Essay 4+(4×(4−4)):
4 Cool probability problems that came to me in my bed (Yes really, well the first one the rest not so much but I digress) (Truly I only need to start making weird books to become 21st century Lewis Carrol (reference to the numberphile video about him and problems that came to him while in bed))

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Problem 1:

If someone were to pick 2 random different primes from let's say 1 to 100 what's the probability the sum of those 2 primes is odd?

Problem 1.5:

Same rules as the previous problem but this time the 2 primes can be the same now what's the probability for the sum to be odd?

Problem 2:

Let's say you have a box with any number of objects that number we'll call N. Now you want one specific object from that box, what's the minimum number of objects in relation to N that have to be taken in order to have at least 50% chance of getting the object you want? (Keep in mind that the objects once taken out don't go in again so the maximum objects you can take is N)

Problem 2.5:

Same as the problem above but this time the objects you take you put them in again now in relation to N what would be the number of objects to have at least 50% of having taken it?

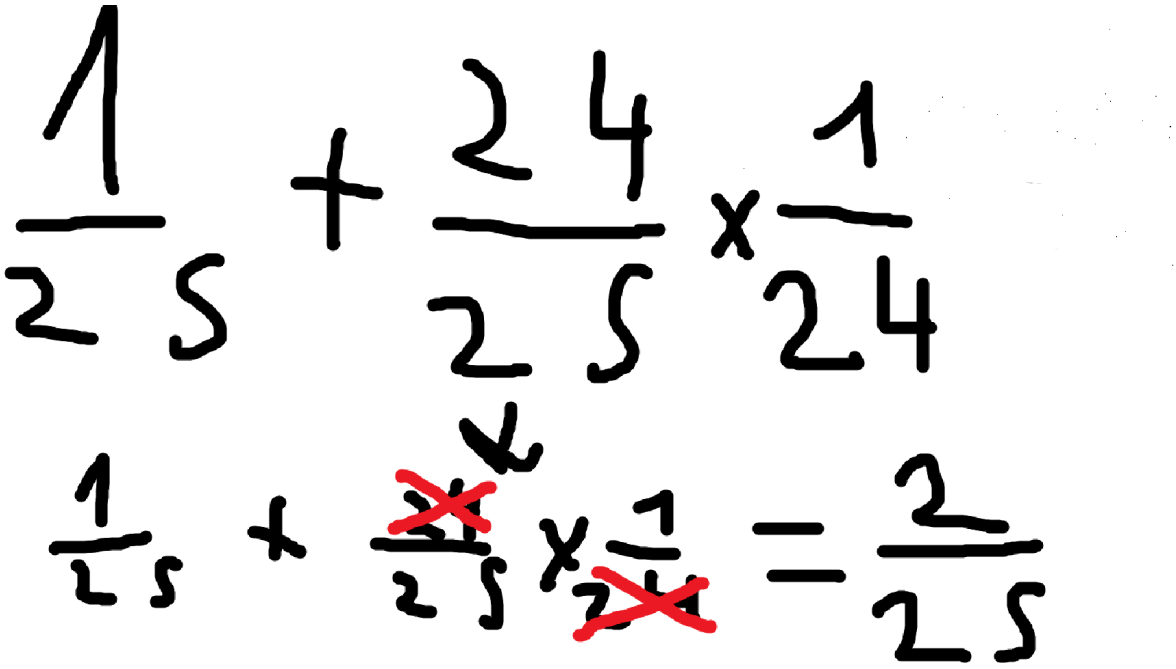
Now one thing you should know about these problems is that they're meant to be solved in order, basically meaning that they go from easy to hard and that they build upon the logic of the previous ones. With this out of the way let's get into the answers for them starting with the ones without .5 in their number.

Solution to problem 1.

This one is actually pretty simple. First we need to know how many primes there are between 1 and 100 a quick internet search will tell us that there's 25 of them and that 2 is one of them which is quite useful because, the only way to get an odd number by adding 2 numbers is by adding an even one and an odd one, and since 2 is the only even prime number we now know that to get by 2 random different primes an odd number we'll need to

get 2 and any other number, so now the problem we have is: What is the probability of getting an specific object if we get objects 2 times from 25?

This can be solved with the operations below which gives us this:

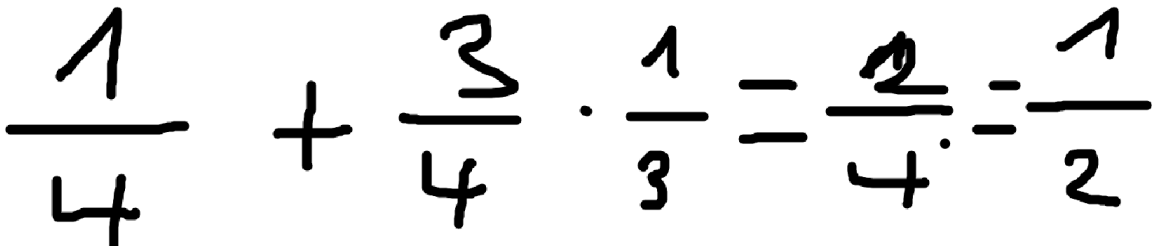


(The first 1/25 is just because it's the chance of getting what we want on the first try and since the second thing we get after that doesn't matter with just this it's enough. The second though is for the chance of getting what we want on our second chance which as can be seen below gets canceled into 1/25 which makes the answer 2/25 and it cannot be reduced)

Solution to problem 2:

This one is way harder than the previous one but once you see the answer you'll understand how it really works.

