The Something That Fills the Universe and Can't Be Seen

Dark Energy, Infinity, and the Paradoxes of a Boundless Cosmos

I. A Universe Full of Something We Cannot See

In 1998, two separate research projects - the Supernova Cosmology Project and the High-Z Supernova Search Team - made a completely bewildering scientific discovery. Both teams measured the luminosity of Type Ia supernovae as standard candles on cosmological scales. Distant supernovae were observed to be consistently dimmer than predicted. Not only did the universe appear to expand, but it was also expanding at an accelerating rate. Some unseen power was causing the expansion to accelerate further and further away.

The discovery by Saul Perlmutter, Brian Schmidt, and Adam Riess that won them the 2011 Nobel Prize in Physics brought back a concept Albert Einstein used in 1917 and then discarded as "the greatest blunder" he ever made: the cosmological constant, or λ (Lambda), today referred to as dark energy. According to measurements from the ESA's Planck satellite (2018), dark energy accounts for about 68.3 percent of the total energy density of all observable space. Baryonic ordinary matter makes up only 4.9 percent,t and dark matter is estimated to make up 26.8 percent. Dark energy is not just lar; it has some unusual structural properties. It doesn't clump together due to gravity. It doesn't dissipate into less dense regions as space expands. Its equation of state parameter (w) is -1; therefore, it exerts a form of negative pressure (cosmic anti-gravity), and, regardless of how much space there is, its energy density is relatively stable at about 6×10^{-10} J/m^3. As space continues to expand, new dark energy appears with every additional region of space and thus maintains a consistent density everywhere.

II. Galileo's Paradox: When Infinity Breaks Common Sense

In his essay from 1638 titled "Discourses and Demonstrations in Mathematics" (Two New Sciences), Galileo Galilei posed a very simple yet one of the most important mathematical problems that had never been previously addressed. We begin with the set of all natural numbers: {1, 2, 3, 4, 5, ...}. Now let us consider only the perfect squares from that set: {1, 4, 9, 16, 25, ...}. By common sense - and as stated by Euclid in one of his axioms, "the whole is greater than a part" - we would believe that there cannot be the same number of perfect squares as there are natural numbers, because most natural numbers are not; they are not perfect squares.

However, Galileo has shown us that we would naturally match each natural number (n) with a perfect square (n^2), and vice versa, since every perfect square also has an associated integer, its square root. Therefore, there is a one-to-one relationship (i.e., correspondence) between the two sets; that is, neither set has any surplus elements - thus the two sets have equal cardinality!

Galileo did not solve the paradox of apparent differences between the two infinite sets; he stated simply that "greater than" "less than", and "equal" cannot be applied in the case of infinite numbers. The paradox, therefore, was left "unresolved."

Georg Cantor’s formalized theory of infinite sets settled this conundrum 2½ centuries later. In Cantor’s definition of two sets possessing identical cardinalities (identical sizes of ‘infinity’), two sets possess the same cardinality if they can be matched/bijective. Using this definition, the integers and the perfect squares are both the same size (cardinality = $\aleph_0$ – aleph null – smallest infinite cardinal) and are both countably infinite. Using his (famous) diagonal argument, Cantor showed the real numbers are at least one level larger than the integers. That means that there are an infinite number of levels of infinity that are ordered in a hierarchy with no upper limit. The infinite is not one large landscape but an immense variety of infinitely sized landscapes.

III. Hilbert's Grand Hotel: Infinity as an Active, Living Structure

Cantor created an underpinning theory and David Hilbert provided the conceptual backbone of his setup. One of the most recognized methods to visualize and clarify the concept of infinite cardinality happened around 1924 when Hilbert proposed a thought experiment that so far is the best definition of infinite cardinality.

If you have a hotel with an infinite number of rooms numbered 1,2,3,4,... and every room is full, can you change that to accommodate a new guest?

A finite-size hotelier would say; "Sorry! We're full!" An infinite-size hotelier would say, "Welcome! Please give me a moment." The hotelier asks every guest in room n to move to room n + 1 so all guests will have moved down one room. Now room number one will be empty for occupancy by the new guest. Thus, the hotel is full of guests, and still, it has one new guest occupying room number one.

Now, let's consider an infinite number of new guests arrive simultaneously at the hotel. The hotelier would ask all current guests to move from room n to room 2n, with the result that now all even-numbered rooms are filled with guests. When current guests move to their new rooms, that results in all odd-numbered rooms being empty. Therefore, the hotel can comfortably accommodate each new guest without displacing any current guests.

What the hotel demonstrates operationally is the defining property of an infinite set: The countable infinite set can be matched in a one-to-one relationship with a proper subset of itself, which is mathematically named Dedekind-infinite. This property cannot happen with finite sets, yet it is a defining property of infinite sets.

That is not a trick or a logical error. It is the arithmetic of infinity — a system that operates by rules radically unlike the mathematics of finite. And as we see, it is also, quite precisely, the arithmetic of dark energy.

