Calculus, from a simple idea to modern innovation-related applications

The Language of Change: How Calculus Shaped the Modern World

Imagine trying to describe a speeding car using only arithmetic. You may make some calculations to describe it as average speed for an interval of time, such as saying X kilometers per second, which does not mean that the car was moving with this speed at every moment during the interval of time. But if you want to calculate the speed of the car at a single, precise moment like the instant of breaking or the exact point at which it reaches its peak velocity, ordinary mathematics falls silent. For thousands of years, these questions were unanswerable. Then, in the seventeenth century, two men working independently, in different countries, and in a bitter rivalry, invented a new language. That language was calculus, and it would go on to shape almost every corner of the modern world.

Ancient Seeds

The story does not begin with Newton or Leibniz. Long before them, the ancient Greeks were wrestling with the same deep problem: how do you measure something that is continuously changing? Around 250 BC, the mathematician Archimedes developed what he called the "method of exhaustion". To find the area of a circle, he filled it with hundreds of tiny triangles, each one smaller than the last, until the triangles "exhausted" the space inside. The more triangles he used, the closer he got to the true answer.

This was a breathtaking idea using infinity to reach precision. But Archimedes had no formal framework for it. He could apply the trick to circles and spheres, but not much else. Even though the true breakthrough would have to wait nearly two millennia, this idea was the seed of calculus's most fundamental concept: the limit. The notion that you can approach a precise answer by making something — triangles, rectangles, intervals — infinitely smaller without ever quite reaching zero is exactly what Newton and Leibniz would later formalize. Archimedes had the intuition two thousand years ago. He simply lacked the language to complete it.

The Invention

By the mid-1600s, Europe was in the middle of a scientific revolution. Astronomers like Kepler and Galileo were overturning centuries of received wisdom, and mathematicians sensed that something new needed a tool powerful enough to describe motion, change, and the physical world in full, more precisely.

Sir Isaac Newton, a quiet and obsessive Englishman, found it first. Working during a plague-forced retreat to his family farm in Lincolnshire between 1665 and 1666, Newton developed what he called "the method of fluxion". His central insight was elegant: if you want to know the rate at which something changes at one exact instant, you zoom in on that moment until the change becomes vanishingly small. What you are left with is a "derivative," a precise measure of instantaneous change—the speedometer of mathematics. To see what this actually means geometrically, we need to go slightly deep into the mathematical definition of "Rate of Change".

Motion in math is often represented by a displacement–time graph. To calculate the speed of an object over a time interval, we mark the starting and ending points on the curve and draw the line connecting them — this is called the secant line. Its slope, calculated by dividing the change in displacement (Y-axis) by the change in time (X-axis), gives us the average rate of change.

But the instantaneous rate of change is different. To find it, we bring the two points closer and closer together until the distance between them approaches zero. The line connecting them stops being a secant and becomes a tangent — a line that touches the curve at exactly one point. Its slope is the instantaneous rate of change: not the average speed over an interval, but the exact speed at one precise moment. That was Newton's discovery.

At almost the same time, the German philosopher and mathematician Gottfried Wilhelm Leibniz arrived at the same idea through a different route. Where Newton thought about motion and physics, Leibniz thought about geometry and area. He developed an "integral," a method for adding up infinitely many infinitely thin slices to find a total quantity, such as an area or a volume. His notation, the elegant elongated "S" symbol (∫), that we still use today, but what does integration geometrically mean? Where the derivative asks *"how steep is this curve at this point?"*, the integral asks a different question: *"how much area lies beneath this curve?"*
Imagine plotting the speed of a car over time — a curve rising and falling as the car accelerates and brakes. The area trapped between that curve and the time axis represents the total distance travelled. But curves are not rectangles, and finding their area precisely seems impossible.
Leibniz's insight was to do what Archimedes had done two thousand years earlier — but with a formal language. Slice the area beneath the curve into very thin vertical rectangles. Each rectangle has a width so small it is almost nothing, and a height equal to the curve's value at that point. Add all of them up, and you get an approximation of the area. Now make the rectangles thinner and thinner — more and more of them, each one narrower than the last — until their width approaches zero and their number approaches infinity. What you are left with is not an approximation but an exact answer. That is the integral.
The elongated *S* symbol that Leibniz chose — ∫ —was deliberate. It stands for *summa*, the Latin word for sum. It is a reminder that integration is, at its heart, infinite addition made precise.

The dispute over who deserved credit became one of the most notorious feuds in the history of science, dividing British and continental mathematicians for over a century. But history's verdict is generous to both: Newton and Leibniz each discovered calculus independently, and together they gave humanity one of its most powerful intellectual tools.

