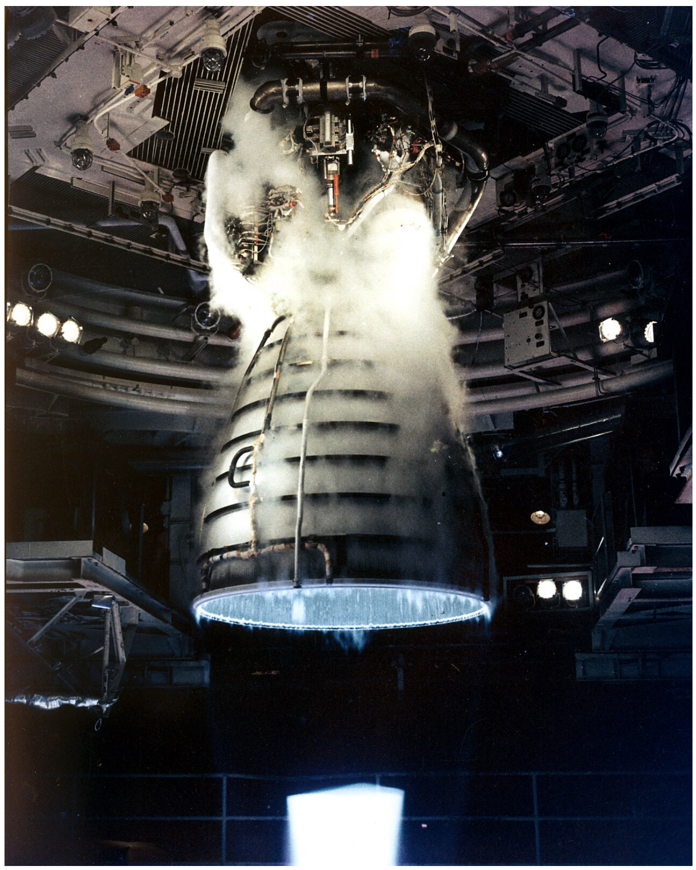
Fluid Dynamics of Rocket Engine Nozzles



Ali Juma

April 2026

Introduction

How do you cause a flame to reach supersonic speeds without any moving parts? The shape of a rocket engine nozzle is designed to exploit the properties of fluid dynamics in order to force the combustion gases out at these high speeds, making the engine efficient enough to actually lift the rocket off into space. Throughout this essay I will build up an understanding of fluid dynamics and explain how the most common shape, the de Laval nozzle, is designed, from first principles.

1 Water hoses

Imagine you have a short hose with water flowing through it at a constant flow rate. What happens if you pinch the hose at the end, reducing its radius and thus its cross-sectional area? Its velocity would, of course, increase. In fact, its cross-sectional area and velocity are inversely proportional:

$$A_1 v_1 = A_2 v_2$$

This is true firstly because it has a constant flow rate—in fact, the flow rate is $Q = Av$ in the first place, so it has to stay constant. But the other important reason is the fact that water is *incompressible*. This means that its density is effectively always the same—note that this is not true, however, for combustion gases coming out of a rocket.

Another not so obvious thing which happens is that pressure decreases. This might seem strange, but it happens because that pressure is converted to kinetic energy, and thus velocity. Although in school we are taught that $Pressure = \frac{Force}{Area}$, it helps to think of $Pressure = \frac{Energy}{Volume}$, which is equivalent. This pressure change can be explained by Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

Where ρ is density and v is velocity. The term $\rho g h$ stays the same in our situation (height doesn't change) so we can safely ignore it. The term $\frac{1}{2}\rho v^2$ refers to the *dynamic pressure*, which is the same as saying the kinetic energy per unit volume; you may recognise its similarity to the kinetic energy equation $KE = \frac{1}{2}mv^2$. The other important term is P, referring to the *static pressure*, which is the simple water pressure you would measure with a pressure gauge.

When you pinch the hose, increasing its velocity, $\frac{1}{2}\rho v^2$ increases, since it has v^2. Because the sum is constant, P must decrease—this is why (static) pressure decreases. In fact, this static pressure is converting into dynamic pressure, which is what increases the velocity. If we pinched the hose hard enough, we could reduce this static pressure all the way down to the outside atmospheric pressure as it leaves the end, extracting the maximum velocity from it.

You can see how reducing the static pressure of the water to the outside pressure gives it maximum efficiency—extracting all of its stored pressure energy into kinetic energy—this is a very important principle which also carries over to rocket engine nozzles.

