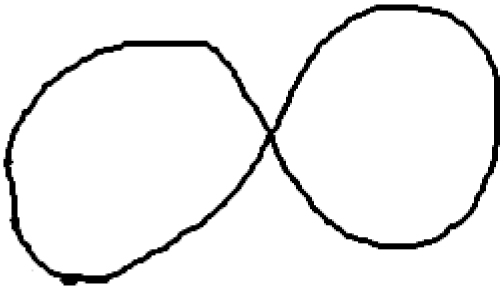
Essay ***e***(2.738...) :
How to make a **DIGON** (kinda it's complicated and a lot of semantics) (And also how a shape/graph with exactly 6 ways of being drawn in one go doesn’t exist lol) (Also this is part of a duology of essays on graph theory lol)

By:Jon Aguiar

Okay if you’ve read that and know anything about geometry (specifically euclidean) you’ve probably already discarded this as the ramblings of a mad man — in this case a teenager, but please sit down, relax and let me explain.

First, what do I mean by digon? Well sorry (not really lol) to get started with the semantics so early but by that, I mean that I know a shape (drawing, graph, whatever) that has exactly 4 ways of being drawn in one go (any time I say draw or drawn just put those last three words there except obviously of course if i say otherwise just putting this here to not repeat myself to much kay?) and you may ask, “Jon what does the amount of ways to draw a shape have to do with it being a digon that doesn’t seem very related” well, think of how many ways are there to draw a triangle, a square, or any other regular polygon (that just means all of its angles and sides are equal) really with one special condition: you’re only allowed to start at the corners because, if you were allowed to start anywhere there would be infinite ways of drawing any regular polygon since after all they’re all a loop and, since technically all loops are circles (Exercise for the reader: Show that this shape can be rearranged into a circle (by that I mean at points where multiple lines meet separate them in such a way everything is still connected)

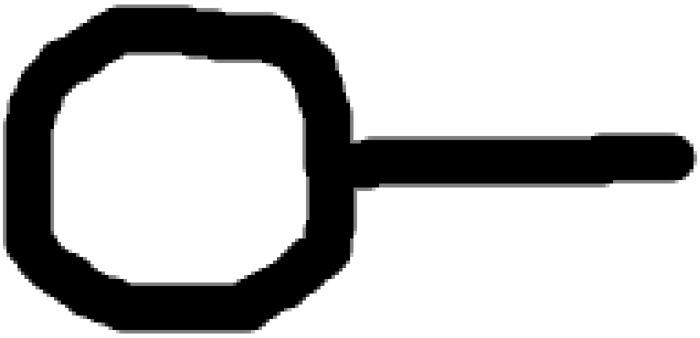


(I did this in paint with a mouse cut me some slack please)

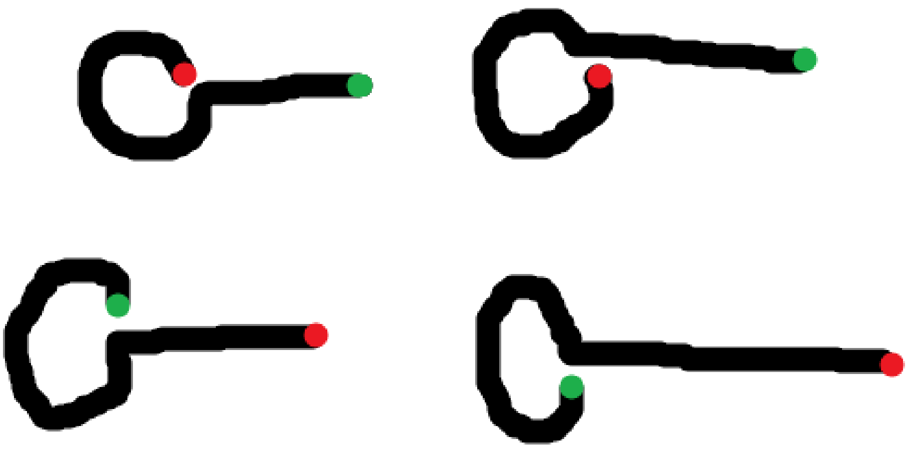
they have infinite ways of being drawn so yeah start at the corners please.

So if you did what I asked of you and counted the number of ways to draw any regular polygon you'd quickly realise that the number of ways to draw it is the number of corners which I'll go now and call them points —it will make sense later don't worry about it, times 2 so it would follow that a digon is a shape that can be drawn in exactly 4 ways and oh wouldn't you look at that it's just what I said at the beginning so yeah, let's see the mythical digon!!!!

