I Bet My Friend £10 That Every Number Reaches 1

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1 The Bet

It started over coffee.

My friend and I were killing time in Costa before a lecture that neither of us wanted to attend. I'd pulled up a video on my phone about an unsolved maths problem, and the rules fit on the back of a receipt. Pick any positive integer. If it's even, divide by 2. If it's odd, multiply by 3 and add 1. Keep going.

$$T(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ 3n+1 & \text{if } n \text{ is odd} \end{cases}$$

The claim is that every starting number eventually reaches 1. Nobody has ever found one that doesn't.

"That's it?" he said.
"That's it."
He put a tenner on the table. "Find me one that doesn't."

We started on the back of the receipt. I tried 6:

$$6 \to 3 \to 10 \to 5 \to 16 \to 8 \to 4 \to 2 \to 1$$

Eight steps. He tried 7, which spiked briefly to 52 before coming back down in sixteen steps. Then 15, 31, 42. All reached 1.

I suggested 27.

Twenty-seven. Smaller than most people's age. Three cubed. Nothing interesting about it. But under these rules, 27 shoots up to 9,232 before wandering back down to 1 in 111 steps. Meanwhile, 28 is finished in eighteen. We ran out of receipt somewhere around step 40.

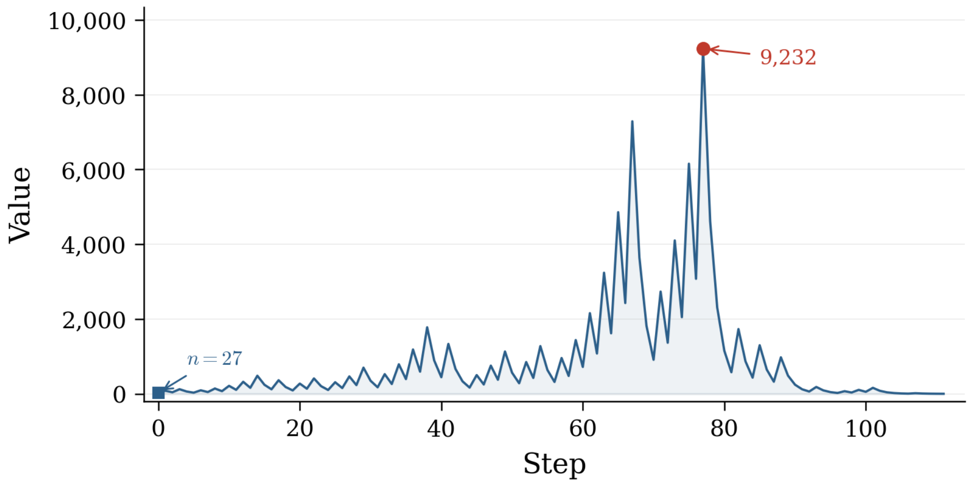

Figure 1: The trajectory of $n = 27$. It peaks at 9,232 before returning to 1 in 111 steps.

He stared at the graph on my phone. “What is *wrong* with that number?”

I didn’t have an answer. That was sort of the point.

The bet ate at me. That weekend I wrote a Python script to chase every integer from 1 to a million through the same rules. The whole thing fit in about twenty lines:

```
def collatz(n):
    steps = 0
    peak = n
    while n != 1:
        if n % 2 == 0:
            n = n // 2
        else:
            n = 3 * n + 1
        peak = max(peak, n)
        steps += 1
    return steps, peak

longest, record_n = 0, 0
for n in range(1, 1_000_001):
    steps, peak = collatz(n)
    if steps > longest:
        longest, record_n = steps, n

print(f"Longest: {record_n} ({longest} steps)")
```

Forty minutes on my laptop. Twenty-one megabytes of trajectory data. And every single number reached 1.

The longest holdout was 837,799. Five hundred and twenty-four steps. Peak altitude: 2,974,984,576. Nearly three billion, from a starting value that fits on a Post-it note. And it still came down. Meanwhile 837,800, the literal next integer over, took 144 steps. Two numbers sitting shoulder to shoulder, and one needs more than three times longer to reach the same place.

I texted him: *ran a million. all reach 1. pay up.*

He replied: *prove it for ALL of them and i’ll pay.*

Lothar Collatz proposed this conjecture in 1937 [5]. Eighty-nine years on, nobody has a proof. Paul Erdős said mathematics "is not yet ready for such problems." Terence Tao proved in 2019 that the conjecture holds for "almost all" numbers [1], which sounds like the end of the story until you realise "almost all" still leaves infinitely many unaccounted for.

So I can't claim the tenner. But I wanted to understand *why* every number I tested came down.

