Build Your Own Memory Palace

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1. Introduction

Say you have an awful memory... Not necessarily an ideal trait for a mathematician. Ramanujan, Gauss, John von Neumann: just a few of many Hall-of-Fame names that would undoubtedly agree.

Thus, we introduce memory palaces: a technique of mentally storing information in different areas of some geographical location. Our goal is to find the optimal memory palace to store decades of crucial pieces of life to come.

The 3 criterion (which we refer to by [1], [2] and [3] respectively throughout the paper) of memory palaces that we aspire to adhere to, follow:

1. A compact system to ensure all information is compactly stored, and not overflowing.
2. Seamless spatial connectivity to establish relationships between ideas.
3. Unbounded space for distinguishable loci to allow for infinite memory to be stored.

In this paper, we will discover potential forms of memory palaces, each better than the last, from the Taj Mahal to the sphere arising from the Poincaré Conjecture. We ultimately aim to prove that the Hilbert Curve fixes all prior limitations.

2. Taj Mahal

The Taj Mahal, a masterpiece in geometric construction, contains 2 primary elements that contribute to its arguably successful representation of a memory palace: its symmetry and use of the golden ratio.

The bilateral symmetry across the central axis, alongside its magnificent design, ensures that the structure is easy to envision, especially since we can chart certain types of information to be in similar positions, supporting [2].

However, the crucial limitation of using the Taj Mahal as our memory palace is that it is ultimately a finite space, within which there is a bounded number of loci, hence posing a challenge to [3]. Moreover, we consider that memorizing the vastly different decorations in the intricate building does not necessarily provide us with a compact space, therefore not fully meeting the purpose of [1].

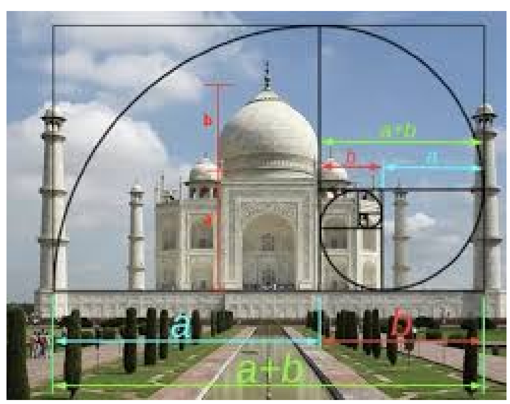

Figure 1: Taj Mahal Golden Ratio

Still, the Taj Mahal follows the beauty of the golden ratio- considered the 'divine proportion'- that can help rectify this problem.

The golden ratio is the ratio of a line segment cut into pieces such that the ratio of the whole line to the longer segment is equal to the ratio of the longer segment to the shorter segment. The unique property of the golden ratio is that, regardless of the length of the line we draw, in order to ensure that $\frac{a}{b} = \frac{a+b}{a}$, we require that $\frac{a}{b} = \phi$, whose exact value is expressed as: $\frac{1+\sqrt{5}}{2}$.

It also has the fascinating property of being the only number for which one more than itself is that number squared: $\phi^2 = \phi + 1$.

The golden ratio even has an intrinsic connection to the Fibonacci sequence, which begins as 1,1,2 from which every subsequent term is the sum of the two previous terms. Observing the ratios between any two consecutive terms of this sequence, we find that the ratios approach the precise value of as we move further along the sequence.

Ultimately, phi simply relates two pieces of information that frequently interact. Seeing the golden ratio in such architecture means that we can break down the building into smaller proportional sections of related information, with the use of a golden rectangle supporting [2], strengthening your ease in traversing the information in the palace.

Figure 2: Golden Ratio Fractal

3. Poincaré Conjecture

However, as mathematicians, we seek the geometric shape that appears to optimises utility by dealing with our prior limitations. So, we consider an all-too-simple shape: a sphere. Thus, we explore the **Poincaré Conjecture.**

This conjecture, the only one of 7 Millennium Problems to have been solved, by Grisha Perelman, states that any finite 3D manifold that is path-connected, with every loop contractible to a point is homeomorphic to a 3-sphere. This problem was a special case of a much larger conjecture, Thurston's Geometrisation Conjecture, proved in 1982 by William Thurston, who was awarded the Fields Medal for that considered 8 geometries. This conjecture claimed that any closed 3-manifold can be decomposed into a new particular geometric structure.

