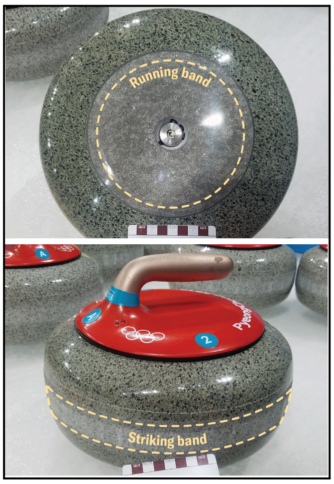
CURRLLLLLLL!!!!! - By Samuel Jarvis

As a collective we can all agree that the Winter Olympic curling has ‘swept’ across our lives appearing almost every time the television was turned on. Whilst not necessarily pure maths, more an applied area of mathematics, I was intrigued as to the many theories on how a curling stone moves along with how we can use maths to predict the outcomes of collisions based on many factors.

However, it would be a boring essay to read for you if there was no question to answer; so what is my question? There was a very large controversy surrounding the Canadian curling team due to them pushing the stone with their finger after letting go of the handle. Which leads into my goal of finding out whether this slight force added would result in a significant enough change to the stones movements rendering the gold medal winners ‘cheats’.

Firstly, we must understand the elements of curling separately before understanding how they interact. To begin, we will look at the stone.



The stones are made of two types of granite and have a mass of 20kg. To minimise friction part of the bottom is concave so only the part labelled running band can come into contact with the ice. To work out the possible contact area we will perform the calculations shown below

$$\text{area of annulus} = \pi\left[r^2 - (r-x)^2\right]$$
$$= \pi\left[0.13^2 - (0.13 - 0.006)^2\right]$$
$$= 0.0048\,m^2$$

As we are working with the area between two concentric circles we can use the formula for the area of an annulus to calculate the surface area of the running band which is 0.0048m^2. However, approximately only 1% of this surface area comes into contact with the ice (Pressure sensitive film data from Nyberg et-al Wear) due to the surface of the ice being covered by tiny microscopic bumps known as asperities so only 0.000048m^2. This means the pressure applied to the surface is 100x greater. We use force/area to work out pressure which gives us 4MPa of pressure due to these asperities.

Olympic ice is kept at -4.5 degrees celsius which is the best temperature to maintain the ‘pebbles’ - asperities on the surface. If it were colder then the stones would curl significantly less, whereas if it was warmer the asperities would melt very easily causing the surface area contact between the ice and stone to increase and as such a decrease to the pressure.

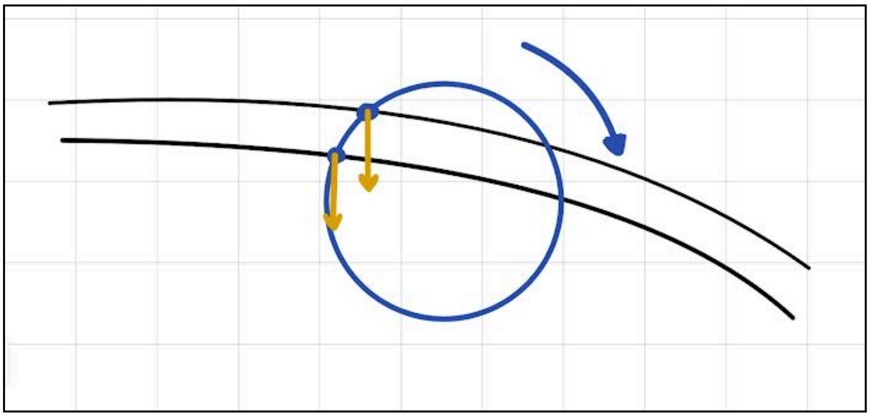
Before we look at the interactions we must understand one more aspect - sweeping. Most people assume it is used to melt the ice, and so did I. However the friction caused by sweeping does not generate enough heat to melt the ice or asperities, so what is it actually used for?