2 The Fingerprint

If the Collatz map were just shuffling numbers randomly, the leading digits of all trajectory values should follow something called Benford's Law:

$$P(d) = \log_{10}\left(1 + \frac{1}{d}\right)$$

This says that leading digit 1 shows up about 30% of the time, and digit 9 only about 4.6%. If you flip to a random page in an atlas, the population of whatever city you land on is more likely to start with a 1 than a 9. It sounds wrong. It is not. For digit 3, the prediction is $P(3) = \log_{10}(4/3) \approx 0.125$, or about one in eight. River lengths, stock prices, electricity bills all follow the same pattern. And Kontorovich and Miller proved in 2005 that Collatz trajectories should obey it too, at least in the long run [2].

They mostly do. Mostly.

When I checked the leading digits across all my trajectory values, digit 3 was under-represented by 6%. The chi-squared test,

$$\chi^2 \;=\; N \sum_{d=1}^{9} \frac{\big(f_{\text{obs}}(d) - f_{\text{Ben}}(d)\big)^2}{f_{\text{Ben}}(d)} \;=\; 61{,}154$$

needs about 15 to clear statistical significance. Mine was four thousand times that.

Leading digit	Observed	Benford prediction	Deviation
1	0.2979	0.3010	−1.0%
2	0.1734	0.1761	−1.5%
3	**0.1174**	**0.1249**	**−6.0%**
4	0.1146	0.0969	+18.3%

Table 1: Leading digit frequencies across over 10 million intermediate trajectory values.

And why 3? Think about what the odd step does. It multiplies by 3 and adds 1. That multiplication shoves values away from leading digit 3 and piles them into 4 and above. Where digit 3 loses, digit 4 gains. The conjecture's own constant, leaving a fingerprint on every number it touches.

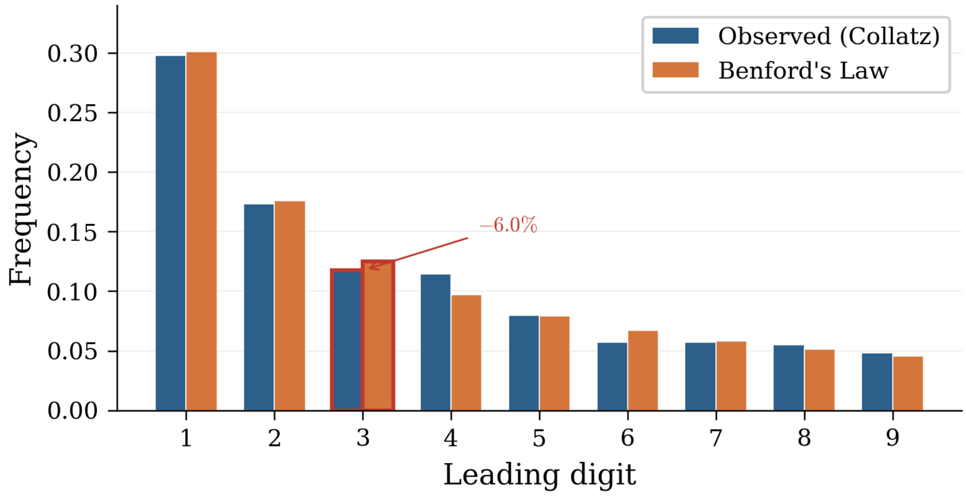

Figure 2: Observed leading digit frequencies (blue) vs. Benford's Law (orange). The deficit at digit 3 and the excess at digit 4 are both visible.

3 Acceleration Lanes

The finding I keep thinking about, though, is about what happens right after each odd step. When you compute $3n + 1$, you always get an even number. The question is *how* even.

The 2-adic valuation, written $v_2(n)$, is really just asking: how many times can you divide n by 2 before it stops being even? If $v_2 = 1$, you halve once and probably land on something odd again. If $v_2 = 4$, you get four halvings in a row and the number drops by a factor of 16.

Here is what that looks like. Take $n = 5$. The odd step gives $3(5) + 1 = 16$. Sixteen is 2^4, so it halves four times in a row: $16 \to 8 \to 4 \to 2 \to 1$. Done. Four halvings erased the damage from one multiplication, and the trajectory is finished.

For a random even number, the geometric distribution predicts how often each v_2 value should occur:

$$P(v_2 = k) = \frac{1}{2^k}$$

So divisibility by 16 should happen about 6.25% of the time. In actual Collatz trajectories, $v_2 = 4$ shows up 8.9% of the time. That's 42% above what the geometric model predicts.

v_2	Observed	Geometric prediction	Ratio
1	0.5001	0.5000	1.000
2	0.2374	0.2500	0.949
3	0.1241	0.1250	0.993
4	**0.0890**	**0.0625**	**1.424**

Table 2: Distribution of v_2 after each odd step across Collatz trajectories. The excess at $v_2 = 4$ is the largest deviation from the geometric baseline.