First, let us clarify a fundament of topology, the field within which this problem lies. Topology is the study of homeomorphism, where there exist invariant properties of geometric objects through repeated deformations, given a constant genus. In other words, we can contort and change a shape to become another in as many ways as we like so long as we don't tear or add to it, similar to the ancient art of origami. For instance, consider the torus below. Though the mug (Object 1) and the donut (Object 4) appear different to us, the total number of holes (the genus) remains one, as it is deformed through the process displayed.

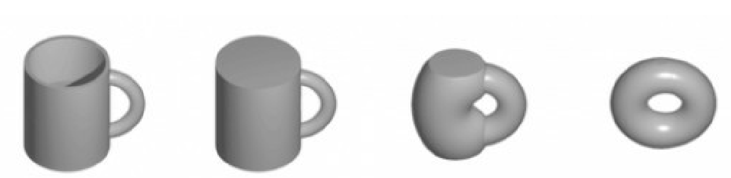

Figure 3: Homeomorphic torus of genus 1

A function is a **homeomorphism** if it is bijective and both f and f^{-1} are continuous.

Let X and Y be topological spaces of the memory palace and the set of information you would like to remember, respectively.

A function $f : X \to Y$ is continuous if $f^{-1}(V)$ is open in X for each open set V $\in Y$.

Homeomorphism's property of continuity emphasises [2], as we are able to traverse a path that is unbroken, so that we can seamlessly move between different pieces of interlinked information.

To prove the Poincaré Conjecture, Perelman significantly utilises the Ricci Flow. To conceptualise the Ricci Flow in 3D, we begin by introducing the idea of the Curve Shortening Flow.

Consider the following 1D surface in the 2D plane.

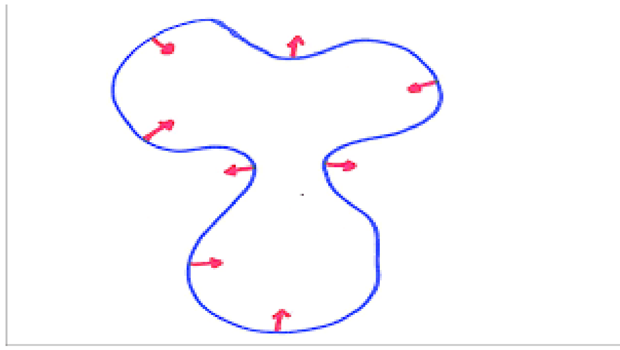

Figure 4: Curve Shortening Diagram

We take tangents at all points along the curve. The amount moved perpendicular to the bisector of this tangent is proportional to the amount of curvature.

Now consider a 2D surface in the 3D plane. We adapt this concept into the mean curvature flow, using Riemannian geometry. The metric g acts as a weighted matrix that tells you how to calculate length at a specific point.

If the metric tensor is the identity matrix $g_{ij} = \delta_{ij}$, the underlying space is Cartesian, with no curvature or space-time deformation, where the distance metric simplifies to the standard Pythagorean theorem:

$$ds^2 = \sum (dx^i)^2$$

Here, the basis vectors are orthogonal to each other and have unit length: $v_i \cdot v_j = \delta_{ij}$

The tensor relates size to curvature by considering 3 significant properties: distance, angle and area. A metric tensor, g_{ij} is a function that assigns some arbitrary length orthogonal to the plane tangent to the manifold.

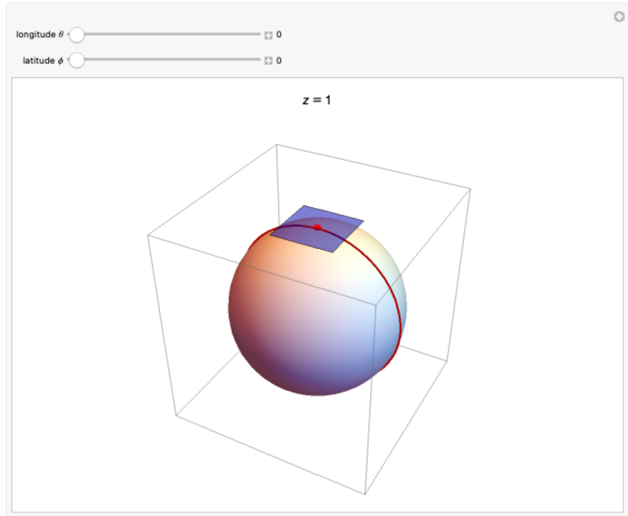

Figure 5: Tangent plane to a sphere

We then calculate the dot product of the tangent vectors (the derivative of a particular point on the curve). The Frenet-Serret Frame represents the two orthogonal vector fields: tangent (red) and normal (blue) vectors.