2 Supersonic

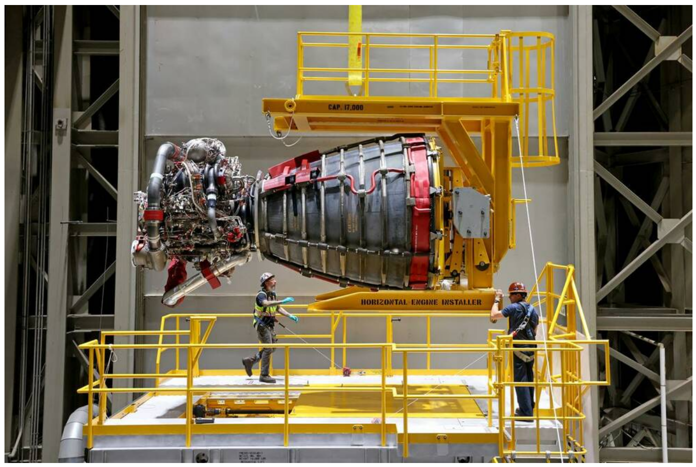


Figure 1: An RS-25 engine being attached to the Artemis II rocket. It has a de Laval shaped nozzle.

I have chosen the everyday example of a water hose since it links nicely with rocket nozzles, having many of the same principles. However, rocket nozzles have a few important differences, the largest being that they feature *supersonic* flows of gas. At this point Bernoulli's equation completely breaks down and we need a better equation.

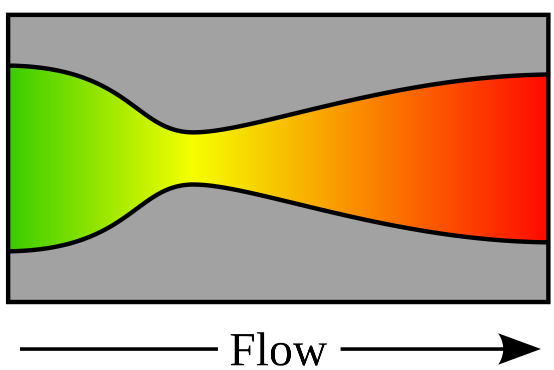


Figure 2: Cross section of de Laval nozzle.

The above diagram shows a de Laval (bell-shaped) nozzle. The fuel and oxidiser mixture have already been mixed together on the left side and start flowing similarly to our water hose example, at subsonic speeds. The part where the nozzle is converging and the area is getting smaller acts like our pinch, increasing the velocity and decreasing the pressure. By the throat of the nozzle, the gases are moving at Mach 1, or the speed of sound.

After this point, with the combustion mixture at supersonic speeds, fluid dynamics behaves quite differently. You might expect that the larger cross-sectional area after the neck would decrease the velocity, but actually at supersonic speeds the complete opposite thing happens—velocity increases. I will build up the mathematical reason why the fluid behaves like this from the basics, arriving at the more complex equation

$$dP\left(1 - M^2\right) = \rho v^2 \left(\frac{dA}{A}\right)$$

The *Mach number* is a unit of speed relative to the speed of sound (e.g. Mach 3 is three times the speed of sound) and features in many relevant equations. It can be expressed as $M = \frac{v}{c}$ where c is the speed of sound (not to be confused with light) and v is velocity. Thus all $M > 1$ are supersonic speeds.

Another difference compared to our water hose example is the fact that the combustion gases are a compressible fluid, especially at mach numbers $M > 0.3$, so the density is not constant. Because of this, our simpler equations which assume constant density will no longer work, and we have to rely on certain other conservation laws to explain the fluid dynamics in rocket engine nozzles.

Firstly, due to conservation of mass, the flow rate of mass $\dot{m}$ is constant. We can express this as:

$$\dot{m} = \rho A v = constant$$

A is cross-sectional area, ρ is density and v is velocity all at a certain point along the nozzle. All of these variables change along the nozzle, but we know that their product must be a constant. If it wasn't, then mass would have to be created or destroyed, violating the conservation of mass. Differentiating both sides with the product rule we get:

$$vA\,d\rho + \rho A\,dv + \rho v\,dA = 0$$

Divide through by $\rho v A$

$$\frac{d\rho}{\rho} + \frac{dv}{v} + \frac{dA}{A} = 0 \tag{1}$$

In this equation, $d\rho$, dv and dA just mean very small infinitesimal changes in ρ, v and A, and it shows that any change in one affects the others.