(This is the cue to prepare yourselves to be disappointed at least a little because yeah the shape I'm calling a 'digon' obviously really isn't one because it doesn't have straight edges and all of that but yeah it really is a shape with 4 ways of being drawn. In fact I believe it's the only shape, graph, etc that can only be drawn in 4 ways.



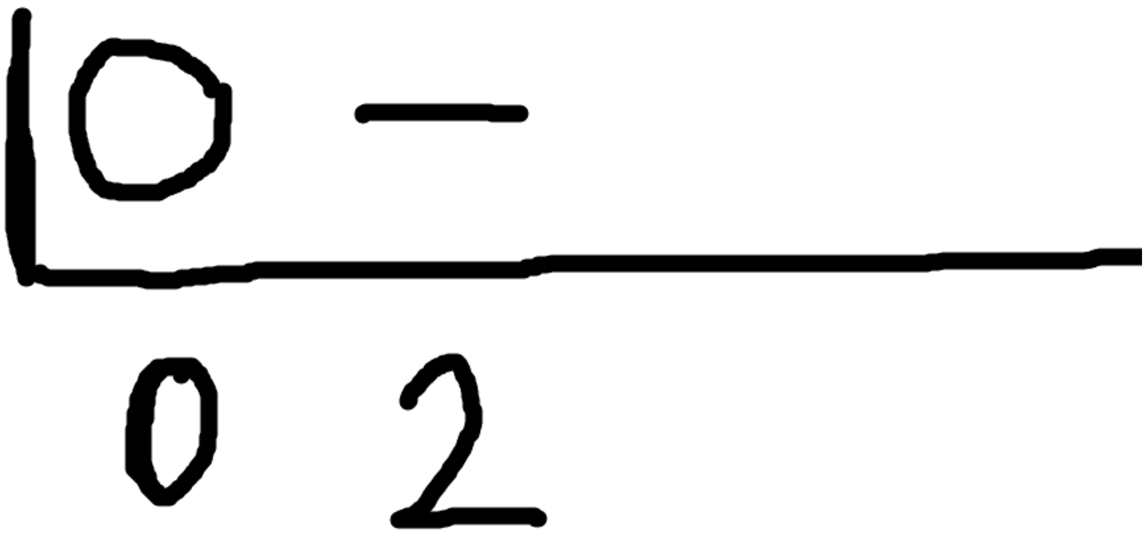
Here it is right above isn’t it cool anyways right below are all 4 ways to draw it (the green and red points are just to differentiate them also they’re not fully connected so you can fully see the route each way follows (what the green and red points truly mark will be shown later don’t worry))



So now I explained what I meant by digon but hey the title of the essay still has like 20 unaddressed words don’t worry I’m going to talk about that now.

So first I need to explain what can be called as a new numbering system or something like that (At least I think so I’d love to be proven wrong though and to not be the first person to think of this) so when I first mentioned loops and all that it was because honestly I structured this badly but anyways it works for what I'm going to say: If you know anything about graph theory you probably know about how to know if you can go through every connection once in one go which is by counting the number of lines that come out of every vertex/crossing point and seeing if there's either zero crossings with an odd number or exactly two with an odd number — which as a cool aside, did you know, that it's impossible to have an odd number of odd crossings it's because every connection that you add adds exactly two connection points and to have an odd number of odd crossings would mean you have an odd number of connection points which as you can tell is impossible to get since all connections add 2 no matter what which also means that it's impossible to have an odd number of connection points in any geometry system or anything really (again I'd love to be proven wrong with this but eh) end of the aside. The first case I specified of all crossings being even we'll call a loop since you must end exactly where you start and the second case we'll call a line since you always go from point A to point B.

Now that we have this we can assign numbers to the simplest cases of each that being a simple loop for the former and a straight line for the latter to which we'll assign the numbers 0/infinity and 2 respectively which is the number of ways to draw each shape:

