Title:Bloons,Bananas and mathematical survival

Introduction:

Why does round 63 in Bloons TD 6 feel so much harder than round 40? Why does placing one Banana Farm early on sometimes feel like a mistake, but later become the smartest move in the game? These are not just questions about game strategy. They are questions about mathematics.

At its heart, Bloons TD 6 is a game about making decisions under pressure. Every round introduces a new challenge, every tower has a cost, and every dollar spent has an opportunity cost. The player is constantly balancing defence against income, short-term safety against long-term growth, and certainty against risk. That is what makes the game so interesting mathematically: it is not only about surviving bloons, but about solving an optimisation problem.

Normal Bloons

Type	Health	Cash	Children	Speed Relative to Red Bloon	RBE	Total Cash	Total Cash with 0-0-3 Village	Abilities
Red	1	1	None	1	1	1	1.6	
Blue	1	1	Red	1.4	2	2	3.2	
Green	1	1	Blue	1.8	3	3	4.8	
Yellow	1	1	Green	3.2	4	4	6.4	
Pink	1	1	Yellow	3.5	5	5	8	
Black	1	1	2x Pink	1.8	11	11	17.6	Immune to explosions
Purple	1	1	2x Pink	3.0	11	11	17.6	Immune to fire, energy, and plasma
White	1	1	2x Pink	2.0	11	11	17.6	Immune to freezing
Lead	1	1	2x Black	1	23	23	36.8	Immune to sharpness (Darts)
Zebra	1	1	Black, White	1.8	23	23	36.8	Immune to explosions and freezing
Rainbow	1	1	2x Zebra	2.2	47	47	75.2	
Ceramic	10	1	2x Rainbow	2.5	104	95	152	
M.O.A.B.	200	1	4x Ceramic	1	616	381	609.6	
B.F.B.	700	1	4x M.O.A.B.	0.25	3164	1525	2440	
Z.O.M.G.	4000	1	4x B.F.B.	0.18	16656	6101	9761.6	
D.D.T.	400	1	4x Ceramic +C, R	2.75	816	381	609.6	Immune to explosions and sharpness Has Camo
B.A.D.	20000	1	2x Z.O.M.G., 3x D.D.T.	0.18	55760	13346	21353.6	Immune to slowing and knockback

Fortified Bloons

Type	Health	Cash	Children	Speed Relative to Red Bloon	RBE	Total Cash	Total Cash with 0-0-3 Village	Abilities
Lead +F	4	1	2x Black	1	26	23	36.8	Immune to sharpness (Darts)
Ceramic +F	20	1	2x Rainbow	2.5	114	95	152	
M.O.A.B. +F	400	1	4x Ceramic +F	1	856	381	609.6	
B.F.B. +F	1400	1	4x M.O.A.B. +F	0.25	4824	1525	2440	
Z.O.M.G. +F	8000	1	4x B.F.B. +F	0.18	27296	6101	9761.6	
D.D.T. +F	800	1	4x Ceramic +C, R, F	2.75	1256	381	609.6	Immune to explosions and sharpness, has Camo
B.A.D. +F	40000	1	2x Z.O.M.G. +F 3x D.D.T. +F	0.18	98360	13346	21353.6	Immune to slowing and knockback

The rules of the system

The structural integrity of *Bloons TD 6* rests upon a foundation of deterministic logic and a rather brutalist application of macroeconomic principles. Rather than a chaotic influx of enemies, the game deploys Bloons in discrete, pre-programmed sets, forcing the player to engage in a high-stakes exercise of **Constraint Satisfaction**. We see this most clearly in the Red Balloon Equivalent (RBE) metric, a summation defined as $RBE = \sum(ci \times Li)$, which dictates the absolute minimum workload a defensive system must perform. However, the game is rarely a simple matter of raw power. Each Bloon property acts as a mathematical hurdle; the

Lead Bloon, for instance, functions as a binary gate that returns a **`Null`** value for any damage type within the "Sharp" or "Laser" sets. This necessitates a diversified portfolio of towers. By Round 59, the system presents the "Camo-Lead" problem—a specific intersection of two distinct sets of invulnerabilities. If the player's defensive algorithm hasn't accounted for this specific variable, the game's state-machine reaches an abrupt and unforgiving termination.