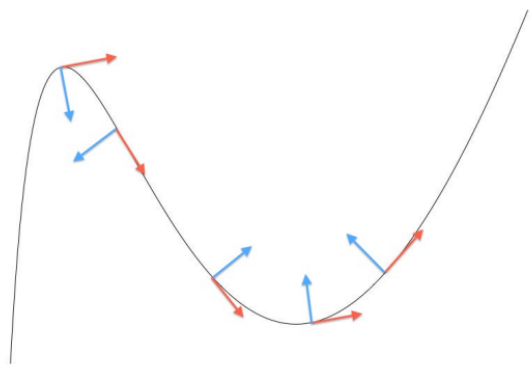

Figure 6: Frenet-Serret Frame

This length and angle between two given vectors a and b are given by:

$$||\mathbf{a}|| = \sqrt{g(\mathbf{a},\mathbf{a})}$$

$$cos(\theta) = \frac{g(\mathbf{a},\mathbf{b})}{||\mathbf{a}||\,||\mathbf{b}||}$$

Smaller lengths cause contractions, larger lengths cause expansions around that point, thus dictating the new curvature of the manifold.

When this is applied to 3D curves, we can consider the mean curvature flow. As the radius decreases to zero, when the surgery is repeated, the curvature becomes infinite, which we can find the distance of through the following equation:

$$ds^2 = \sum_{i,j} g_{ij} dx^i dx^j$$

The Ricci Flow equation equates the derivative of the metric tensor, which represents the relationship between the manifold and time, to the Ricci tensor. The Ricci tensor acts as an average of the metric tensor.

It measures the curvature of the structure through the change in the dot product of vectors across a manifold over time to smoothen the shape.

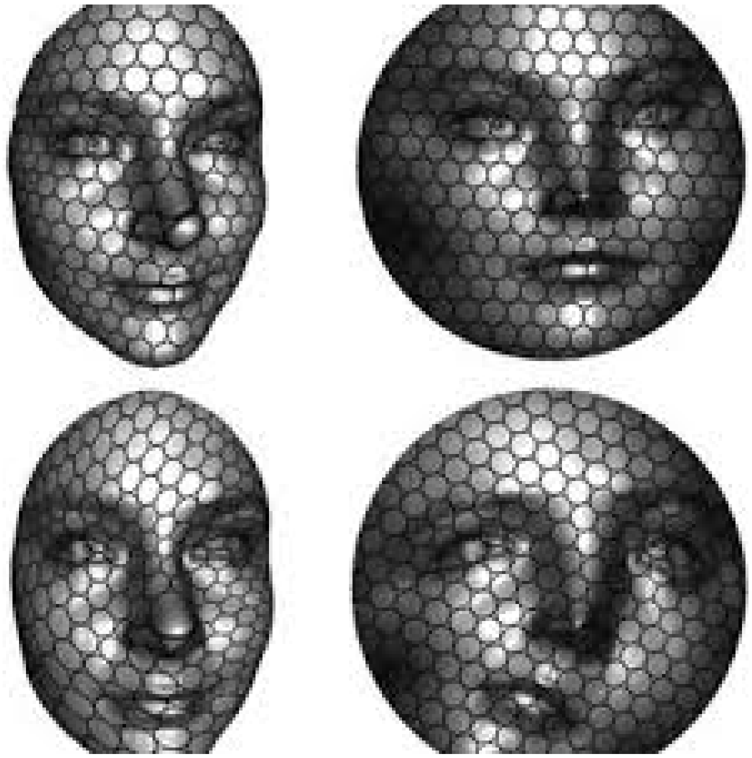

Figure 7: Ricci Flow Effect

Therefore, the arbitrary length chosen by the metric tensor is 'smoothed out' by the metric tensor, as it reaches an equilibrium over time.

The Ricci Flow Equation

$$\frac{\partial}{\partial t} g_{ij}(t) = -2 \ \mathrm{Ric}(g(t))$$

Why do we have a negative coefficient for the Ricci tensor? Our objective is to smooth out the curvature into a sphere, getting rid of anomalies. If an area of the manifold is highly curved (for a positive R_{ij}), the metric is forced to shrink. The opposite is true for the converse .i.e.

$$If\ R > 0, \frac{d}{dt}(g) < 0$$

$$If\ R < 0, \frac{d}{dt}(g) > 0$$

The core reasons this is effective are as follows:

1. There is never an end to the memory palace, and everything loops back to each other. The palace is therefore a closed mental map, satisfying [1].
2. We can use the concept of the Ricci Flow to reinforce unity between ideas. Thus, everything is connected, satisfying [2].