Newton's second law leads to the famous equation $F = ma$, but this can also be written in another form. Imagine a very small slice of fluid along the length of the nozzle, with the full cross sectional area A, and having an infinitesimally small length dx. This slice would thus have the mass $\rho A\,dx$, and the only force pushing it would be the pressure difference between its front and back face. Remember that pressure acts in all directions, and $Pressure = \frac{Force}{Area}$, so the force PA on the left face would push it rightwards and the force $(P + dP)\,A$ on the right face would push it leftwards. Therefore the resultant force on the slice, representing any point along the fluid, would be

$$F = PA - (P + dP)\,A$$

$$F = -dP \cdot A \tag{2}$$

From the mass $\rho A\,dx$ we also know that

$$F = \rho A\,dx \cdot a \tag{3}$$

Equating (2) and (3)

$$-dP \cdot A = \rho A\,dx \cdot a \tag{4}$$

The acceleration of the slice of fluid $a = \frac{dv}{dt}$ by definition, but this depends on time, so using the reverse chain rule

$$\frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt}$$

And $\frac{dx}{dt} = v$ so

$$a = v\frac{dv}{dx} \tag{5}$$

Substituting (5) into (4) we get

$$-dP \cdot A = \rho A\,dx \cdot v\frac{dv}{dx}$$

Cancelling A and dx from both sides

$$dP = -\rho v\,dv \tag{6}$$

This equation is another expression of Newton's second law, and easier to work with in our case.

Lastly we need the equation for the speed of sound, c, which for an ideal gas with no heat transfer is

$$c^2 = \frac{dP}{d\rho}$$

$$dP = c^2 d\rho \tag{7}$$

Where P is pressure and ρ is density anywhere in the gas. We now only need to put everything together. Equating (6) and (7)

$$d\rho = \frac{-\rho v dv}{c^2}$$

Substituting into (1):

$$\frac{-v\,dv}{c^2} + \frac{dv}{v} + \frac{dA}{A} = 0$$

Rearranging

$$\frac{dv}{v}\left(1 - \frac{v^2}{c^2}\right) = -\frac{dA}{A}$$

Substituting mach number $M = \frac{v}{c}$

$$\frac{dv}{v}\left(1 - M^2\right) = -\frac{dA}{A} \tag{8}$$

Dividing both sides of (6) by $-\rho v^2$ to get

$$\frac{dv}{v} = -\frac{dP}{\rho v^2}$$

and finally substituting this $\frac{dv}{v}$ into (8)

$$dP\left(1 - M^2\right) = \rho v^2 \left(\frac{dA}{A}\right) \tag{9}$$

And that's how you derive the equation for the area-velocity relation in a rocket engine nozzle. A lot more complicated than $A_1 v_1 = A_2 v_2$, that's for sure.

From this equation the term $(1 - M^2)$ almost jumps out at you since it perfectly explains why at supersonic speeds, increasing area actually decreases pressure. At subsonic speeds, for $M < 1$, $(1-M^2)$ is positive so dP and dA have the same sign, so the beginning of the de Laval nozzle gets smaller to decrease pressure. But after the neck, when $M > 1$, the nozzle gets *bigger* to decrease pressure.

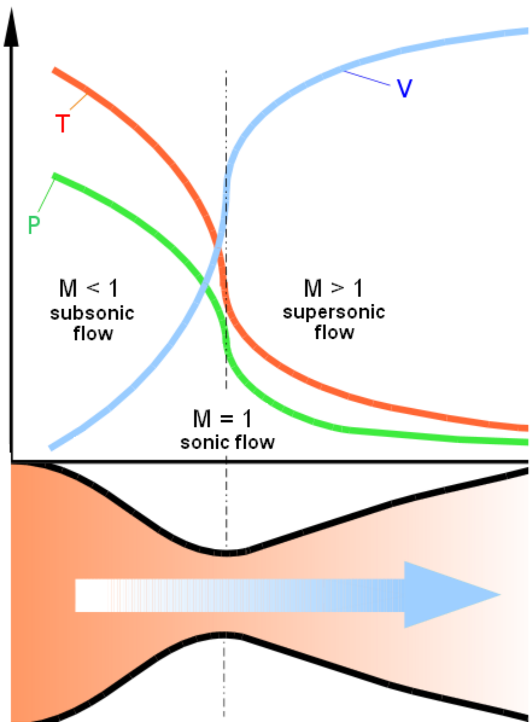


Figure 3: Velocity, pressure and temperature throughout a de Laval nozzle.

Substituting (6) into (9) we can explain velocity as well:

$$\frac{dv}{v}\left(1 - M^2\right) = -\frac{dA}{A}$$

When $M < 1$ before the neck, the cross-sectional area gets smaller to increase the velocity, and when $M > 1$ after the neck the area gets bigger to *also* increase the velocity of the combustion gases. Also, we know that the throat is *exactly* Mach 1 because $dA = 0$ there—it has the minimum cross-sectional area—so either $dv = 0$ or $M = 1$.

In an optimal de Laval nozzle pressure is decreased all the way to atmospheric pressure at the end, extracting the maximum possible velocity and therefore thrust. Since rockets have to operate at a wide range of altitudes, engineers often design a compromise to work well at all altitudes.

Conclusion

I hope you may appreciate the wonder of rocket engines as much as I did writing this. The applications of maths in real-life are fascinating and are what makes maths so interesting, in my opinion. As much as I wanted to include more, such as how temperature and thermodynamics play a role, this essay would unfortunately get too long.

Thank you for reading my essay.