The system's true sophistication, however, is revealed in the transition to "Freeplay" after Round 80. Here, the game abandons linear progression for a regime of hyper-inflation and artificial scarcity. The health of MOAB-class Bloons begins to scale according to the function $H(R) = Hb \times [1 + 0.05(R - 80)]$, while the "Central Bank" of the game executes a series of aggressive interest rate hikes on player income. To illustrate this "Economic Squeeze," one can calculate the **Efficiency Gap** (Eg), or the ratio of Bloon Health to Income per pop. At Round 80, with an income of $0.20 per pop and a MOAB health of 200, the ratio sits at a manageable 1,000:1. Yet, by Round 120, the health has tripled while the income has plummeted to a measly $0.02. This drives the Efficiency Gap to 30,000:1.Mathematically, the burden on the player's towers increases by 2,900% in a mere forty rounds. This is the **Law of Diminishing Returns** in its most distilled form. As the **Marginal Utility** of every dollar spent—the additional popping power gained per unit of currency—reaches an asymptotic limit, the player is forced into a frantic, recursive battle against a system designed to ensure that the math eventually wins. You aren't just playing a tower defense game; you are fighting an uphill battle against a collapsing currency and an enemy whose durability is effectively decoupled from your ability to pay for its destruction.

1.Systematic Scaling and RBE

The Red Bloon Equivalent (RBE) represents the total damage required to fully clear a wave
It is defined as the summation of all Bloon types (c) multiplied by their total health (L), including all nested children:

$$RBE = \sum (ci \times Li)$$

- ci is the count of a specific Bloon type
- Li is the total layers (health) of that Bloon type

2.MOAB-Class Health Scaling

Post-Round 80, the game institutes a linear scaling factor. For any round R where $81 \leq R \leq 100$:

$$H(R) = Hbase \times (1 + 0.05(R - 80))$$

Note: ***The growth remains linear but the coefficient increases significantly after Round 100 (0.5), leading to a functional 'difficulty wall'.***

3.Hyperbolic Income Decay $I(R)$

The game uses a piecewise step function to reduce the cash per pop. These are the exact game constants:

$$I(R) = \begin{cases} 1.00 & 1 \leq R \leq 50 \\ 0.50 & 51 \leq R \leq 60 \\ 0.20 & 61 \leq R \leq 85 \\ 0.10 & 86 \leq R \leq 100 \\ 0.05 & 101 \leq R \leq 120 \\ 0.02 & R > 120 \end{cases}$$

4.The Efficiency Gap Formula

We define the Efficiency Gap (Eg) as the ratio of enemy durability to financial reward:

$$Eg = \frac{H(R)}{I(R)}$$

- $H(R)$ is the health scaling
- $I(R)$is the income per pop

To find how much harder the game gets between Round 80 and Round 120, we calculate the percentage increase in the Efficiency Gap:

$$\text{Increase}=\left(\frac{Eg(120)-Eg(80)}{Eg(80)}\right) \times 100$$

- Round 80 Efficiency: $\frac{200}{0.2} = 1,000$
- Round 120 Efficiency: $\frac{600}{0.02} = 30,000$

$$\text{Increase} = \left(\frac{30,000-1,000}{1,000}\right) \times 100 = 2,900\% \simeq 3,000\%$$

Result: The 2,900% increase in the Efficiency Gap serves as a mathematical proof of the **Law of Diminishing Returns**. Every dollar earned in the late game requires thirty times more "effort" from the player's towers than in the mid-game.

