Chaos Theory and the inaccuracy of Ballon D’or rankings

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April 2026

Situation

Each year there is immense discourse around one particular voting event that captures the eyes of all sports fans from across all continents. The Ballon d'Or ceremony is a cause of controversy each time of occurrence, notably in recent years Messi v Lewandowski and Ronaldo v Ribery. While seemingly down to narrow margins created by voter bias and luck, many situations perhaps can be modelled mathematically.

Chaos Theory

Chaos theory most simply is a study of underlying patterns within seeming random systems that are in practice deterministic. Arguing chaotic systems are not truly random, it also focuses on the sensitivity of these system regarding their initial conditions. The phrase "the flap of a butterfly's wings in Brazil set of a tornado in Texas" is now embedded into the modern lexicon, originally derived from Edward Lorenz's work in the 1960s, where a minute change in input data (rounding a decimal) led to totally different weather predictions. Such as in the example of weather above, chaotic behaviour exists in many natural systems such as fluid flow, heartbeat irregularities, and climate; as well as some artificial systems: traffic.

Application within Ballon d'Or context

When applied to the Ballon d'Or, chaos theory helps explain why seemingly minor events can have a disproportionate decisive impact on the final rankings. A single missed penalty, an injury, or one exceptional performance in a game of high importance can significantly alter how a player is perceived. A high impact performance in a latter round of the champions league can alone carry more weight in the collective image of voters than a few weeks of consistent showcase. Essentially, relatively small events when considered all in equality can the cause massive narrative shifts and therefore vastly different voting outcomes.

Similarly, it is important to record the non-linear weighting of performance. There is no direct relationship between input and output, hence scoring 25 goals is not necessarily valued higher than 20. This further complicates the system, causing behavioural 'chaos' as performance is evaluated in a much broader context. Chaos theory is particularly well suited to describing such systems, where inputs and outputs are not proportional and small contextual factors amplify results.

Furthermore, other important dimensions such as voter bias and narrative amplification can skew reliable outputs; with individual awards and performances valued at different rates while in modern days, media can often dynamically shift the appearance of a player, often based on likeability and media conformity. Considered altogether, these principles ca explain several commonly observed features of Ballon d'Or outcomes. Results can appear unpredictable or even unfair because of the disproportional weight of small yet incredibly significant evets.

A simple chaotic style model

1)We may model a players Ballon d'Or outcome by these following criteria:

-P: performance (e.g g/a)
-T: trophies (team and personal)
-M: media narrative
-I: important games impact

Therefore achieveing,

$$S=f(P,T,M,I)$$

Where 'S' is the overall score a player receives from a voter
2)We can add chaos (sensitivity to initial conditions)

We introduce this dynamic:

$$M_{t+1}=rM_t(1-M_t)+\eta I_t$$

Where M_t is the media narrative at a specific point in time

-r is used as an amplification factor for media narratives, while the $(1-M_t)$ is used to limit growth.

-η is used as a factor for how much a big game performance adds to the narrative.

Disclaimer: for the modelling of this next part A.I was used to choose reasonable starting values as well as amplification factors. It was also used to generate suitable values for early season form; UCL period; Copa America and Late season.

3)Application

While grossly oversimplified, we can run a small model to simulate Robert Lewandowski vs Lionel Messi in the 2021 race for the Ballon d'Or

-r=3.5 - strong media/social feedback
-η=0.8 -big matches are watched by millions, performance is highly considered

Starting narrative form early 2021:
-Messi: N_0=0.55 (prominent name in world football)
-Lewandowski: N_0=0.60 (dominant 2020 campaign)

t	Event	Messi I_t	Lewandowski I_t
1	Early season form	0.1	0.2
2	UCL period	0.1	0.15
3	Copa América	0.9	0
4	Late season	0.2	0.2

t = 0:

Messi:
-Self-growth: 3.5×0.55×(1-0.55)=0.866
-moment impact: (0.8x0.1)=0.08
N_1≈0.946

