How do patterns in nature relate to mathematical rules?

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Patterns in nature are recurring structures found across biological and physical systems, from the stripes of animals to oscillations in population dynamics. These patterns are not merely aesthetic, but arise from underlying mathematical rules that govern how systems are structured and how they evolve. Mathematics provides a framework not only for describing such patterns, but for explaining their origin. In many cases, these patterns are generated directly from simple rules, such as optimisation principles or feedback mechanisms, revealing that both the structure and behaviour of nature are deeply mathematical.

“To bee or not to bee” may be a playful phrase, yet the structures created by bees reflect a deeper mathematical principle: nature often adopts optimal solutions. A honeycomb utilises a hexagonal tessellation pattern in order to create an efficient storage unit for honey, pollen or larvae (MathWorld, 2024). Bees face a tiling problem motivated by maximising the area of each cell while simultaneously minimising the amount of wax and energy required to construct it.

To determine which shapes can tile the plane with complete efficiency, we consider the interior angle condition. For a regular n-gon, the interior angle is

$$\frac{180(n-2)}{n}.$$

For tessellation, angles around a point must sum to 360°, giving

$$\frac{360}{\frac{180(n-2)}{n}} = k, \quad k \in \mathbb{Z}^+,$$

which simplifies to

$$\frac{2n}{n-2} = k.$$

Solving:

$$2n = nk - 2k$$

$$2n - nk = -2k$$

$$n(2 - k) = -2k$$

$$n = \frac{-2k}{2-k} = \frac{2k}{k-2}$$

$$n = \frac{2k - 4 + 4}{k-2} = 2 + \frac{4}{k-2}$$

Since $n \in \mathbb{Z}$, we require $k - 2 \mid 4$.

$$k - 2 \in \{-4, -2, -1, 1, 2, 4\}$$

$$\frac{4}{k-2} \in \{-1, -2, -4, 4, 2, 1\}$$

$$n = 2 + \frac{4}{k-2} \Rightarrow n \in \{1, 0, -2, 6, 4, 3\}$$

Since $n \geq 3$ for a polygon:

$$n = 3, 4, 6.$$

Solving this yields $n = 3, 4, 6$, corresponding to the equilateral triangle, square, and hexagon. Thus, the set of possible tiling shapes is mathematically constrained, showing that the structures observed in nature arise from strict geometric conditions rather than arbitrary choice.

We can utilise an isoperimetric-type argument to justify why a hexagon is the most efficient shape. It is known that, for a given perimeter, a circle maximises area. However, a circle cannot tessellate without leaving gaps. Hence, we require the largest polygon that satisfies the tessellation condition. The larger the polygon inscribed within a circle, the closer it approximates the circle. Therefore, among the candidate shapes, the hexagon provides the most efficient tiling.

To formalise this, we compare the ratio of area to perimeter for each candidate. For a regular hexagon of side length x,

$$\frac{\text{Area}}{\text{Perimeter}} = \frac{\frac{3\sqrt{3}}{2}x^2}{6x} = \frac{\sqrt{3}}{4}x.$$

For a square,

$$\frac{\text{Area}}{\text{Perimeter}} = \frac{x^2}{4x} = \frac{1}{4}x,$$

and for an equilateral triangle,

$$\frac{\text{Area}}{\text{Perimeter}} = \frac{\frac{\sqrt{3}}{4}x^2}{3x} = \frac{\sqrt{3}}{12}x.$$

Thus, the hexagon maximises area per unit perimeter among tessellating shapes. This shows that the structure of the honeycomb arises directly from optimisation, demonstrating how mathematical rules constrain and generate efficient natural patterns.

Figure 1: Honeycomb structure illustrating hexagonal tessellation in nature.

The familiar expression that a zebra "never changes its stripes" captures an essential feature of many natural patterns: their permanence. Beneath the surface of simplicity lies a complex mathematical order shaping their formation and structure. A Turing pattern forms when two chemicals interact in a developing organism. One chemical, the activator, encourages the production of pigment, whilst the other, the inhibitor, spreads to prevent pigment from forming nearby. The interaction between these opposing processes generates patterns that we recognise as the stripes of a zebra.

Such patterns reveal that the beauty of natural structures arises not from randomness, but from mathematical constraints that enforce order, symmetry, and efficiency. This suggests that our perception of beauty may stem from a cognitive recognition of underlying mathematical order.

Figure 2: Zebra stripe pattern as an example of a static natural pattern.

While static patterns reveal beauty through symmetry and structure, many natural systems exhibit patterns that evolve over time. In such systems, mathematics governs not only form, but change itself, giving rise to patterns that are dynamic rather than fixed. Predator–prey interactions, described through the Lotka–Volterra model, clearly demonstrate this (Britannica, 2024).

The model is given by

$$\frac{du}{dt} = Au - Buv, \qquad \frac{dv}{dt} = Cuv - Dv,$$

where u represents prey and v represents predator populations. The terms Au and Buv represent prey reproduction and predation respectively, while Cuv and Dv represent predator growth and natural decline.

