Does he love me?

By Ella Baddick

I - alongside many others - have spent a fair amount of time playing the wonderful game of ‘does he love me?’. The game focuses on pulling the petals off a daisy one by one, each time alternating between the phrase “he loves me” and “he loves me not”. The purpose being that whichever phrase is spoken as the last petal is pulled is the truth.

Unsurprisingly, I never thought it a coincidence that I was “loved” the majority of the time. I now realise that was not solely down to my luck and charm. (If you find the maths boring the way to increase your odds of love is at the end! (section 3.0))
(The more complicated maths comes later, check section 4.1)

This essay explores how the game “does he love me?” is so successful, and expands upon the role of the Fibonacci sequence in a plant’s growth.

1.0 Fibonacci Sequence

1.1 Sequence

source 12

The Fibonacci sequence is formed by adding the two previous numbers to form the next.

Ex: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 et cetera

You will notice a pattern, ⅔ of the numbers are odd.

Ex. even, odd, odd, even, odd, odd, even, odd, odd, even, odd, odd, even

1.2 Proof

Let us explain this pattern.

An integer can be represented by a constant, n.

1.2.1 Even Numbers

An even number can be represented by $2n$.

Even numbers can also be represented by $(2n+2)$ or $(2n+4)$ et cetera.
This is as even number + even number = even number.
Ex. we have established $2n$ is even. $2n + 2n = 4n$. $4n$ is divisible by 2 ($=2n$) and $2n$ is even. Therefore even + even = even.

To further prove this, take $2n+2$
Ex. if n is 3, $2n = 2(3) = 6$.
So, $(2n + 2)$
$= (2(3) + 2)$
$= (6 + 2) = 8$.
8 is an even number

1.2.2 Odd Numbers

A similar theory can be applied to odd numbers.
An odd number can be represented by (2n + odd number).

Let us demonstrate this.
Ex. $n = 3, (2n + 1)$
$= (2(3) + 1)$
$= (6 + 1) = 7$ (odd)

1.2.3 Some more addition

Fibonacci Pattern

Ex. even, odd, odd, even, odd, odd, even, odd, odd, even, odd, odd, even

Let us work through the sequence together.

even + odd = odd

To begin, **even + odd**
Ex. even, odd, odd, even, odd, odd, even, odd, odd, even, odd, odd, even

Even = $(2n + 6)$
Odd = $(2n + 1)$

$(2n + 6) + (2n + 1) = 4n + 7$

This can be rewritten as $2(2n + 3) + 1$ | $4n + 6 + 1 = 4n + 7$

$2(2n+3)$ must be even, as it is an integer $(2n+3)$ that has been doubled.

This means $4n + 7$ is one more than an even number, making it odd.

odd + odd = even

Next, **odd + odd**
Ex. even, odd, odd, even, odd, odd, even, odd, odd, even, odd, odd, even

Odd = $(2n + 1)$
Odd = $(2n + 5)$

$(2n + 1) + (2n + 5) = 4n + 6$

This can be rewritten as $2(2n + 3)$

$2(2n+ 3)$ must be even as it is an integer that has been doubled.

odd + even = odd

Next, **odd + even**
Ex. even, odd, odd, even, odd, odd, even, odd, odd, even, odd, odd, even

As is with all addition, the order in which numbers are added does not affect the outcome.
Therefore, the proof above for even + odd remains, and results in an odd number

1.2.4 The Pattern

As we can see, the pattern is destined to continue repeating even, odd, odd.
Therefore, it is important to note ⅔ of the numbers in the sequence will be odd.

So how can this relate to my belief that some people were destined for me?

2.0 Daisies

2.1 Playing the Game

In the game, you always begin with the phrase "he loves me".
This means you will end on "he loves me" if there is an odd number of petals.

Example
Petal 1: he loves me
Petal 2: he loves me not
Petal 3: he loves me
Petal 4: he loves me not
Petal 5: he loves me

2.2 Why a daisy?
source 15

Of course, for a guarantee of love, a buttercup may be a wiser choice as they commonly have 5 petals.

However, "daisies tend to have 34, 55, or 89 petals".
Can you spot the link? All these numbers fit into the Fibonacci sequence.

Notice there is one even number (34) and two odd numbers (55, 89).

3.0 Conclusion
source 15

We can therefore assume that if we pick a daisy and begin on the phrase "he loves me", approximately ⅔ of the time we will end on "he loves me".

SCROLL

for the best bit!

4.0 Bonus

Where else is the Fibonacci sequence?