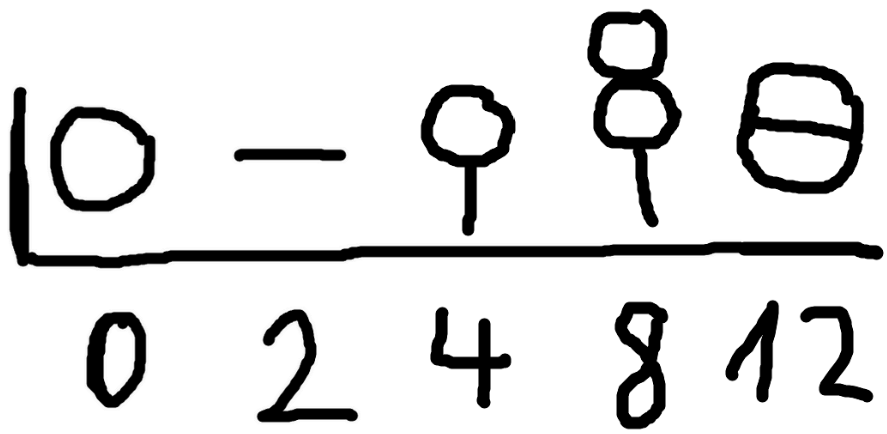


As you can see there is no way to make a shape with only one way to be drawn in the conditions we have which are to be concrete and for completeness sake:

1. The whole shape must be connected, that means that we can go from any point on it to any other point on it.

2. The whole shape must be able to be drawn in one go that is because if you allow breaking of the line without finishing the shape it makes so there's an endless amount of ways to draw the shape since you can start and end from anywhere just like with loops which we already established have an infinite amount of ways to be drawn so yeah it must be drawn in one go.
3. The shape cannot have one ways by that I mean that it can't be directed so if you can go from point A to point B you must be able to also go from point B to point A this condition is added to make calculations simpler basically this is just saying that we're going to work with undirected graphs instead than with both directed and undirected.

Now that we have these three conditions we can continue filling out our table of numbers:

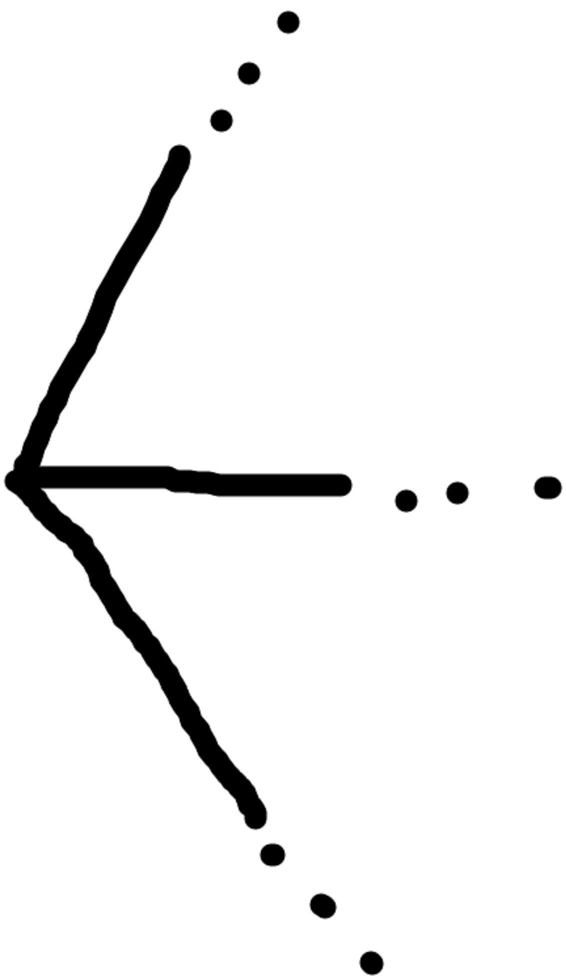


(This is all I've managed to get to as you can see there are some gaps I'm going to explain the first right now because that's as far as I've gotten with this)