The Economy :

In *Bloons TD 6*, Banana Farms can be treated as financial assets, because the player has to decide whether to spend money now in the hope of gaining more later. The clearest way to judge a farm is through banana efficiency, which measures how much income it produces compared with its cost. In simple terms, it is a rate of return per round, so it gives a fair basis for comparing farms of different prices. A farm with a higher efficiency gives more income for every dollar invested, which makes it a stronger short-term purchase. Closely linked to this is the payback period, which shows how long it takes for the farm to recover its original cost. The two ideas move in opposite directions: higher efficiency means a shorter payback time. That relationship matters because it helps explain why the Monkey Bank sits near the top of the banana efficiency table, at about 10%. Its return is high relative to its cost, so it earns back its investment quickly. That said, the table is not just there for decoration. It shows how the numbers actually support the rankings. A Monkey Bank with a cost of about $3650 and an average return of roughly $365 per round gives an efficiency of around 0.10, or 10%, which matches its position in the table. By contrast, farms such as the Marketplace and the standard Banana Farm sit lower, around 6–7%, because they generate less income relative to what they cost. In a real game, this matters around rounds 20 to 25, when a player is deciding whether to invest in farming or save for defence. If a farm costs $1250 and earns $100 per round, it pays for itself in 12.5 rounds, so a farm bought on round 20 would roughly break even by round 33. That makes the choice feel real: keep the farm running for profit, or sell it at the right moment to fund a stronger defence before a difficult round. Since farms can usually be sold for about 70% of their value, the player is not completely trapped by the decision. Banana efficiency, then, is not just a neat formula. It is a practical way of thinking about risk, timing, and opportunity cost.

Variable Definitions

Banana efficiency — the income generated per unit cost (rate of return per round).

$$\epsilon$$

Banana payback period — the number of rounds required to recover the initial cost.

$$\rho$$

Sellback payback period — the number of rounds needed to recover the *unreturned* portion of cost when accounting for selling (e.g. 30%).

$$\rho sell$$

Banana compound efficiency — the effective growth rate per round when income is continuously reinvested.

$$\chi$$

Cost of the Banana Farm (initial investment).

$$c$$

Income generated per round by the farm.

$$g$$

Withdrawal period — number of rounds between when income becomes available (e.g. δ=1\delta = 1δ=1 for normal farms, δ>1\delta > 1δ>1 for Monkey Banks).

$$\delta$$

Initial amount of money (starting capital).

$$S0$$

Total money after rrr rounds.

$$Sr$$

Number of rounds elapsed. r

The Banana Efficiency Formula

$$\epsilon = \frac{g}{c}$$

$$\rho = \frac{c}{g} = \frac{1}{\epsilon}$$

$$\rho sell = \frac{0.3c}{g}$$

Example 1 (Monkey Bank):

$$c = 3650(cost\ of\ farm),\ g = 365\ (average\ income\ per\ round)$$

$$\epsilon = \frac{365}{3650} = 0.1 = 10\%$$

Example 2 (Standard Banana Farm bought around round 20):

c = 1250 (cost of farm), g = 100 (income per round)

$$\epsilon = \frac{100}{1250} = 0.08 = 8\%$$

$$\rho = \frac{1250}{100} = 12.5 \text{ rounds}$$

$$\rho sell = \frac{0.3\times1250}{100} = 3.75\ rounds$$

Banana Efficiency Rankings

Farm Type	Banana Efficiency
Monkey Bank (increased production)	~10%
Banana Research Facility (ValuableBananas)	~7.4%

Marketplace	~7%
Increased Production	~6.5%
Greater Production	~6.5%
Basic Banana Farm (No upgrades)	~6.4%
Valuable Bananas	~4.3%
Banana Central	~4.3%

Key Insight: Monkey Bank pays itself in only **~10 rounds,** while Valuable Bananas takes **~23.5 rounds.**