Lewandowski:
-Self-growth: 3.5×0.6×(1-0.6=0.84
-moment impact: (0.8x0.2)=0.16
N_1≈1.00

t = 1

Messi:
-Self-growth: 3.5×0.946×0.054≈0.1793.5
-moment impact: 0.08
N_2≈0.259
Large drop (chaotic effect)

Lewandowski:
-Self-growth: 3.5×1×0=0
-moment impact: 0.12
N_2=0.12
Both score lower—shows system instability

t = 2

Messi:
-Self-growth: 3.5×0.259×0.741≈0.672
-moment impact: 0.72
N_3≈1.39 (caped at 1)
Large increase due to Copa America Performance

Lewandowski:
-Self-growth: 3.5×0.12×0.88≈0.37

-moment impact: 0
$N_3 \approx 0.37$

t = 3

Messi:
-Self-growth: 3.5×1×0=0
-moment impact: 0.16
$N_4 = 0.16$

Lewandowski:
-Self-growth: 3.5×0.37×0.63≈0.815
-moment impact: 0.16
$N_4 \approx 0.975$

<u>Results:</u>

While Lewandowski ends up higher, the system is chaotic ad so timing dependent, leading to the Messi's peak at the Copa America having much greater gravitas relatively within award season. At this time, t=2, Messi was at 1.39 compared to 0.37 for Lewandowski, highlighting the media narrative being much greater and therefore enriching voter bias as journalists gravitate towards newer successes.

$S \sim N_{\text{voting period}}$

Why this matters:
This small simulation explains what may be considered unfairness or 'voter bias'. What is seemingly unfair can be explained by nonlinear weighting and conditions and timing.

Estimating the Lyapunov exponent

The Lyapunov exponent is a measure that quantifies a system's sensitivity to small changes in initial conditions. It characterises the exponential rate of change in distance between two infinitesimally close trajectories; essentially how nearly identical situations diverge over time due the small difference in their starting information. It can be used to measure a system's stability, with positive exponent indicating chaotic divergence that increases the higher the exponent, causing systems to be increasingly unpredictable. A negative exponent means the system shrinks, becoming more stable and hence predictable.

<u>Application</u>

The Lyapunov Exponent

$$\lambda = \lim_{n \to \infty} \frac{1}{n} \sum_{t=1}^{n} Ln \left[\frac{dM_{t+1}}{dM_t} \right]$$

Using our general equation,

$$M_{t+1} = rM_t(1-M_t) + \eta I_t$$

We can find,

$$\frac{dM_{t+1}}{dM_t}$$

Let $M_{t+1} = y$
Let $M_t = x$

We can now rewrite as

$$y = rx(1-x) + \eta I_t$$

$$y = rx - rx^2 + \eta I_t$$

Therefore,

$\frac{dy}{dx}$ =r+2rx

=r(1+2x)

And so,

$\frac{dM_{t+1}}{dM_t}$ = r(1-2M_t)

Our Lyapunov exponent becomes:

$$\lambda = \lim_{n\to\infty} \frac{1}{n} \sum_{t=1}^{n} Ln[\mathrm{r}(1 - 2\mathrm{Mt})]$$

We can input our previous results for Messi's trajectory

M_t≈[0.55,0.946,0.259,1.0]

Giving us λ = -1.05,1.14,0.53,1.25

Finding an average:

$\lambda \approx 0.47$

What this means

The high positive value of λ shows a strong chaotic system. It represents the large effect of small differences within the narrative, e.g. Messi's Copa America win. It also shows how these moments grow in value exponentially, especially when combined with the correct timing, and ca quickly dominate the arguments for a players success and the eventual outcome. While this is only an estimate over 4 periods within a season, it can show roughly trends within behaviour of the narrative, where in conclusion timing and moment amplification has outvalued consistency and linear performances and statistics.