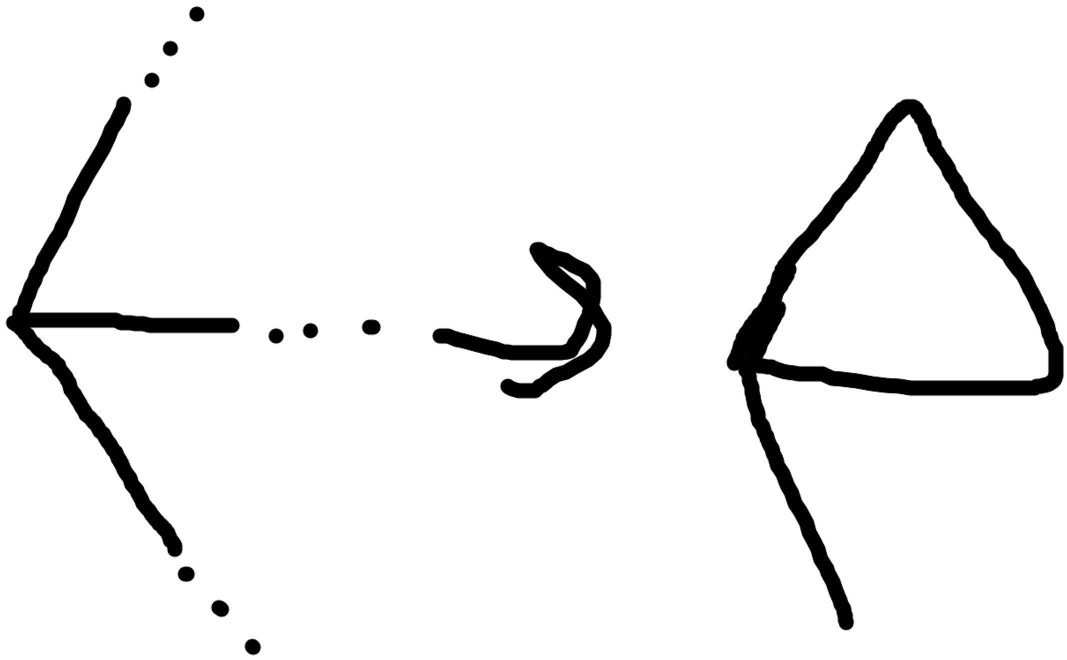
The shape above the four is the one I called before as a digon and the one for the eight is the same shape but with another loop above that's because, every time you add an extra loop you multiply by at least 2 that's because every time you add a loop you add an extra choice, choices that are in fact what we use to get a specific number because the number of ways to draw a shape can be broken down the same way you factorize normal numbers so for example the line equals 2 because you have a choice on where to start and that choice has two possibilities which makes it 2. The same goes for the shape for the 4, you have one choice of two possibilities that being where to start and then you have another that being when you get to the loop of what direction to go which there's 2 and so on. The shape for the

12 in this case is one choice of 2 possibilities then one of 3 then another one of 2 again which multiplied together equals 12. (Exercise for the reader: figure out how exactly those 3 choices work and look like —Yes this is because I was running out of words to explain lol)

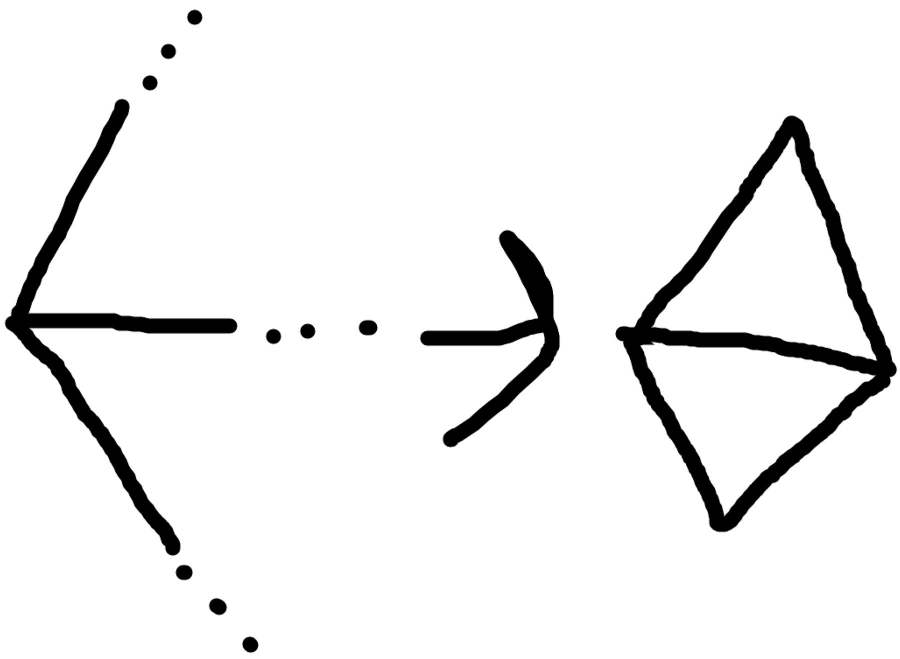
Since the amount of ways to draw the shape is decided by the choices we have and the first x2 is forced (choosing where to start) to find a shape with 6 ways we have to find a shape that has a branching path with 3 valid exits something like this:



And only this but if you think about it you'll quickly see the problem here. First if you draw under our established conditions a shape like this that is a single point with 3 or more spokes two or multiple of the spokes must connect or else the shape is impossible to draw so let's say we connect two of the spokes what do we get?



We get the shape that we established has 4 ways to be drawn and although they don't look the same they're both composed of a loop and a line which makes them the same in essence. So now let's try and connect the 3 spokes, what do we get?