4.1 Fibonacci in Growth

source 15

Figure 1

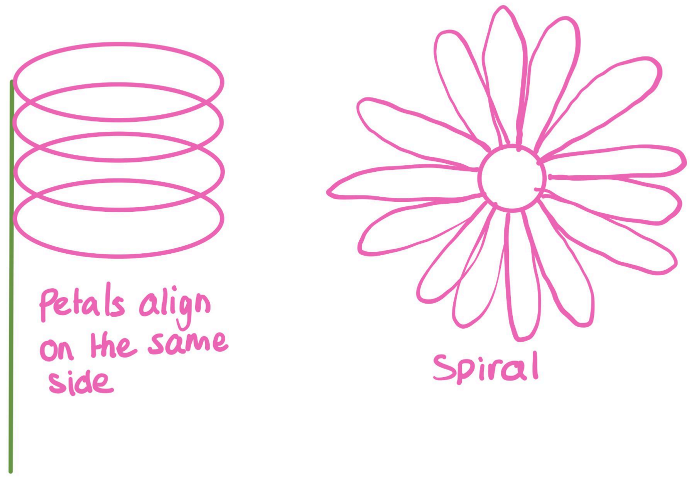


Figure 1 shows the difference between growth at a 1:1 ratio versus the spiral pattern

Plants grow from shoots. At the tip of the shoot (the meristem) cells grow. The meristem produces cells one at a time, rotating in a spiral pattern. Only certain intervals between each growth during rotations produces a viable plant.

Imagine if a petal was grown once per rotation, all the petals would align on the same side.
The ratio of petals : number of turns is commonly the ratio of numbers found within the Fibonacci sequence.
An example of this would be a willow, with a ratio of 3:8.

source 13

Another way of approaching this idea would be to look at how the spirals extend from the centre.
Each time a spiral begins it extends either left or right, alternating each time.
The number of spirals extending to the left can be put in ratio to the number of spirals extending to the right.
Commonly this ratio will be composed of two consecutive numbers from the Fibonacci sequence.
A great example of this is the Aloe Polyphylla, which has a ratio of 2:3.

4.1.1 Divergence Angle

source 4

Again, imagine if a petal was grown once per rotation, all the petals would align on the same side (Figure 1). So, there must be an angle or ratio that allows for the best possible distribution of petals/leaves.

Looking at a plant from above, if a line was drawn from the stem to a leaf, and then the stem to the next leaf, the angle is generally fixed. This is the divergence angle (most prevalently 137.5*).

Assuming sun and rain are orthogonal to the plane of the leaf, you would need an angle such that higher leaves block lower leaves as little as possible.

4.1.2 The most irrational number

source 3

(phi written as ϕ and φ)

Bringing back the idea of rotations, when a rotation is rational each turn, petals end up stacking on top of each other. This forms straight line spokes.

However, we want the leaves to spread out as much as possible, so the most irrational number will produce the best result. (Figure 3)

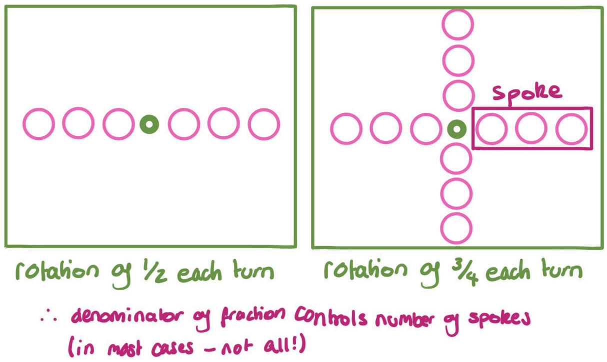
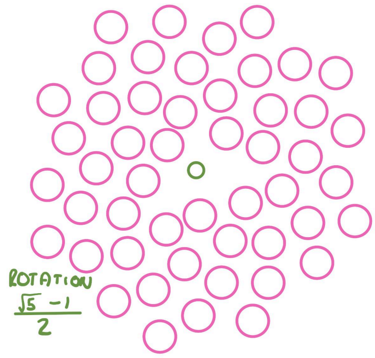


Figure 2



I created this by tracing an image used in the video "The Golden Ratio (why it is so irrational)" by Numberphile on YouTube.

Figure 3

So how do we find the best result?

Continued Fractions

A continued fraction is when you take a value, for example π.

$\pi = 3.14... = 3 + \text{a fraction}$

From this, you take the "fraction" and make it 1/value.

$3.14... - 3 = 0.14...$

$(0.14...)^{-1} = 7.06...$

This will give you a value which you can repeat the process with.

$7.06... - 7 = 0.06...$

$(0.06...)^{-1} = 15.9...$

If the beginning number is irrational, this process can be repeated infinitely.

Continued fraction for π

$$3+\cfrac{1}{7+\cfrac{1}{15+\cfrac{1}{1+\cfrac{1}{292+\cfrac{1}{\text{...(and so on)}}}}}}$$

As you can see, there is a jump from a small to a large value.
As it is 1 / value, 1/292 + etc will produce a very small number.

This means very little accuracy is added at this point.
This demonstrates that π is approximated well, very early on.

Therefore, we can say the most irrational number would be the one with the continued fraction that has **no large numbers**.

(As large numbers on the bottom of the fraction indicate little accuracy added, meaning the original value is easier to approximate and is "less" irrational.)