IV. The Big Bang: An Infinite Beginning, or a Finite One?

Cosmology is required to exercise extreme caution because there is a pervasive and powerful misunderstanding about the beginning of the universe: the Big Bang was initiated by an infinite amount of energy. The actual story has more subtleties. In the case of general relativity, it can be shown (via calculations) that $t = 0$ represents the time of the Big Bang, when the scale factor of the universe was zero ($a(t) \to 0$). As such, the energy density (mass/volume) and the curvature of spacetime (the elasticity of spacetime) diverge from one another, i.e. both go to infinity in the equations at that time. This is what is described as the singularity of the universe at the moment of creation. Nonetheless, physicists do not regard this "infinity" as a meaningful description of reality, rather it is important to understand that divergence from a physical law indicates that a physical theory has been used in an inappropriate manner.

General relativity is a classical theory. At the Planck scale (less than the Planck length of about 1.6×10^{-35} m and prior to the Planck time of about 5.4×10^{-44} s), the framework of quantum gravity must be applied to fully understand physics. In this way, we can view the Big Bang singularity as an incomplete mathematical construct: it is not an actual event, but rather a point in time beyond which our current understanding of physics is incomplete. The moment of the Big Bang singularity represents a point of Galilean nature: that point in time at which we must agree our mathematical theories cannot provide an answer.

In a scientific sense, it can be stated that the original condition for the universe has an exceptionally high physical density of energy and temperature. Moreover, a notable hypothesis based on General Relativity was proposed to establish whether the Universe's total quantity of energy may equal zero. In this conception, the Universe is viewed as being "zero net energy" (the positive energy contained in all matters and radiation, and the negative gravitational potential energy), and therefore all energy is negated or cancelled; the physical structure of the Universe is extremely large yet has zero cost from an energetic perspective. It was created from an immense fluctuation or variance of the vacuum state with no physical justification for cost associated with its creation.

V. Dark Energy and the Hotel That Never Fills

The cosmological constant Λ appears in Einstein's field equations of general relativity as a term representing the energy density of empty space. Its observed value is:

$$\boldsymbol{\Lambda \approx 1.1 \times 10^{-52}\ \mathrm{m}^{-2}}$$

This extremely small value plays a significant role in determining the fate of the universe long-term. To understand why, look at how different kinds of energy scale with the expansion of the universe. The scale factor of the universe is denoted as a(t). The energy density of ordinary baryonic matter scales like a^-3 (it becomes less dense as the volume increases) and the energy density of radiation scales like a^-4 (becoming less dense as volume increases). Dark energy with a value of w=-1 has a constant energy density regardless of the size of the universe.

One way to think about this is that it is analogous to Hilbert's hotel but in the physical universe. Each cubic meter of new space has dark energy inside of it at the same energy density of ~6 × 10^-10 J/m^3. No previous cubic meter of space has lost energy from this influx of new cubic meters; the hotel is always full yet always expands and has room for more. When new cubic meters of space are created, they already have energy from dark energy inside them, even before those new cubic meters are created.

The mathematical implication here is quite clear: Once the universe continues to expand forever (as predicted via current models where $\Lambda > 0$ and $w = -1$), the total amount of dark energy will continue to increase indefinitely. At the limit where $t \to \infty$, the total amount of dark energy will also approach an infinite amount. Thus, as the universe expands indefinitely, it will asymptotically approach a de Sitter space which is an expanse of spacetime that is maximally symmetrical, expanding exponentially due to the cosmological constant, emptying out all matter eventually except for the dark energy itself (which is infinite and cold) via cooling, and being filled everywhere uniformly with something which is not visible to anyone.

This leads to a larger problem than just a number crunching issue, but rather a philosophical abyss concerning the cosmological constant. For example, quantum field theory predicts that the vacuum energy density for the universe should be approximately 10^{96} kg/m^3 due to the energy resulting from quantum fluctuations; however, actual observations indicate that it is much, much lower (~10^{-27} kg/m^3). This discrepancy is therefore one of the largest discrepancies in physics (on the order of 10^{123}—123 orders of magnitude difference) as it is described to be the greatest quantitative prediction error in physics history.

VI. Two Paradoxes, One Universe

Galileo's paradox shows that in infinity, parts can equal their whole dark energy prove this physically by never diluting while matter and radiation fades to zero. Hilbert's Hotel shows infinity as an endless, self-filling process just like dark energy, which pre-fills every new cubic meter of space without ever depleting.

The pairing of these two paradoxes gives rise to an interesting possibility: Dark energy may be the physical representation of infinity itself within the Universe. Not the disorganized infinity of the Big Bang singularity, but rather the ordered and consistent infinity of the systems described by Cantor and Hilbert. Furthermore, just as $\aleph_0$ and $\aleph_1$ are two unique groups within the infinite hierarchy, dark energy is in a different group than matter with respect to its ultimate dominance in the Cosmos.

Just as $\aleph_0$ and $\aleph_1$ belong to different, incomparable tiers of the infinite hierarchy, dark energy and matter belong to different tiers of cosmic dominance. Matter fades toward zero as the universe expands, while dark energy grows without bound; they are simply not comparable in the long run.