How It Changed Physics

Calculus became a powerful tool that Newton and other scientists used to prove and state several equations.

Understanding the surrounding universe was the first application of calculus. Newton's three laws of motion — the foundation of classical physics- are written in terms of calculus. Without derivatives and integrals, they cannot even be properly stated, let alone used.

Newton proved Kepler's laws of planetary motion followed directly from his law of gravity, also by means of calculus. Consider the orbit of a planet. It does not travel in a straight line; it curves continuously around the Sun. To calculate where it will be in six months, you need to track how its velocity and direction change at every single moment along the way. That is precisely what calculus does. It was a stunning unification of earthly and celestial mechanics that had never been achieved before.

The consequences cascaded through the centuries. The first prediction for Neptune and the first space trip to the moon were made thanks to calculus. By the 1800s, the French mathematician Urbain Le Verrier used calculus to predict the existence and location of Neptune purely from irregularities in Uranus's orbit before anyone had pointed a telescope at it. In the twentieth century, engineers at NASA used the same principles to calculate the trajectories of Apollo missions, threading a spacecraft through the vast emptiness of space to land on the Moon with extraordinary precision. Every rocket launch, every satellite, every GPS signal reaching your phone depends on calculus.

How It Changed Medicine

Physics was only the beginning. In the twentieth century, calculus quietly became one of the most important tools in medicine and biology.

When a doctor prescribes medication, they need to understand "pharmacokinetics," how a drug moves through the body over time. After you swallow a tablet, the concentration of the drug in your bloodstream rises, peaks, and then falls as your body metabolizes it. These rises and falls are described by a differential equation, an equation involving derivatives. By solving it, pharmacologists can calculate exactly how large a dose is to give, and how often, to keep the concentration within the safe and effective range. Too little, and the drug does nothing. Too much, and it becomes toxic. Calculus finds the balance.

During the COVID-19 pandemic, the same mathematics moved into the public eye. Epidemiologists used differential equations to model how a disease spreads through a population, tracking the rate at which people move from being susceptible to infected, to recovered. The famous "flattening the curve" idea was calculus in action: by slowing the rate of change of infections (the derivative of the infection curve), health systems could avoid being overwhelmed. Millions of decisions about lockdowns, vaccines, and hospital capacity were informed by these models.

How It Changed Technology

Perhaps the most surprising reach of calculus is in the digital world. Computers, after all, deal in discrete numbers of ones and zeros. Yet the algorithms that power modern technology are soaked in calculus.

Machine learning, the technology behind everything from facial recognition to large language models, relies on a technique called "gradient descent". To train an AI, you define a function that measures how wrong the model's predictions are. You then use derivatives to find the direction in which that error decreases most steeply, and you nudge the model's parameters in that direction, again and again, until the errors shrink to a minimum. Without calculus, there would be no modern AI.

Video games and animated films offer another example. When a studio renders a scene, it needs to simulate how light bounces off surfaces, how cloth ripples in the wind, and how water splashes and flows. All these simulations are governed by differential equations. The lifelike world of a modern video game is, at its mathematical heart, a vast calculus problem solved many times per second.

Even the signal reaching your phone is shaped by calculus. The mathematics of waves used in radio, Wi-Fi, and mobile networks was developed in the nineteenth century by Joseph Fourier, whose integral transforms decompose any signal into its component frequencies. This reflects the importance of calculus for modern technology innovations.

Conclusion

Calculus was born from a simple but profound question: how do we describe the world as it actually is — in motion and in flux — rather than frozen at a single moment? But what makes calculus truly extraordinary is not just that it answered that question. It is that the answer turned out to be universal. The same operation that describes a planet curving around the Sun also describes a drug fading from your bloodstream, a disease spreading through a population, and an artificial intelligence learning from its mistakes. Differentiation and integration — Newton's fluxions and Leibniz's sums — are not merely tools for physicists. They are the grammar of change itself, and change is the one thing everything in the universe has in common.

There is something quietly humbling in that. Two men, working alone in the seventeenth century — one retreating from a plague, one filling notebooks with geometric diagrams — accidentally wrote the language that would one day guide spacecraft to the Moon, hold pandemics in check, and teach machines to think. They argued bitterly over who had seen it first. But the deeper truth is that neither of them fully saw how far it would reach.

Calculus does not just describe the world. It revealed that beneath the apparent chaos of a changing universe, there is structure, precision, and pattern — and that the human mind, given the right language, is capable of reading it.