Continued fraction for the "most irrational number"

Step 1 (*source 3, 6)*

$$x = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\ldots}}}}}$$

...(and so on)

Step 2

As you can see, the red can be valued as x as well.

This means we can rewrite x as

$$x = 1 + \frac{1}{x}$$

this can now be solved as a quadratic

$$x^2 = x + 1 \quad | \quad x^2 - x - 1 = 0$$

Step 3

now to complete the square

$$x^2 - x - 1 = 0 \rightarrow (x - 0.5)^2 - 1.25 = 0 \rightarrow (x - 0.5)^2 = 1.25$$

$$\rightarrow x - 0.5 = \pm\sqrt{\frac{5}{4}} \rightarrow x = 0.5 \pm \sqrt{\frac{5}{4}} \rightarrow \mathbf{x = 0.5} \pm \frac{\sqrt{5}}{2}$$ = most irrational number (= ϕ, phi)

And so, we can determine that a rotation of $0.5 \pm \frac{\sqrt{5}}{2}$ per turn would give the best distribution possible. (As seen in Figure 3).

Linking back, we can arrive at the divergence angle (137.5°). *source 4*

As $360 - \frac{360}{\varphi} = 137.5$ or $\frac{360}{\varphi^2} = 137.5$

4.1.3 Useful Values

source 5

surd	representation	decimal (3dp)
$\frac{1+\sqrt{5}}{2}$	φ	1.618
$\frac{1-\sqrt{5}}{2}$	φ	- 0.618
$\frac{-1+\sqrt{5}}{2}$	$\frac{1}{\varphi}$	0.618
$\frac{-1-\sqrt{5}}{2}$	$-\varphi$	- 1.618
$1 + \frac{1-\sqrt{5}}{2}$	$1 - \frac{1}{\varphi}$ OR $\frac{1}{\varphi^2}$	0.382
$1 + \frac{1+\sqrt{5}}{2}$	φ^2	2.618

4.1.4 The Phyllotactic Ratio and the Fibonacci Sequence

source 4

The phyllotactic ratio is defined as the fraction of a circle a leaf (or petal) turns from the previous leaf (or petal).

$$Phyllotactic\ Ratio\ =\ \frac{1}{\varphi^{2}}\ \approx\ \frac{Fn}{Fn+2}$$

So, this will explain how plants use the Fibonacci sequence to reach the phyllotactic ratio and maximise distribution of leaves.

Take the ratio $\frac{Fn}{Fn+2}$ and the sequence

source 7

F_n | n =	0	1	2	3	4	5	6	7	8	9	10	11	12
value	0	1	1	2	3	5	8	13	21	34	55	89	144

<u>when</u>

n = 4

$$\frac{F4}{F4+2}\ =\ \frac{3}{8}\ =\ 0.375$$

n = 10

$$\frac{F10}{F10+12}\ =\ \frac{55}{144}\ =\ 0.381944...$$

n = 15

$$\frac{F15}{F15+2}\ =\ \frac{610}{1597}\ =\ 0.3819661866$$

source 4

As n (*number of leaves - we established earlier they are typically values in the Fibonacci sequence in section 2.2*) approaches infinity, the phyllotactic ratio converges to the value of $\frac{1}{\varphi 2}$ $(=\ \frac{1}{(\frac{1+\sqrt{5}}{2})^{2}}\ \approx\ 0.382)$.

And the value of $\frac{1}{\varphi 2}$ allows us to approach the divergence angle (360 / ϕ^2 = 137.5).

Therefore, some plants use the Fibonacci sequence to reach the phyllotactic ratio and maximise distribution of leaves.

4.2 Benefits of Fibonacci

source 13

The benefits of the distribution we just determined are as follows.

<u>Reproductive Success</u>

The example of sunflowers is beneficial here, with their seeds being packed in a spiral pattern. This maximises the space used for seeds, helping to optimise the success of new sunflowers being produced.

<u>Maximising Growth</u>

When focusing on leaves, we can see that the spiral arrangement of leaves also maximises their exposure to sunlight, benefiting their ability to photosynthesize.

4.3 So is the universe derived from maths or is this all by chance?

It is only natural (get it?) to question how things can grow so precisely.

However, Minjoo Kim eloquently described that she thinks the best answer to whether maths was invented or discovered is that "We invented math as a language and as symbols to communicate the patterns that we discover in the universe." *source 1*

Personally, I believe that the Fibonacci sequence and the most irrational number do not appear by chance. I believe they are there to allow the most successful game I have ever played to be created. And yes, he did love me in the end.

Sources

3. video

Figure 1, Figure 2, Figure 3 drawn by me.

4. Phi

5. Phi

6. Phi

1. Tedx

2. define phyllotaxis

7. video

8. video

9. Phi

10. Phi

11. Paper

12. Website.

13. Website.

14. Article

15. Article

16. PowerPoint

17. video

18. video