$

The result is a pressure estimate with a **confidence interval**,not just "ICP is 22 mmHg" but "ICP is 22 mmHg ± 2.5 mmHg with 95% confidence." That confidence interval is itself clinical information: a narrow interval means the model is certain, a wide one flags that the patient's lamina may be unusually stiff or soft.

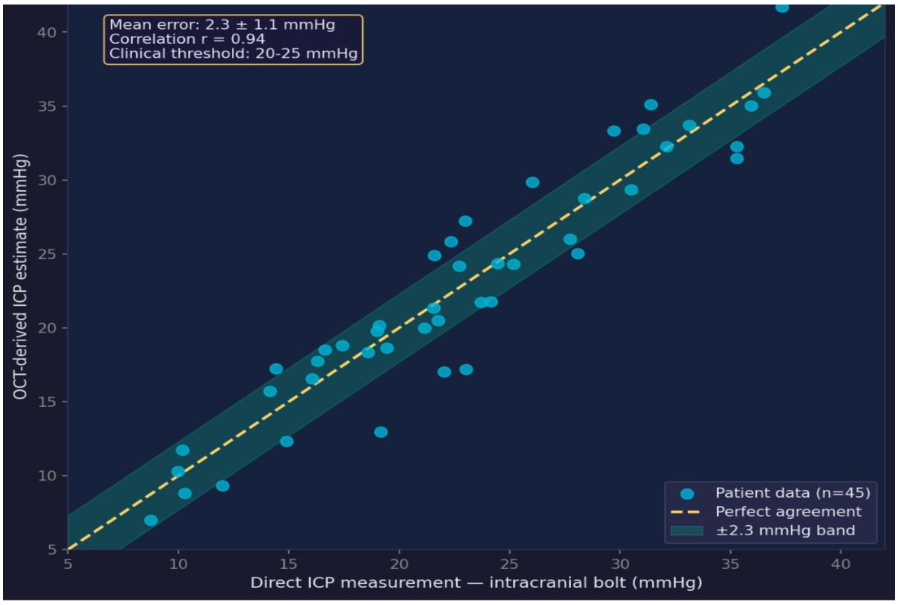


Does It Actually Work?

A 2023 study from University College London measured intracranial pressure in patients using both the gold standard direct method -an intracranial bolt, and the mathematical OCT pipeline simultaneously.

The result:

$$\left|\widehat{P_{OCT}} - P_{direct}\right| = 2.3 \pm 1.1 \text{ mmHg}$$



The OCT-derived estimate was within **2.3 mmHg** of the direct measurement on average. Clinical threshold for raised intracranial pressure is typically 20-25 mmHg,so an error of 2.3 mmHg is well within the margin needed for diagnostic confidence.

Surgeon: *So, the mathematics works.*

Mathematician: *The mathematics was always going to work. The only question was whether anyone would think to ask it the right question.*

CONCLUSION:

Surgeon: *And all of this from a retinal scan. So the retina isn't just a window into intracranial pressure. It's a window into the entire brain.*

Mathematician: *Into the entire central nervous system. The eye is not a diagnostic tool we invented. It is a diagnostic tool that was always there. We just didn't have the mathematics to read it.*

Surgeon: *Until now.*

Mathematician: *Until now.*

Fourier did not know he was building a retinal scanner. The engineers who developed OCT did not know they were solving an inverse problem in neurology. The ophthalmologists who first noticed papilledema in brain tumour patients did not know they were observing a fluid mechanics equation made visible.

Mathematics does not always know what problem it is solving. It builds its tools,the Fourier transform, the elastic cylinder model, Bayesian inference. It is what mathematics is: the language in which the universe writes its constraints, waiting for someone to read them.

Surgeon: *I have one last question.*

Mathematician: *Go ahead.*

Surgeon: *The patient from the beginning. The one whose tumour came back. If we had this pipeline six months earlier, would the mathematics have caught it?*

Mathematician: *The pressure would have been rising for weeks before the tumour was visible on any scan. The lamina cribrosa would have been deflecting. The OCT would have measured it. The inverse problem would have flagged it.*

Surgeon: *So yes.*

Mathematician: *Yes.*

Surgeon: *Then we have work to do.*

Mathematician: *We always did. We just needed the right equation.*

BIBLIOGRAPHY:

1. *Fourier, J.B.J. (1822). Internet Archive (full scan) · Gallica / Bibliothèque nationale de France*
2. *Huang, D. et al. (1991). Optical coherence tomography. Science, 254(5035), 1178–1181. → DOI: 10.1126/science.1957169 · PubMed · PMC full text*
3. *Soh, Z. et al. (2023). Non-invasive ICP estimation via OCT. British Journal of Ophthalmology. PMC9312000*
4. *Jonas, J.B. et al. (2015). DOI: 10.1016/j.preteyeres.2015.01.002 · PubMed*
5. *Burgoyne, C.F. (2011DOI: 10.1016/j.exer.2010.09.005 · PubMed · PMC full text*
6. *London, A. et al. (2013). PubMed*
7. *Tham, Y.C. et al. (2014). Global prevalence of glaucoma. Ophthalmology, 121(11), 2081–2090. → DOI: 10.1016/j.ophtha.2014.05.013 · PubMed · Full PDF (AAO)*

ACKNOWLEDGEMENT:

I would convey my deepest thanks to Sir Tom from Oxford or lovingly known as TomRocksMaths for reading and reviewing this essay. I really hope you like the concept I have chosen and worked upon. I am so grateful to you for giving us this opportunity.

I would like to thank the mathematicians, Fourier, Bayes, Lamé, who built tools without knowing what doors they were opening. And the clinicians who looked through the right window at the right moment.

THANKING YOU,

Anushka Verma.