Banana compound efficiency matters when the player stops thinking only about immediate return and starts thinking about growth. That is where *Bloons TD 6* becomes genuinely interesting, because the best strategy is often not just to earn money, but to reinvest it fast enough that the money begins to multiply. In practice, this means players do not simply collect income and let it sit there. They use it to buy more farms, and that creates a compounding effect. Income starts to snowball. This is why banana compound efficiency is more revealing than basic efficiency when comparing farms such as the Monkey Bank and the Banana Research Facility. The Monkey Bank may look excellent at first glance, because its banana efficiency is high, but it stores money and delays access to it. That delay slows reinvestment, and in a game like this, time is everything. Every round that money is locked away is a round where it is not building more income. The Banana Research Facility, on the other hand, produces income every round, which means the money can be put straight back into the economy. That is exactly why it rises to the top of the compound efficiency table at about 7.4%. Its advantage is not just that it earns well, but that it earns steadily and can be reinvested immediately. The Monkey Bank drops under compound efficiency because delayed cash has less value than cash that can be used right away. This is why the rankings change so much between the two tables. Banana efficiency rewards raw return. Compound efficiency rewards return *plus timing*. That distinction becomes crucial in the middle and late game, especially around rounds 40, 63, 80, and 90, when the player is trying to build an economy fast enough to keep up with stronger bloons. A farm that pays out sooner can sometimes outperform a farm with a higher headline return, simply because it gets back into circulation faster. That is the real logic behind compounding, and it is what makes farming such a powerful part of the game.

The Banana Compound Efficiency Formula

$$\chi = (1 + \epsilon)^{\frac{1}{\delta}} - 1$$

$$Sr = S0(1 + \chi)^{r}$$

Example 1: Banana Research Facility:

$$\epsilon = 0.074, \delta = 1$$

$$\chi = (1.074)^{1} - 1 = 0.074 = 7.4\%$$

Example 2: Monkey Bank:

$$\epsilon = 0.10, \delta = 12$$

$$\chi = (1.10)^{\frac{1}{12}} - 1 \approx 0.00797 \approx 0.8\%$$

That shows why the Monkey Bank can be excellent for banana efficiency but much weaker for compound efficiency: the money arrives later, so it grows more slowly in practice.

Compound Efficiency Rankings (With Reinvestment)

Farm Type	Banana Compound Efficiency
Banana Research Facility	~7.4%
Marketplace	~7%
Increased Production	~6.5%
Greater Production	~6.5%
Basic Banana Farm	~6.4%
Monkey Bank (All Variants)	~Poor
IMF Loan	~1.9%
Valuable Bananas	~Still terrible

Critical Insight: When reinvesting, **Monkey Banks become the worst farms** because their 12-round period destroys compound growth. Withdrawing after just 1 round and reinvesting beats the bank interest.

At this point, one conclusion becomes almost impossible to ignore: the *Valuable Bananas* upgrade path is, quite consistently, the mathematical underperformer. On paper, it sounds promising — increasing the value of each banana should mean more money, right? But the numbers tell a different story. The upgrade increases income by a fixed percentage, yet it comes with a disproportionately large increase in cost. When efficiency is calculated using $\epsilon = \frac{g}{c}$, the result is lower than competing upgrades, meaning less income is generated per dollar invested. This already places it near the bottom of the banana efficiency rankings. It gets worse under compound efficiency. Since $\chi = (1 + \epsilon)^{\frac{1}{\delta}} - 1$, a lower base efficiency directly translates into weaker long-term growth, and no clever reinvestment strategy can fully compensate for that initial disadvantage. In practical terms, while other farms are quietly snowballing into a late-game economy, *Valuable Bananas* is essentially falling behind, round after round, like that one player who insists on saving money but never quite invests it properly. It is not completely useless — but mathematically speaking, it struggles to justify its cost in almost every scenario. The result is almost ironic: an upgrade called ***Valuable Bananas turns out to be one of the least valuable investments in the entire farming system.***

The defence: efficiency

The defence of Monkey Meadows relies on a structured hierarchy of towers, each fulfilling a specific tactical role across primary, military, magic, and support categories. These towers are not simply placed; they are selected as solutions to increasingly restrictive conditions imposed by the bloons. A Dart Monkey may provide efficient early-game coverage through rapid projectile output, while higher-tier towers such as the Archmage introduce layered utility through pierce, range, and camo detection. However, the real difficulty lies not in damage output alone, but in the emergence of bloon properties such as Camo and Lead, which effectively invalidate entire categories of defence if left unaddressed.

