But does 2 + 2 really equal 5?

In an era where 1984 by George Orwell is feeling increasingly relevant, ideas and quotations from it are resurfacing. That book has shaped my worldview in many ways, but one thing that lingers, years later, is how the government asserts that "2+2=5".

Obviously, within our modern-day arithmetic system, 2+2 does not in fact equal 5. But what if it did? 1984 is a book about control, about how the authorities control a society, and it begs the question: is maths controlled by others who enforce rigid rules to prevent questions?

A common conversation in mathematical communities is, "Is maths discovered or invented?" If maths is invented, then surely, we, as humans have power over the rules and how it works, so our creativity is our limit.

I want to take you on a ride of mathematical exploration and creativity to show how Maths is really an unruly and creative subject, despite what you may have been led to believe.

Gödel's Incompleteness Theorem

My story begins with David Hilbert: a greatly influential 19th and 20th century mathematician and philosopher who aimed to provide a secure foundation for all mathematics. He lived within a crisis in the foundations of mathematics as paradoxes (statements that seem to contradict themselves despite appearing completely logical[1]) kept being uncovered, particularly within set theory (a branch of mathematics that studies collections of object called sets). Hilbert wished to formalise mathematics within axiomatic systems (set of rules that are taken as basic truths, and all other rules are built upon them). He also wanted to prove that these axiomatic systems didn't contradict each other (known in mathematics as consistency). He dreamed of a unified mathematics with a solid foundation. It was a tempting idea: after all, mathematicians are stereotyped for wanting order and clear rules, and Hilbert's program was the perfect chance to obtain this order.

From Hilbert's program, important steps in mathematical logic arose. Bertrand Russell and Alfred North Whitehead developed type theory in their *Principia Mathematica*, which was a landmark text for those working on Hilbert's program. Various other mathematicians developed Hilbert's own ideas, such as Bernays and von Neuman.

Then came along Gödel, who shattered these dreams of order in 1931. He encoded the words "this statement has no proof" in numbers, which had to be numerically true or false. If it was false, the system has proof of an untrue statement, which shouldn't be possible. Therefore, the statement is true, meaning that the system has a true but unprovable statement. He showed that any logical system powerful enough to describe arithmetic is either incomplete or inconsistent. He stated that no complete set of rules for arithmetic could ever be written down or generated by a computer, as every system would contain gaps, and you'd never be able to plug these gaps.

This shattered the hopes of the Hilbert program, and caused consternation among contemporary mathematicians, who dreamed of creating a unified mathematics. Maths relies on all true statements being provable, so Gödel's Incompleteness Theorems threatened the

[1] Definition from Brilliant.org

neatness and order of mathematics. It's understandable that this was worrying for contemporary mathematicians when the rules-based way they wanted to view the subject that they'd devoted their lives to was shown to be wrong.

But personally, I find Godel's Incompleteness Theorems more comforting. After all, real life very rarely strictly adheres to one set of rules and fits within nice boxes: it's often messy, and that's part of what makes it beautiful. If maths is inextricably linked to the universe, surely it wouldn't make sense for it to be as simple and orderly as those mathematicians wanted it to be. The theory implies that not everything can be discovered through structured, rigorous processes. To me, I see it as an encouragement to see Maths more creatively, to think outside of the systems that we've been conditioned into and see what we might find.

So, two questions arise:

1. Is maths discovered or invented?
2. But what does imagination mean in the context of maths?

1) Is Maths discovered or invented?

Considering the philosophy of how maths originated, and what it is at its core, is important when we consider how to be creative within the subject.

One piece of evidence for Maths being invented is how many times we've extended our number system: from the positive natural numbers (1, 2, 3, 4, 5...) to all natural numbers (..., -3, -2, -1, 0, 1, 2...) to the rationals (these can be expressed as a fraction but aren't necessarily finite e.g. 1/3). Then we progressed to the reals (all numbers that can be expressed as a finite decimal, like 256.1209 or pi), and most recently the complex numbers (involving the square root of -1). This suggests that if there isn't already a way to talk about something mathematically, we just make one up. Therefore, we could perceive that Maths exists within us; it could be argued that by talking about Maths, we are really talking about ourselves, so there's no gap to bridge between our world and the mathematical world.