To simplify the system, we non-dimensionalise by rescaling variables so that the constants can be normalised to unity, reducing the number of parameters without changing the qualitative behaviour of the model. This gives

$$\frac{du}{dt} = u - uv = u(1 - v), \qquad \frac{dv}{dt} = uv - v = v(u - 1).$$

The steady states are found by setting

$$\frac{du}{dt} = 0, \qquad \frac{dv}{dt} = 0,$$

giving

$$u(1 - v) = 0, \qquad v(u - 1) = 0,$$

and hence

$$(u, v) = (0, 0), \quad (1, 1).$$

The nullclines $u = 1$ and $v = 1$ divide the phase plane into regions. From

$$\frac{du}{dt} = u(1 - v), \qquad \frac{dv}{dt} = v(u - 1),$$

it follows that if $v < 1$, then $\frac{du}{dt} > 0$, while if $v > 1$, then $\frac{du}{dt} < 0$. Similarly, if $u > 1$, then $\frac{dv}{dt} > 0$, whereas if $u < 1$, then $\frac{dv}{dt} < 0$. These sign changes determine the direction of motion, producing closed trajectories around the equilibrium $(1, 1)$.

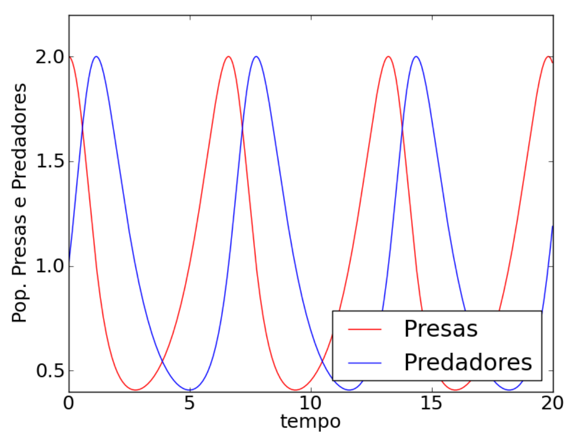

Figure 3: Oscillatory predator–prey populations over time.

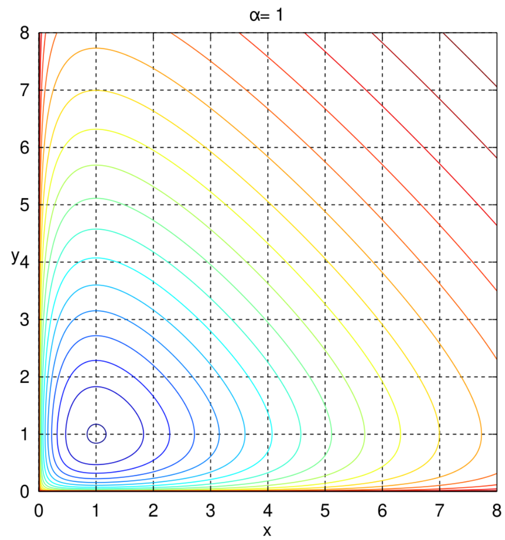

Figure 4: Phase plane showing cyclic trajectories around equilibrium.

The significance of this model lies in the behaviour it predicts. The system possesses a steady state in which both populations coexist. However, rather than converging to this equilibrium, the system evolves in continuous cycles around it. This occurs due to the feedback mechanism within the equations: an increase in prey leads to a delayed increase in predators, which subsequently reduces the prey population, causing predator numbers to decline and the cycle to repeat.

When represented with respect to time, these interactions produce oscillatory patterns characterised by regular cycles of growth and decline, with predator populations consistently lagging behind those of prey. In natural systems, populations rarely remain constant, instead exhibiting recurring cycles of growth and decline. These cycles arise from the mathematical structure of the model, which governs the interaction between species and constrains their behaviour to follow repeated patterns over time (Khan Academy, 2020).

This behaviour is evident in the phase plane, where the system traces closed orbits around the coexistence equilibrium, showing that oscillations arise naturally from the structure of the system.

Beyond their scientific significance, these oscillations exhibit a form of dynamic beauty. Rather than arising from fixed symmetry, this beauty emerges through rhythm and balance over time. The system maintains equilibrium through continuous motion, with opposing forces (predator and prey) coupled in a sustained cycle. This suggests that beauty in nature is not limited to structure, but also arises from the unfolding of mathematically governed processes.

Unlike static patterns, this form of beauty emerges through change itself, suggesting that mathematics governs not only the structure of nature, but the processes through which it unfolds.

This raises a deeper question: is beauty an inherent property of nature, or is it imposed by the human mind? The patterns observed in nature are governed by mathematical rules regardless of human perception, yet it is our ability to recognise order, symmetry, and regularity that allows us to perceive them as beautiful. It may be that mathematics acts as a bridge between the external world and human cognition, with patterns appearing beautiful precisely because they align with the way we process structure and predictability. In this sense, we do not simply observe beauty in nature, but recognise it through an innate sensitivity to mathematical order.

In conclusion, patterns in nature are not arbitrary, but arise from underlying mathematical rules that govern both structure and change. From the efficient geometry of honeycombs to the oscillatory behaviour of predator–prey systems, mathematics acts as both a descriptive language and a generative force. It shapes not only how natural systems are organised, but how they evolve over time. Ultimately, the connection between mathematics and nature extends beyond explanation, offering insight into why such patterns are perceived as beautiful, and suggesting that the order we observe in the natural world is fundamentally mathematical in origin.

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