An Infinite Loop in Your Head: The Math behind Earworms

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A Loop You Never Chose

There's a song in your head right now. It's not one you chose, not one you even like. It's just one that appeared and refused to leave. It loops, repeating the same few seconds over and over again, and it feels as if your brain has been hijacked by a broken playlist. This is what's called an earworm - that annoying (but catchy) bit of music that gets stuck in your head and plays on repeat.

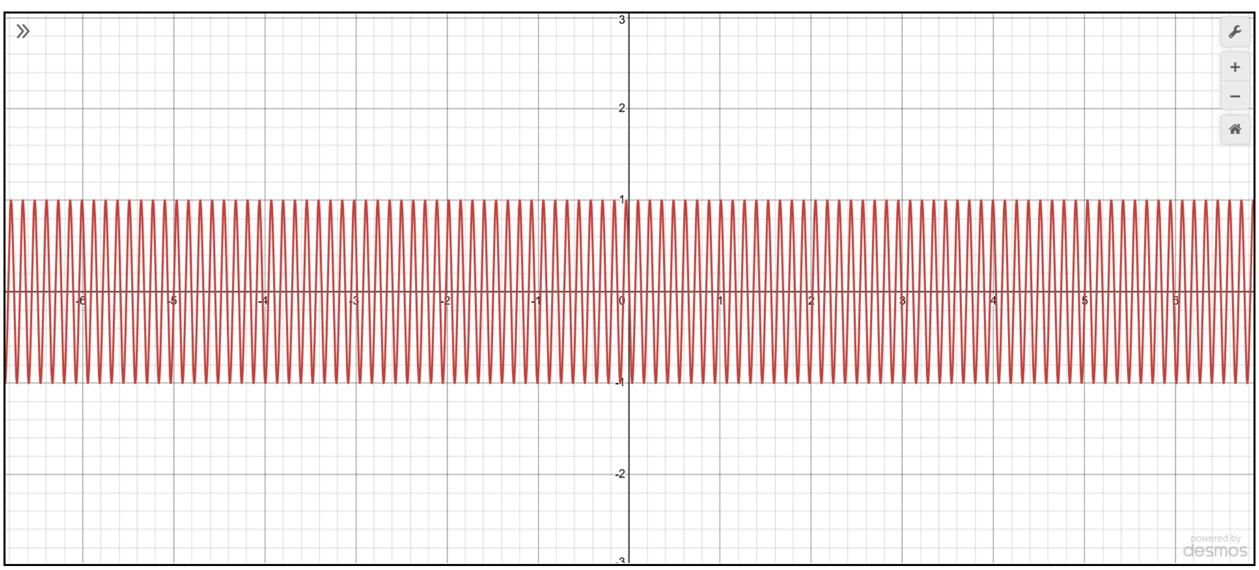
What if this isn't just a glitch? Your brain is simply doing what it does best. It recognises a pattern, learns it, and then runs it over and over again.

Music as a Function

Music is not just random sound - it has structure. Every note corresponds to a frequency and every melody is a sequence of these frequencies.

Mathematically, this can be seen as a periodic function, i.e. something that repeats and that can be described.

For example, the note A (which is commonly used for tuning) has a frequency of 440 Hz. This means that the sound wave completes 440 cycles per second. In its simplest form, it is a sine function described as $y = \sin(2\pi \times 440x)$



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This represents a pure tone - a pure tone is a single frequency with no additional complexity. Here, the spacing of the waves determines the pitch we hear.

However, real music is never this simple. When a note is played, it is accompanied by various additional frequencies. These frequencies are called harmonics - harmonics are integer multiples of the fundamental (original) frequency. These harmonics combine to form a richer sound. This means that even a single note is actually a superposition of multiple waves and what is perceived as one note is a structured mathematical pattern. When these patterns are simple enough (i.e. repetitive and predictable), our brain does not just hear them. It learns them and replays it to us - this forms earworms.

Breaking Music Apart

While a note can be modelled as a combination of sine waves, there is a deeper idea being used - the Fourier Transform. This states that any complex sound wave can be broken down into a sum of simple sine and cosine waves, each with different frequencies and amplitudes.

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-i\omega t}$$

Therefore, even the most complicated piece of music can be represented in the form of a collection of pure tones. This is how digital music works - songs are stored as numerical data describing waves.

Power of Repetition

The most prominent feature of an earworm is repetition. It is a short segment of music that repeats with precise consistency. In mathematics, this feature is called periodicity - a function (f) is periodic if there exists a value (T) such that $f(x+T) = f(x)$.