Sweeping does heat up the ice and as such means that the tensile strength, shear strength, penetration hardness and fracture toughness all decrease. The main thing we will focus on is the penetration hardness and fracture toughness as these are what affect the curl in the scratch guide curling model which I will explain later.

If the asperities' fracture toughness decreases, they are more likely to break when passed over by the stone. This then means that there are sharp tips which change the course of any future stones that pass over it. The penetration hardness decreasing means that the rough surface of the stone - which is sanded to be rough - indents more into the ice, which affects the curl - covered later.

<u>The Scratch-Guide Model:</u>
There are still many theories as to why a stone may curl; however after researching many this is the one I believe to make the most sense.

Asperities on the ice along with indentions left by previous passing stones mean that there are small divots on the ice. This means that as a rotating stone passes down the track the rough surface of the running band comes into contact with these divots causing a lateral force to act on the stone causing it to curl. A clockwise rotation would cause the stone to curl right, anticlockwise vice versa.
As shown in the diagram, we can see the bumps on the running band coming into contact with the scratches leading to a lateral force causing the curl. I have given a simplified version of this model, however, I would definitely suggest reading some of the research papers on this as it is a very interesting topic.



Whilst the curl is arguably the most important part of 'curl'ing it is not that important in answering our question. This is because in all the videos of the 'cheating' we see the Canadian player push from the centre so our rotational velocity would not change whatsoever. Therefore the average rotational velocity(1.09 rad/s) would not change so regardless of the push it will travel 0.5-1.5m laterally in the 30m it usually travels. (https://pmc.ncbi.nlm.nih.gov/articles/PMC7692518/#Sec1)

We will now look at how stones interact with one another in both 1d and 2d collisions.

To ensure we obey the laws of physics there are multiple things that must be conserved in a collision. Momentum must be conserved in elastic collisions - which due to the ice and the stones curling is almost perfectly elastic.

Two stones hitting head on(1d collisions):

Kinetic Energy: $$\frac{1}{2}mv_{1i}^2 + \frac{1}{2}mv_{2i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

$\div \frac{1}{2}m$

$$v_{1i}^2 + v_{2i}^2 = v_{1f}^2 + v_{2f}^2$$
$$v_{1i}^2 - v_{1f}^2 = v_{2f}^2 - v_{2i}^2$$
$$(v_{1i} + v_{1f})(v_{1i} - v_{1f}) = (v_{2f} + v_{2i})(v_{2f} - v_{2i})$$

Momentum: $$mv_{1i} + mv_{2i} = mv_{1f} + mv_{2f} \quad \div m$$
$$(1)\ v_{1i} + v_{2i} = v_{1f} + v_{2f}$$
$$v_{1i} - v_{1f} = v_{2f} - v_{2i}$$

Both momentum and kinetic energy must be conserved in order for us to have an elastic collision. Therefore, the sum of initial momentums must equal the sum of the final momentums, similarly with kinetic energy, hence the equations to the left. As mass is equal we have just given it value m instead of m_1 and m_2. This makes cancelling very easy as we have just been able to cancel the m's out of our expressions. The kinetic energy expression can be rearranged to get a difference of two squares expression which can be factorised as shown.

Kinetic Energy: $$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$
$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \quad (2)$$

Calculating V's: $$v_{1i} + v_{2i} = v_{1f} + v_{2f} \quad (1)$$
$$v_{1i} - v_{2i} = -v_{1f} + v_{2f} \quad (2)$$