There is a reason for this, and it is not subtle. The v_2 of $3n+1$ is completely determined by n mod 16. Work through all eight odd residue classes: four of them ($n \equiv 3, 7, 11, 15$)

give $v_2 = 1$. Two ($n \equiv 1, 9$) give $v_2 = 2$. One ($n \equiv 13$) gives $v_2 = 3$. And one, $n \equiv 5 \pmod{16}$, sends $3n + 1$ straight to a multiple of 16, giving $v_2 \geq 4$. You can check: $3(5) + 1 = 16$, $3(21) + 1 = 64$, $3(37) + 1 = 112 = 16 \times 7$. Every time. If odd numbers in Collatz trajectories were spread evenly across these eight classes, the geometric distribution would hold. They are not spread evenly. The map creates correlations between successive odd values, steering trajectories toward the residue classes that trigger longer bursts of halvings.

The $3n+1$ operation does not produce random even numbers. It manufactures numbers divisible by large powers of 2, far more often than they should be. Four halvings in a burst. A number north of a billion crashes below a hundred million before the next odd step even arrives. I started calling these acceleration lanes. Chutes on a bumpy slope that keep pulling everything downward, firing at nearly one in eleven odd steps. The stochastic models that predict Collatz convergence assume this divisibility is random [3]. It is not. The map rigs its own dice.

4 The Nuclear Option

Next time we got coffee, he tried a different angle. "What if you change the 3?"

Fair point. So I went home and ran the same analysis with $5n + 1$ and $7n + 1$ instead.

With $5n + 1$, 97% of starting values diverge to infinity. They climb and never come back. With $7n + 1$, it is 99.8%. The trajectories don't just grow slowly. They rocket upward and never return.

Variant	Convergence to a cycle
$3n + 1$	100%
$3n - 1$	100% (three distinct cycles)
$5n + 1$	3%
$7n + 1$	0.2%

Table 3: Convergence rates for Collatz-type maps with different odd-step multipliers.

The critical number turns out to be the ratio $a/2$, where a is the multiplier. For $3n+1$, that ratio is $3/2 = 1.5$. For $5n + 1$, it is $5/2 = 2.5$. Push past 2 and everything blows up.

Why 2? Count the steps. In my trajectories, about 31.5% of steps are odd and 68.5% are even. That works out to roughly two halvings for every one multiplication, which is exactly the expected value of the geometric distribution:

$$E[v_2] \;=\; \sum_{k=1}^{\infty} \frac{k}{2^k} \;=\; 2$$

So each full round of the game multiplies by a and then divides by about $2^2 = 4$. The net factor per round:

$$\frac{a}{4}$$

When $a = 3$, that is $3/4$, and the number shrinks each round. When $a = 5$, it is $5/4$, and the number grows forever. More precisely, the trajectory contracts on average when

$$f_{\text{even}} \cdot \ln\left(\tfrac{1}{2}\right) + f_{\text{odd}} \cdot \ln(a) \;<\; 0$$

With two even steps for every odd step, this reduces to $a < 2^2 = 4$. The knife edge sits right there.

And so does everything I found. The Benford fingerprint at digit 3? Gone. The acceleration lanes at $v_2 = 4$? Gone. All the structure I spent weeks documenting dissolves the instant you swap a 3 for a 5.

The whole conjecture sits on the fact that 3/2 is just barely less than 2. Move that single number, change nothing else, and a million trajectories that all obediently reached 1 now fly off toward infinity and never come home.

I asked if he wanted to double the bet. He did not.

5 The £10

I showed him the variant results over our next coffee. His take: "so 3 is just built different."

Pretty much. The map leaves a fingerprint at digit 3, builds its own fast lanes through divisibility by 16, and falls apart the instant you touch its one constant. Every scrap of evidence I have says these sequences want to come down.

But wanting is not the same thing as proving. Barina verified the conjecture computationally up to 2^{68} in 2021 [4]. That is over 295 quintillion individual numbers, every one of them reaching 1. Still not a proof. Still not enough for the tenner.

He brings it up every time we get coffee. I tell him I'm close. He tells me Erdős said maths isn't ready.

The £10 is still on the table.

References

[1] Tao, T. (2019). "Almost all orbits of the Collatz map attain almost bounded values." *arXiv:1909.03562*.

[2] Kontorovich, A.V. & Miller, S.J. (2005). "Benford's Law, values of L-functions and the 3x+1 problem." *Acta Arithmetica*, 120(3), 269–297.

[3] Lagarias, J.C. & Weiss, A. (1992). "The 3x+1 problem: Two stochastic models." *Annals of Applied Probability*, 2(1), 229–261.

[4] Barina, D. (2021). "Convergence verification of the Collatz problem." *The Journal of Supercomputing*, 77, 2681–2688.

[5] Lagarias, J.C. (1985). "The 3x+1 problem and its generalizations." *American Mathematical Monthly*, 92(1), 3–23.