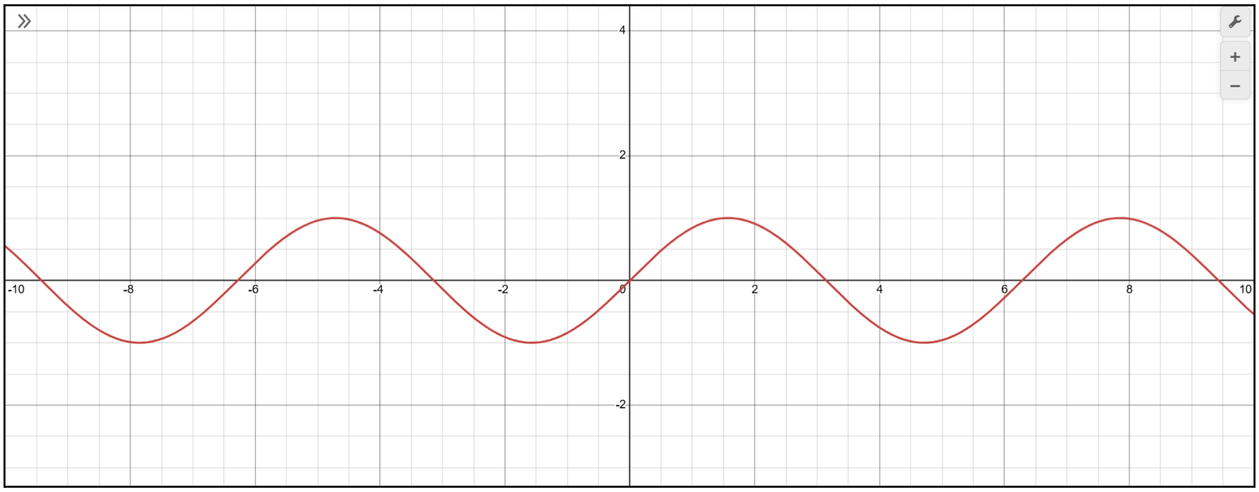
Catchy melodies behave in this way. They form loops which are easy to detect and predict. Once your brain identifies this loop, it doesn't need the rest of the music. It isolates it and keeps repeating it.

Why Short Loops Work Best

Not all patterns which repeat become earworms. The brain prefers patterns that are short enough to memorise quickly, but not so short that they become trivial or boring. This is because of cognitive load - the amount of information your brain can process at once. If a musical loop is too long, it becomes difficult to store. If it is too short, it becomes boring.
The most effective earworms lie in an optimal range:
1. Short enough to repeat easily
2. Complex enough to stay interesting

Predicting the Next Note

But not every loop becomes an earworm. A perfectly predictable signal, like a pure sine wave, i.e. $y = \sin(x)$, is too simple to hold attention.



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There is no uncertainty and no need to think. Earworms are a balance between predictability and surprise.

As you listen to music, your brain is constantly making predictions - what note will come next?

These predictions can be thought of probabilistically. After one note, there is a set of possible next notes, each with a certain likelihood or chance. This is similar to a Markov process - a model where the future state depends solely on the present state and is independent of the past.

The most "catchy" melodies are those where these probabilities are structured - not random, but not too obvious. They give your brain just enough information to learn the pattern, and just enough variation to keep it guessing.

Measuring Surprise in Music

Entropy measures how unpredictable something is - by definition, it measures the uncertainty, randomness, or disorder associated with a set of data or events.

- Low entropy = very predictable, makes it boring
- Medium entropy = balanced, makes it engaging
- High entropy = completely random, makes it chaotic

Earworms are usually in the middle ground (medium entropy). They are predictable enough for your brain to learn, but surprising enough to keep your attention.

When Algorithms Agree With Your Brain

This idea extends beyond your mind. Modern music platforms (like Spotify) analyse millions of listening patterns to determine which songs people replay the most. While they do not directly measure "earworms", they identify similar features. This includes repetition, structure, memorability and many others.

In a way, both your brain and these algorithms are solving the same problem. They detect patterns that persist. One does it through neurons while the other does it through data. But both arrive at the same conclusion - some songs are simply better at looping.

Pattern Recognition in Machine Learning

Modern music recommendation systems do more than just replay counts. They use machine learning algorithms to detect patterns in melody and structure.

These systems analyse features such as -

- Repetition
- Tempo
- Pitch variation
- Listener behaviour

Interestingly, these are the very features that make songs memorable to humans. Both your brain and these algorithms are performing pattern recognition at the core, identifying structures that persist.

Earworms as Iterative Systems

From a mathematical perspective, an earworm can be thought of as an iterative process. You start with a small input (here, a short musical phrase). The brain applies a rule to replay the pattern. Each repetition becomes the input for the next iteration, creating a loop.

This is how simple mathematical rules are able to generate complex behaviour when repeated several times.

The Loop Never Ends

So the next time a song gets stuck in your head, your brain has recognised a pattern, learned its structure and has compressed it into a simple rule.

And now, it just keeps iterating over and over again.

An earworm - an infinite loop, running quietly in your head.