1: COORDINATE GEOMETRY:

- Midpoint of a line segment
- Distance between two points
- Parallel lines
- Perpendicular lines
- Perpendicular bisector

A: MID-POINT OF A LINE SEGMENT:

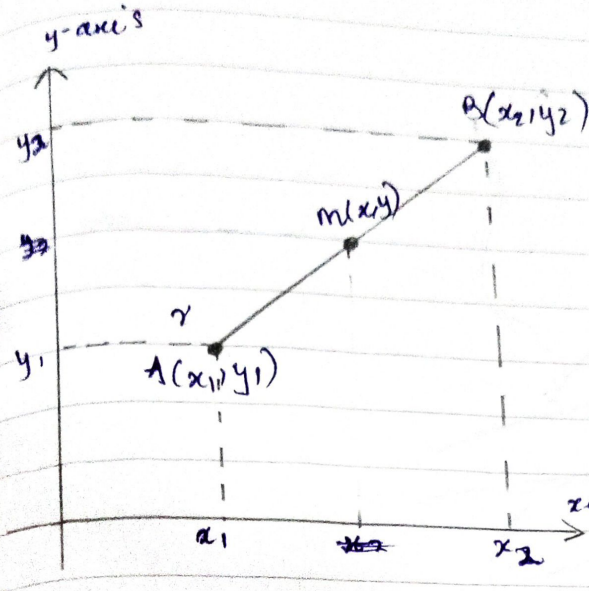
In generally;

* The midpoint of a line segment is given by:

$$M_{(x,y)} = \left(\frac{x_1+x_2}{2}\right), \left(\frac{y_1+y_2}{2}\right)$$

Proof

Consider the, x, y plane below,



Then

$m(x, y) = ?$

for x

$$x = \frac{x_1 + x_2}{2} \quad \text{and} \quad y = \frac{y_1 + y_2}{2}$$

hence

$$m(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\boxed{M_{(x,y)} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)}$$

Examples:

1) To find the midpoint of a line segment AB where: A (3, 4) and B (5, 9)

A (3, 4) $m(x, y) = ?$ B (5, 9)

x_1, y_1 x_2, y_2

from:

$$m(x,y) = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$$

$$m(x,y) = \left(\frac{3+5}{2}\right), \left(\frac{4+9}{2}\right)$$

$$m(x,y) = \left(\frac{8}{2}\right), \left(\frac{13}{2}\right)$$

$$m(x,y) = \left(4, \frac{13}{2}\right)$$

∴ The mid point is $(4, 13/2)$

2. Find the midpoint of a line segment whose end points are A (1, 3) and B (4, 5)

Solution

A(1,3) $m(x,y) = ?$ B(4,5)

from

$$m(x,y) = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$$

$$m(x,y) = \left(\frac{1+4}{2}\right), \left(\frac{3+5}{2}\right)$$

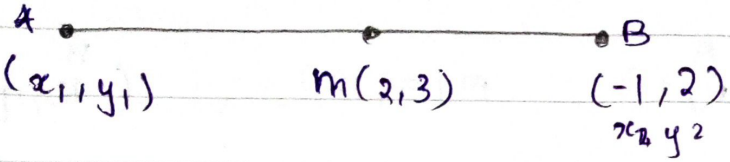
$$m(x,y) = \left(\frac{5}{2}, \frac{8}{2}\right)$$

$$m(x,y) = \left(\frac{5}{2}, 4\right)$$

∴ The mid point is $(5/2, 4)$

5) m is the mid point of a line segment AB. If m is (2, 3) and B is (-1, 2). To find the coordinate of A.

Solution



from

$$m(x,y) = \left(\frac{x_1 + x_2}{2}\right), \left(\frac{y_1 + y_2}{2}\right)$$

$$2, 3 = \left(\frac{x_1 + (-1)}{2}\right), \left(\frac{y_1 + 2}{2}\right)$$

$$2, 3 = \left(\frac{x_1 - 1}{2}\right), \left(\frac{y_1 + 2}{2}\right)$$

By comparing

$$2 = \frac{x_1 - 1}{2} \quad , \quad 3 = \frac{y_1 + 2}{2}$$

$$4 = x_1 - 1 \quad , \quad 6 = y_1 + 2$$

$$4 + 1 = x_1 \quad , \quad 6 - 2 = y_1$$

$$5 = x_1 \quad , \quad 4 = y_1$$

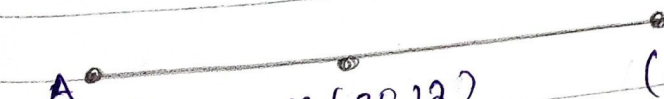
now $x_1 = 5$

now $y_1 = 4$

∴ The coordinate of A is (5, 4)

4) To find the coordinate of P and q : If the mid point of a line segment whose end point are (P, 8) and (-10, q) is (30, 12)

Solution :



from :

$$m(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$m(x, y) = \left(\frac{P + (-10)}{2}, \frac{8 + q}{2}\right)$$

$$30, 12 = \frac{P - 10}{2}, \frac{8 + q}{2}$$

By comparing :

$$30 = \frac{P - 10}{2}, \quad 12 = \frac{8 + q}{2}$$

$$60 = P - 10, \quad 24 = 8 + q$$

$$60 + 10 = P, \quad 24 - 8 = q$$

$$70 = P, \quad 16 = q$$

now

$P = 70$

now

$q = 16$

∴ The coordinate of p and q is (70, 16).