However, mathematicians Frege, Russell and Whitehead suggest that these aren't extensions of the number system at all – it's just an example of us liberalising the term "number". This is an example of mathematical Platonism – the metaphysical view that there are abstract mathematical objects whose existence is independent from us. This suggests that reality extends far beyond the physical world and includes objects that don't have a causal effect. As we know we have mathematical knowledge, the idea that we have knowledge of abstract objects (which must have no causal effect on the physical world) would be an important philosophical discovery and oppose many theories of knowledge which we currently have.

Alternatively, British philosopher Michael Dummett suggests that objects of maths may be "prodded" into existence – as if they already exist somewhat, so they aren't entirely products of our mind, but we have a part in making them exist. However, he admits that this theory is equally challenging, and states that he can't really explain it. What is prodded? Who does the prodding?

Fine argues that the possibilities of extending our domain of mathematics are endless, independent of how we think or what we do. But how we realise these possibilities is up to us. We can extend the system of numbers or sets or any other mathematical object we like to whatever infinite limit we choose. He ends with the powerful comment: "Our only constraint is our imagination and what we find appropriate or pleasing."

2) But what does imagination mean in the context of Maths?

I get it. You've always been told that Maths is this rigid game of arithmetic – adding numbers up quickly, finding the longest side of a triangle. But school hides the hidden joy of Maths. At its core, it's a rich, philosophically fascinating and deeply creative discipline, filled with ingenious theories and beautiful proofs.

Let's consider 1 + 1 – what seems to be a patronisingly basic question.

It could equal 2: one apple and one apple means you have two apples.

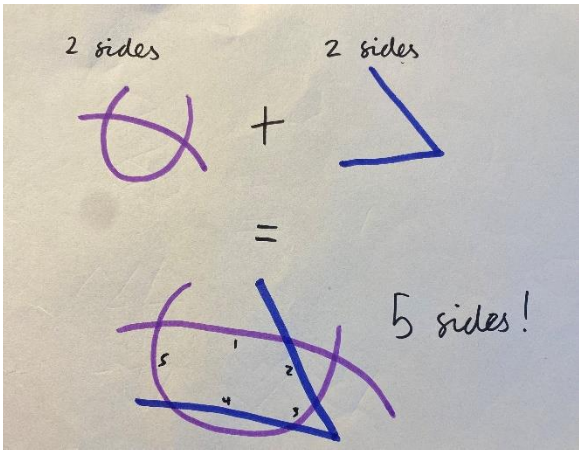
But you might have one apple and one banana. In that group, you have the concept of twoness (the group is of size two), but also the concept of oneness (there is only one of each type of fruit). Therefore, 1 + 1 could equal 1.

1 + 1 could equal 0. If you class 1 as the action of flipping a switch, flipping it twice just brings you back to your original position.

1 + 1 might equal 10, if we're adding within a binary number system.

As you can see, we limit ourselves to just one definition of 1+1, but by opening our mind, we gain a new appreciation of numbers and see the world slightly differently.

This brings me back to my original question: but does 2+2 really equal 5? My answer: it can. I've done a drawing to prove it.



This might make you feel uncomfortable – "but you used 2 curved lines!". And to that I say: 2 curved lines + 2 straight lines = 5-sided shape. We're not talking about polygons now: we're exercising our imagination, so if I decide that I can use curved lines, then I'm allowed to!

Conclusion

Ultimately, I'm trying to prove that we don't always need to follow the systems we've been told to, and we need to be creative: after all, the Greeks were originally extremely suspicious of irrational numbers, and Cardano's work on complex numbers was originally met with resistance. But now, these concepts are taught daily across the world and are taught as truth to students across the world.

The idea that "2+2=5" was originally used in Orwell's 1984 to suggest the corruption of the authoritarian government, to the extent that they could distort people's views about basic arithmetic. The government maintained their power through distorting people's views of reality through methods like this. Maybe, through challenging mathematics, the characters in 1984 may have obtained some intellectual freedom from the regime.

Maths is an overwhelmingly beautiful and creative subject. Even though we sometimes wish it was more orderly, it's messy, incomplete and inconsistent. But that's what makes it more beautiful. It reflects our world perfectly, with its contradictions and philosophical quandaries and multiplicity nestled within beauty and elegance. Through imaginations and objects "prodded" into existence, Maths will continue to uncover revelatory truths, not despite of its imperfections, but because of them.

Key Sources

Cheng – Is Maths Real?

Elwes - Maths in 100 key breakthroughs

Fine – Mathematics: Discovery or Invention?

Zach – Hilbert's Program Then and Now