Perelman, seeking a peaceful life and rejecting the million dollars, may not have needed a palace, but his proof provides us with an intensely useful structure for our own.

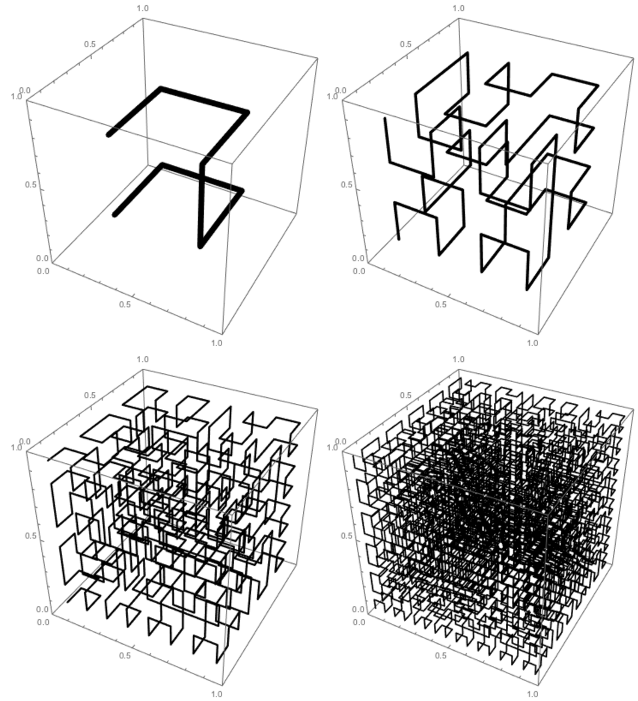

Figure 8: 3D Hilbert Curve

However, though it meets our initial criteria, the general shape does not provide us an accurate understanding of what occurs inside the sphere. A sphere is simple, yes, but perhaps simplicity does not fulfill the needs of a memory palace. Hence, we move to collaborate with the memorable aesthetics of the Taj Mahal, by considering a new optimal solution: the Hilbert Curve.

4. The Best of Both Worlds: The Hilbert Curve

A **fractal** is a geometric pattern that is self-similar at infinite different scales.

We note that the Fibonacci spirals that the Taj Mahal's golden ratio property provided us with belongs to the geometric brand of fractals. Knowing it is useful from our earlier reasoning in section 2, we must now uncover the 'best' fractal for our memory palace.

The **Hilbert Curve** is a single, continuous line that makes 90 degree turns at certain junctions, and ultimately visiting every square without a n x n grid with repeating.

Look at Google's all-too-popular snake game for a good example (that is, if you are successful at playing). It can be considered the optimal space-filling curve for our purpose, as it has the unique property of locality preservation, meaning it is able to map a 1D line to 2D while keeping nearby points close together. By increasing the order n of a fractal, you can generate 4^n units of information, as if you were zooming in to a beautiful pattern, with the structure never changing.

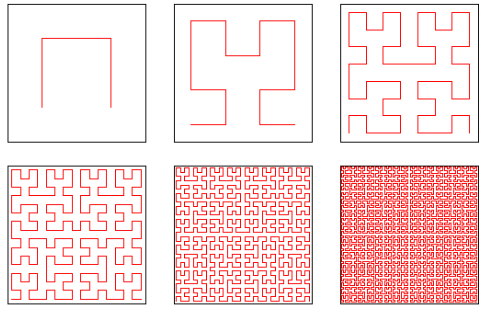

Figure 9: Hilbert's Curve representation

Within a memory palace, we consider its effectiveness, remembering that we intend to consistently add new information to memorise, into this palace. Information is nested neatly within each other, within a rigid boundary [1], but containing infinite information [3]. Similar information would be relatively close to each other, following one unbroken path for you to walk down without overlaps, allowing for seamless connectivity [2], when translating from a line of thoughts to a geometrical manifestation of it. Hence, we prove that this characteristic allows for all 3 benefits.

5. Conclusion

In order to satisfy our quest to find our own personal best model of a memory palace, we have journeyed through the magnificence of real-world structures and the austerity of a sphere, and have unravelled both of their limitations. Ultimately, we conclude that the Hilbert curve within structures such as spheres, where the sphere is internally broken down into units, albeit of differing sizes.

6. References

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