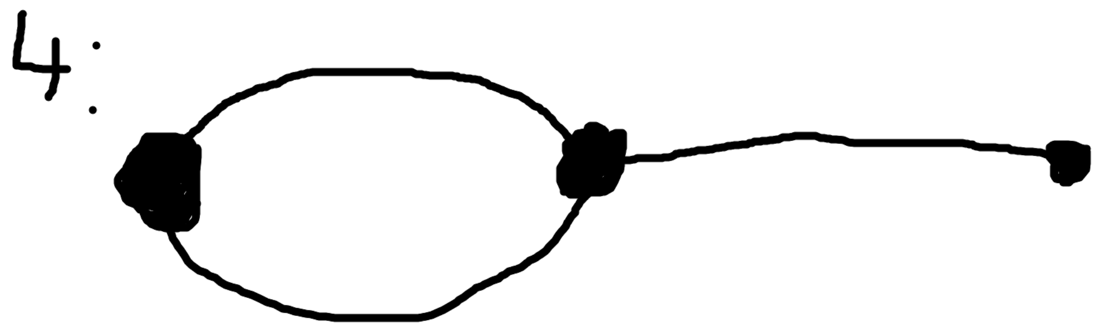
We get the shape for 12 ways. Why? Well because in essence the shape for 12 is a loop with a line that connects at 2 points instead than connecting with just 1 like with 4 and since there's no way for a line to connect decimal times to something we can be pretty sure that there's no way of making a shape with 6 ways of being drawn. — We can be pretty sure about that since adding things only increases the number of ways which if starting from the simplest case already gives us double that adding more will get us further from 6.

Extra portion for math addicts:

If you read the title correctly you'll remember me mentioning graphs well don't worry here's that:

This whole thing about going around every connection in a graph is called an eulerian path and graphs that have one are of course named eulerian graphs (technically graphs are also named that if every vertex in it is of an even order but since that is already a requirement for there being an eulerian path along with the graph being connected but hey that's just semantics baby) this all comes from the seven bridges of Königsberg problem, a problem from 1736 about going through every bridge in Königsberg in one go a problem that planted the seeds for modern graph theory but that whole thing is part of the subject of the other essay in the duology so I won't get to much into that.

Now the thing about graphs is that the shape doesn't matter, what does is the connections between vertices so the cases for 4 and 12 I established before would be like this: (the large dots are the vertices also for the case for 4 technically any number equal or larger to 1 of vertices could be on the left side it's just that they would need to be connected correctly forming a loop)



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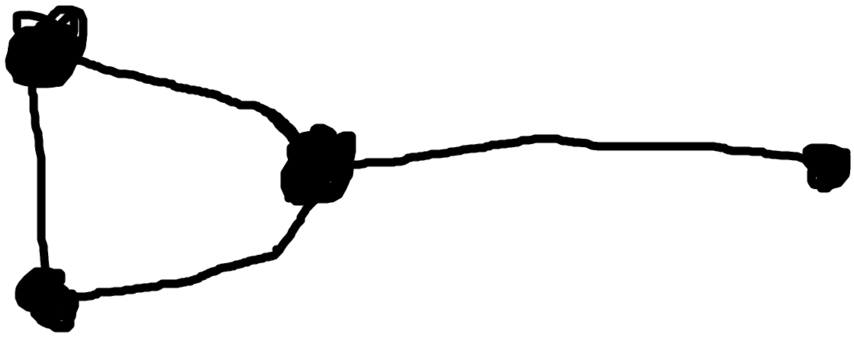

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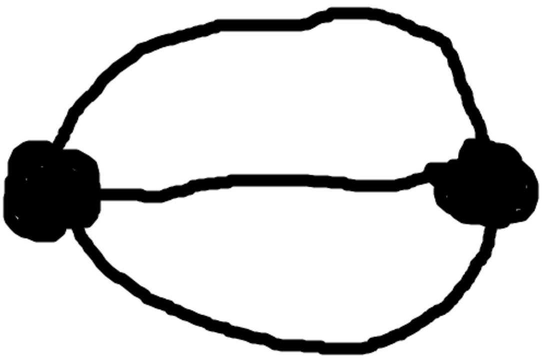

Now another rule that I could establish before reveals itself:

4. Vertices cannot connect to themselves that is because if they could this essay would be a lot longer and I have a word limit.

So surprise I guess this whole essay has been about graph theory baby and yeah that’s about it.

Thanks for reading this far, here’s the link to the other essay of the duology:

Essay π(3.141...): Another way of solving the bridges of Königsberg problem this time …