A Hard Mode run in *Bloons TD 6* is basically an optimisation problem disguised as a game. You’ve got fixed resources and about 2,036,251 RBE to deal with across 80 rounds. So naturally, the goal is to maximise output per dollar. And if I’m being honest, the Submarine, specifically the 4-0-2 Bloontonium Reactor on the way to 5-0-2 Energizer, feels almost unfair in how well it does that.It just covers too much. Camo disappears, lead isn’t an issue, and grouped bloons get cleared without much effort. Most towers are good at one thing. This one casually handles three, which is kind of ridiculous.The math behind it is where it gets interesting. Most towers follow a simple model:
D = Projectiles × Pierce.
That works until rounds like 63, where bloon density spikes and suddenly pierce becomes the limiting factor. It’s not that your towers are weak. They just physically can’t apply damage fast enough.

The Reactor doesn’t really care about that. It applies damage every 0.3 seconds to everything in its radius. So instead of being limited by pierce, it effectively scales with the number of bloons on screen. More bloons actually means more value, which feels like breaking the rules a bit. And then there’s opportunity cost. The 4-0-2 path gives you decamo and lead-popping in one purchase, which means fewer extra towers eating into your budget. You’re simplifying the system instead of constantly patching it. At that point, it’s not just strong. It’s efficient in a way that makes everything else you place work better too.

- Primary role:
  water tower providing
  (decamo + support).
- Radiation (Bloontonium
  pulses roughly every

Cost-effective early–midgame
both DPS and strong utility

Reactor aura): Emits damage
0.28s (very fast tick rate),

dealing 1 damage per pulse and affecting up to ~50 bloons at once in range.

- Decamo ability: The radiation aura removes Camo properties from bloons continuously within its radius, making it strong support for other towers.
- Dart attack (unsubmerged): Fires darts dealing 2 damage per shot, giving solid early-game single-target damage when not fully relied on aura.
- Airburst darts: Each shot can split into 3 sub-projectiles, with each dealing 2 damage and 2 pierce, improving grouped bloon clearing.
- Support buffs: Provides ~15% cooldown reduction to nearby water-based towers, improving overall water-defense efficiency.
- Advanced targeting: With Advanced Intel, it can attack bloons anywhere within range of other towers, increasing effective coverage across the map.

If I were optimising for a solo Hard Mode run, the Mortar Monkey is where the model starts to break down. The main issue is its lack of auto-targeting, which makes its performance dependent on constant manual adjustment. Unlike towers such as the Ninja or Submarine that guarantee damage once bloons enter range, the Mortar operates on probability. Its damage can be expressed as

D = (Damage × Pierce) × P(hit),

and that P(hit) term is highly unstable. Because it fires at a fixed point and bloons are constantly moving, faster rounds like 42 or 63 reduce its effective DPS significantly, sometimes to the point where it contributes very little.

The opportunity cost makes this worse. To deal with camo, you are effectively forced into the 0-2-3 Signal Flare path, which prevents access to higher-damage upgrades like the top path. This creates a trade-off where solving one constraint limits your ability to handle overall RBE. Even with upgrades like Rapid Reload, the Mortar still lacks consistent target tracking, so its damage output remains unreliable. As a result, its pops-per-dollar ratio is low compared to other towers. You are investing heavily in a tower that requires continuous input and still lacks consistent uptime. In a solo context, where reliability is critical, this makes the Mortar Monkey one of the least efficient choices for handling all 80 rounds.

- **Primary role:** focused on decamo reliability as a
- **Explosive shells:** (33% faster than base), base damage with ~25 grouped bloons in a
- **Burny Stuff** a burn effect dealing 4

Cost-intensive support tower and utility, with limited standalone DPS option. Fires shells roughly every 1.5s each explosion dealing low pierce, allowing it to affect fixed area. **(damage-over-time):** Applies damage per tick at an increased rate, adding passive damage after each hit but not enough to carry rounds alone.

