Behind the Doors
Why Switching Wins

The Mathematics of the Monty Hall Problem

Picture this. You are on a game show, sweat on your palms, three identical doors in front of you. Behind one is a brand-new car. Behind the other two, goats. You pick Door 1. The host, who knows exactly what is behind every door, swings open Door 3 to reveal a rather unimpressed goat. Then he smiles and asks: "Want to switch to Door 2?" Most people say it does not matter. Two doors, one car. Surely it is 50-50 now? Switch, stay, flip a coin. Same odds.

They are wrong. And the margin is not small. Switching wins the car twice as often as staying. If you stay, you win roughly 1 in 3 times. If you switch, it is 2 in 3. This is the Monty Hall Problem, and it has been making people furious since 1990, when columnist Marilyn vos Savant published the correct answer and received nearly 9,000 letters disagreeing with her, many from people with PhDs.

The key is to think carefully about what you knew before the host opened anything. When you first picked Door 1, there was a 1-in-3 chance the car was behind it, and a 2-in-3 chance it was behind one of the other two doors. That 2-in-3 probability did not vanish when Door 3 opened. It got concentrated.

Here is the crucial detail: the host is not opening a random door. He knows where the car is, and he will never open that door. His action is deliberate and constrained. So when he opens Door 3 to show a goat, he is telling you something: all that 2-in-3 probability that used to be spread across Doors 2 and 3 now sits entirely behind Door 2.

Think about it from both scenarios. If you originally picked the car (probability 1/3), the host opens either remaining door, and switching loses. But if you originally picked a goat (probability 2/3), the host must open the other goat door, leaving the car behind the remaining door, and switching wins. Since you are wrong with your first pick 2/3 of the time, switching wins 2/3 of the time.

If that still feels slippery, scale it up. Imagine 100 doors, one car, 99 goats. You pick Door 1. The host now opens 98 doors, all goats, leaving just your door and Door 47. Would you switch? Of course. Your original pick had a 1-in-100 shot at being right. The host's 98 carefully chosen reveals have funnelled all the remaining probability into Door 47. The three-door version follows exactly the same logic. The numbers are just close enough together to fool our intuition.

This is a worked example of conditional probability: the idea that probabilities update when you receive new information. The formal machinery behind it is Bayes' Theorem. Apply it here:

> P(Car in Door 1 | Monty opens Door 3)
>
> = [P(Monty opens 3 | Car in 1) x P(Car in 1)] / P(Monty opens 3)
>
> P(Car in 1) = 1/3
>
> P(Monty opens 3 | Car in 1) = 1/2 (random choice between doors 2 and 3)
>
> P(Monty opens 3 | Car in 2) = 1 (must open door 3)
>
> P(Monty opens 3 | Car in 3) = 0 (would reveal the car)
>
> Total P(Monty opens 3) = (1/2)(1/3) + (1)(1/3) + (0)(1/3) = 1/2
>
> Therefore: P(Car in 1 | Monty opens 3) = [(1/2)(1/3)] / (1/2) = 1/3
>
> Door 2 gets the remaining 2/3.

Our brains struggle with this because we treat symmetry as fairness. Two doors, one car, feels like 50-50. But symmetry in the number of doors does not mean symmetry in the information. The host's knowledge breaks the symmetry invisibly, and our gut instinct misses it entirely.

This matters beyond game shows. Conditional probability underlies medical testing (why a positive result on a rare disease often still means you are probably fine), spam filters, weather forecasting, and machine learning. Every time a system updates its confidence based on new evidence, it is doing Monty Hall reasoning at scale. Getting it wrong leads to real mistakes: over-diagnosed patients, mispriced risks, bad decisions made with a false sense of certainty.

The most striking part of the Monty Hall story is the reaction it provoked. Vos Savant's column ran in Parade magazine in 1990, and the backlash was extraordinary. Thousands of readers, including mathematics professors, wrote in insisting she was wrong. Some were condescending. Most were simply baffled. She ran simulations. She explained it multiple ways. She was right every time. The problem has since been verified computationally millions of times over.

So next time someone tells you maths is about memorising formulas, remember the Monty Hall Problem: three doors, one car, and a lesson about how deeply our instincts can mislead us. The maths does not just give a different answer from intuition. It explains exactly why intuition fails, and what to do instead.

Always switch.