(1) + (2) $$2v_{1i} = 2v_{2f}$$
$$v_{1i} = v_{2f}$$

(1) - (2) $$2v_{2i} = 2v_{1f}$$
$$v_{2i} = v_{1f} = 0ms^{-1}$$

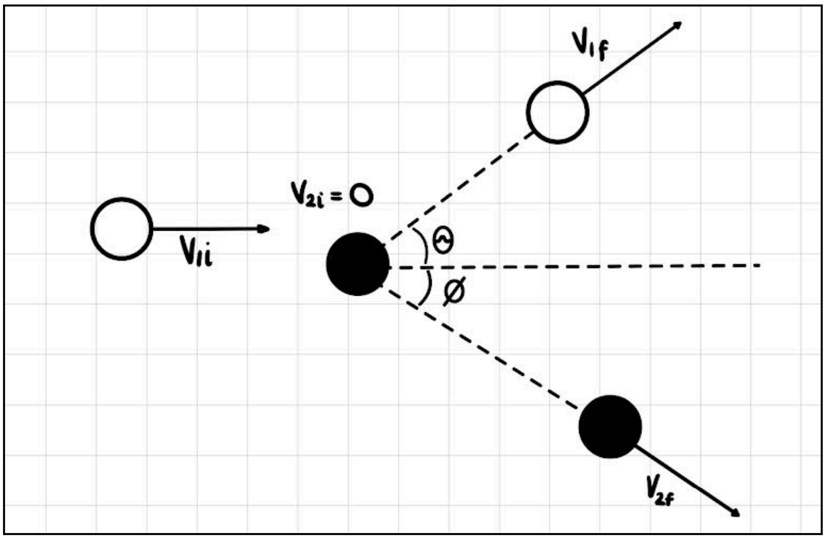
However, due to what is shown by the highlighted sections we have two equivalent brackets on either side. We can substitute in the highlighted equation and cancel out giving equation 2. Using simultaneous equations as shown in the calculations to the left we can eliminate terms. And as the second stone is stationary, its initial velocity is 0, the final velocity of the first stone is also 0. Therefore we know all kinetic energy is transferred to the second stone.

This shot would most commonly be used to clear a stone from the house, therefore returning to the initial question I don't believe that a small increase to the initial velocity of the thrown stone would affect the outcome in a shot like this as their initial stone will end up in the same spot with another continuing onwards most likely out of the scoring zone.

Two stones hitting at an angle(2d collisions):

Curling collisions are practically instantaneous; the change in time is equal to 0 and as a result there is no change in angular momentum for either object involved in the collision. As such the only reason we need to look at the angular velocity of a stone is to look at its trajectory.

Looking at the actual collision, we get the exact same expressions as for 1d essentially with a few minor differences as we separate our velocities into horizontal (x) and vertical (y) components.



As shown to the right, we can use the 1d equations to represent each dimension within our 2d collisions. We know that when an object collides with an object with equal mass at rest the two objects continue to travel in perpendicular directions and as such θ+∅=90 degrees.

Along with that, we also know that increasing the initial velocity will not change the value of these angles and instead will change the magnitudes of the final velocities. As shown, the final velocities can be seen as directly proportional to one another so as one increases so does the other. And using our simplified kinetic energy expression we can clearly see that as initial velocity increases, the sum of the final velocities squared will also increase therefore, the final velocities will increase.

Momentum:

$x \rightarrow \quad V_{1i} + \overset{(V_{2i})}{0} = V_{1fx} + V_{2fx}$

$V_{1i} = V_{1fx} + V_{2fx}$

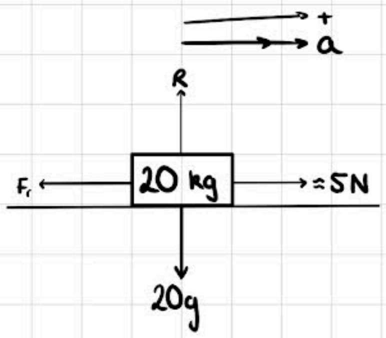
$V_{1i} = V_{1f}\cos\theta + V_{2f}\cos\emptyset$

$y \rightarrow \quad 0 = V_{1f}\sin\theta \underbrace{- V_{2f}}_{\text{opposite direction}}\sin\emptyset$

Kinetic energy $= \frac{1}{2}mv_{1i}^2 + 0 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$