First we know that the number N doesn't really matter to solve the problem so to make it a simple case let's just say it's equal to 4 and see where that takes us:



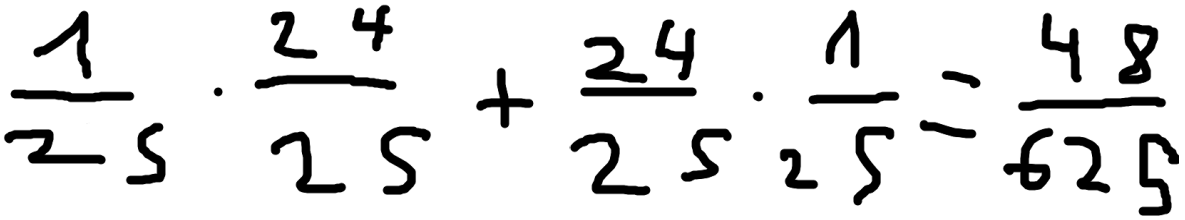
As we can see it took us two tries to get 50% which coincidentally enough is half of 4 but, I don't know if you've noticed but every time we add one + and the probabilities after it we're

really just adding 1/N again. Now the problem is asking for a 50% chance and if every time we add something more we're doing it by 1/N we'll get to 50% once we've added 1/N N/2 times so the answer for this problem is that we'll have to take things out of the box N/2 times where N is the number of objects in the box.

Now I will explain the solution to the problems whose numbers end in .5

Solution to problem 1.5:

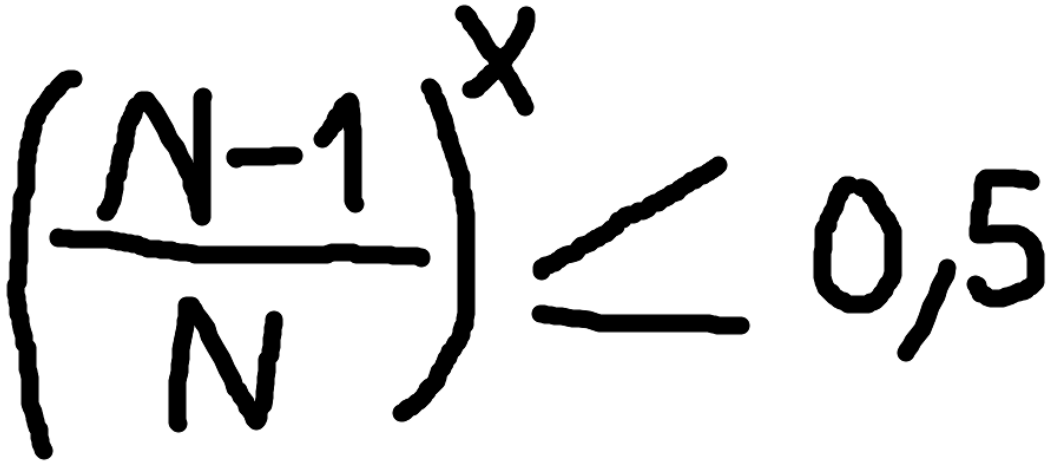
This really is a normal probability problem so the answer for it simply is:



As you can notice there are 25s as the denominator for each fraction because we can choose the same number again so we really can't simplify like before also the first part is for getting the 2 on the first chance and then not getting it and the second is the same but backwards.

Solution to problem 2.5:

Ok this is a hard one there's no tricks to do no secret realizations no nothing like that which means that the answer to this problem is a lot less neat which by that I mean that it's just an inequation where we want to solve for x while using N as a base on which to make our calculations So anyways here's the answer and below the explanation:



The inequation (the middle is a less than or equal symbol that's what makes it an inequation but technically you could substitute the middle symbol by an = and it would still work) is just the fraction of the probability of NOT getting what we want multiplied by itself x times that x being the number of times we have to do it until the probability becomes lower than 0,5. Now you may ask why we're calculating the opposite probability to what we want well worry not the reason is quite simple; it's just because it's easier in fact this is a thing that is done in many cases where instead of calculating the probability we want we calculate the probability of what we don't want then subtract that to 1 to get the probability we want; but since in this case we just x instead of the probability we can just use this function to find what we want.

(Congrats if you managed to understand that because I've written it know what it's meant to say but I'm not sure if I can fully understand it so yeah sorry for that) (by that i mean that I understand the answer what I don't is understand what I've written lol)

Yeah that's about it I think so yeah hope you enjoyed those problems.