- **Decamo utility:** At 0-0-3, removes Camo properties from bloons hit by explosions, providing area-based decamo rather than constant coverage.
- **Defortification support:** At 0-0-4, removes Fortified properties from BFBs and below, reducing the effective durability of incoming bloons.
- **Global range:** Can target any location on the map regardless of placement, giving theoretical full-map coverage.
- **Manual targeting:** Uses "Set Target" instead of auto-targeting, meaning its effective damage depends heavily on player input and accuracy over time.

To determine which monkey is mathematically superior:

The formula changes at Round 81 and again at Round 101.

1. Rounds 1−80: Base Health = 200.
2. Rounds 81−100: Health increases by 5% of the base (10 HP) per round.
3. Rounds 101−124: Health increases by 20% of the base (40 HP) per round.
4. Rounds 125−150: Health increases by 50% of the base (100 HP) per round.
5. Round 151+: The health multiplier increases even faster.

"High Round" multiplier:

$$Multiplier = 1 + (100 - 80)\times 0.05 + (124 - 100)\times 0.2 + (150 - 124)\times 0.5 + (267 - 150)\times 0.7$$

At Round 267 the hp of the moab is **20,140**

If I were to optimize for the highest rate of MOAB elimination at Round 267, the comparison between the base Submarine and the base Mortar becomes almost mathematically unfair. Both towers deal 1 damage per shot, so in a perfectly controlled model, each requires exactly 20,140 shots to deplete a MOAB's health pool. At first glance, that symmetry feels satisfying—like a clean equilibrium in a basic economic model. But the moment we introduce time as a variable, the "equilibrium" collapses. The Submarine fires every 0.75 seconds, leading to a total time-to-pop of 15,105 seconds (about 4.2 hours). Already inefficient, yes—but still operating within a somewhat defensible production rate. The Mortar, however, fires every 2.0 seconds, stretching the total time to 40,280 seconds (11.2 hours). That's a 2.66× decrease in output efficiency. If this were a firm, it would be producing at less than half the productivity of its competitor while somehow expecting to stay in the market.

The issue is reliability. The Mortar introduces projectile travel time, which, at Round 267 speeds, reduces hit probability to nearly zero. So while its theoretical damage output exists neatly on paper, its expected value collapses under real conditions. In contrast, the Submarine delivers consistent, guaranteed damage. And if I were optimizing for actual outcomes rather than theoretical ones, the conclusion is straightforward: the Submarine may be inefficient, but the Mortar is inefficient *and* unreliable—which is, frankly, a terrible combination in any system.

Conclusion:

Bloons TD 6 is more than just a game, it is a dynamic and deeply strategic system where mathematics becomes the language of optimisation, efficiency, and survival. From the deceptively simple act of popping bloons to the layered complexity of RBE calculations, MOAB health scaling, banana efficiency, and compound growth, every decision is grounded in quantifiable structures. This essay has explored the game not merely as entertainment, but as a model of real-world economic and mathematical problems, solvable through ideas such as optimisation, rate of return, and probability.

Yet, what makes Bloons TD 6 genuinely interesting is how this mathematical structure interacts with player decision-making. You do not need a perfect strategy to succeed; rather, the game rewards understanding. A Monkey Bank may appear optimal under simple efficiency, yet fail under compound growth. A Mortar may seem powerful in theory, yet collapse when probability and reliability are considered. These outcomes do not contradict the maths, they reveal a deeper layer of it. The game rewards those who can adapt their model, not just follow it.

In building economies and defensive systems, players effectively become applied mathematicians, navigating constraints, uncertainty, and trade-offs. Concepts like opportunity cost, diminishing returns, and efficiency gaps are not abstract ideas, they actively determine whether a strategy succeeds or fails. Every placement, upgrade, and investment reflects a balance between short-term survival and long-term optimisation.

For me, as someone interested in both mathematics and economics, Bloons TD 6 provides a space where theory becomes practical. It demonstrates how mathematical thinking can transform simple decisions into strategic systems, where logic drives success.

In the end, whether you are calculating the most efficient farm or comparing the reliability of a Submarine against a Mortar, you are engaging in a system defined by precision and constraint. And in doing so, it becomes clear that Bloons TD 6 is not just a game, it is a mathematical model in disguise.

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