$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$

Taking us back to the real purpose of this adventure into the world of curling we need to understand what this push actually does to the stone initially; to determine if the Canadians "cheating" had an impact.



y

$Res\ F = ma \quad a = 0$

$Res\ F = 0$

$20g - R = 0$

$20g = R$

$= 196$

x

$Res\ F = ma$

$5 - 1.96 = 20a$

$a = 0.152\,ms^{-1}$

$F_{max} = \mu \times R$

$= 0.01 \times 196$

$= 1.96N = F_r$

$Res F = 20a$

$0 - 1.96 = 20a$

$a = -0.098\,ms^{-2}$

As shown above, we have a force diagram along with multiple calculations. This force diagram represents the instant that the player is pushing the stone. Initially, we have used Newton's Second Law to calculate the normal contact force which is equal to the weight as there is no vertical acceleration. This then allows us to work out the maximum value of friction using a coefficient of friction of ice of 0.01 (https://www.mdpi.com/2075-4442/10/10/265) which can then be used when working out horizontal acceleration when being pushed, but also deceleration when not being pushed as shown by our two calculations above. We have assumed a small push with one finger is equal to 5N of force.

s	~
u	u
v	v
a	$0.152\,ms^{-2}$
t	≈1s

$v = u + at$

$v = u + 0.152\,ms^{-1}$

To the left we have used SUVAT to work out the new initial velocity given the extra acceleration. This will allow us to work out the distance travelled with this new acceleration comparative to without.

s	?
u	u+0.152
v	0
a	-0.098
t	~

$v^2 = u^2 + 2as$

$\frac{v^2 - u^2}{2a} = s$

$-\frac{(u+0.152)^2}{2 \times -0.098} = s_1$

$\frac{u^2 + 0.304u + 0.152^2}{0.196} = s_1$

Using SUVAT we now have an expression for distance travelled in terms of u with the push.

s	s_2
u	u
v	0
a	-0.098
t	~

$\frac{-u^2}{-0.196} = s_2$

$\frac{u^2}{0.196} = s_2$

Similarly we have one without the push.

$$s_1 - s_2 = \frac{u^2 + 0.304u + 0.152^2 - u^2}{0.196}$$

$$\Delta s = \frac{76u}{49} + \frac{722}{6125}$$

$$= \frac{76(2)}{49} + \frac{722}{6125} = 3.22m$$

We are now able to calculate the difference between these two and therefore the extra distance travelled down the track. Curling stones where distance actually matters (disregarding takeout shots for that reason), travel at approximately 2m/s (https://www.jssm.org/jssm-08-495.xml%3EFulltext). Using my calculations, we know it will therefore travel an extra 3.22m which is quite significant when you consider the scoring zone is only 3.66m. However, the force applied will vary and so will the extra distance travelled.

If we decrease the force the initial acceleration will be lower, therefore, our second initial velocity will be lower. This will then also result in a smaller distance travelled and therefore, a smaller difference, meaning a lower force would have a lesser effect.

However, due to how friction works if the force applied does not exceed the maximum value of friction(1.96N) then there will be no resultant force, hence no acceleration, hence no change in velocity. As every good applied mathematician does we have ignored air resistance, however that would just apply an extra resistive force increasing the deceleration after release along with reducing the acceleration during the push. It would also raise the 1.96N limit.

As the range of a small nudge can vary from 1N-5N it is very possible that no extra acceleration is applied. So, we have worked out the maximum extra distance travelled down the field.

Through this adventure we have found out how the stone interacts with the ice, why the stone curls, and how stones interact with one another. But now for the big question?

Would the push have a large enough effect - positive or negative for Canada - to be considered cheating? Throughout this essay we have found that the only thing the extra push would affect is the distance travelled by the stone and the velocity of the stones it hits after collisions, but ultimately that is just the extra distance of those stone/s.
Personally, I believe regardless of what change - positive or negative for the team it has - it is considered cheating, however, I will allow you to come to your own conclusion with the evidence I have presented in this essay.

Thank you for reading.