Is There a Formula for Feeling?

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March 2026

Some things stay private, locked inside heads. Your experience of red might be nothing like mine, deep down. Science cannot reach into minds to compare those inner sensations. A tomato has a certain glow when seen – that glow means something specific to you. Suppose my version of that sensation matches what you call green. It is not about labels or mistakes; I follow signals correctly, spot ripe fruit, match socks without error. Never do I point at crimson and claim it emerald. Yet beneath correct words, the feeling could twist entirely different. Maybe my inner world looks like a leaf looks to your eyes. No wire links two heads so thoughts can touch directly. That gap defines *qualia* – what burning light or sharp sound truly feels like inside.

Spotting that puzzle changed something quiet in how I see maths. Could sadness weigh three pounds? Does joy stretch seven inches across the skull? Surprisingly far – that is how much ground you can cover. Four solid mathematical frameworks actually tried solving this, each useful in practice, each built with care. Yet what tends to stay hidden until later is this: all four break down. Same spot, without exception. By the end of these lines, I hope it becomes clear – such consistent collapse is not failure. It might be among math's sharpest truths, its quietest elegance.

1 A Weigh In

Strange though it sounds, Ernst Weber had no interest in cracking the mystery of awareness. Working as a German physiologist during the 1830s, his experiments involved handing subjects metal blocks and asking them to judge weight – simple stuff, really. Yet what came out of those dull tests shook things up. Instead of needing the same fixed gap between masses to notice a change, people shifted at a constant ratio. Ten grams in every hundred makes the difference. A thousand needs twenty more to feel different. That link holds steady – change divided by original equals a fixed number:

$$\frac{\Delta I}{I} = k$$

where I is the stimulus intensity, ΔI is the just-noticeable difference (JND), and k depends on the sense – roughly 0.02 for weight, 0.1 for loudness. For lifted weights that value sits near two percent. Sounds need about ten times that shift. Maths found a rule hiding behind how things feel.

It is almost eerie. Even if your room hums softly or blares suddenly, just ten percent more sound reaches the edge of what you might catch – no matter the starting point. All while your hearing recalculates ratios on its own, quietly. Step by step, louder inputs lead to stronger feelings – that part stays predictable. Yet Weber's Law will not confirm whether each jump matches the last one – whether adding two grams at a hundred feels the same as adding twenty at a thousand. Here is when things start to shift. The question of *additivity* calls for an integral.

2 Fechner's Integral: Assume the Feeling

Years passed before Gustav Fechner noticed something in the data worth following. A physicist turned philosopher – an unusual direction even by Victorian standards – he saw that Weber's Law concealed a pattern. Each just-noticeable difference adds up like blocks piled high. When every shift measures $\Delta I = kI$, the accumulated total forms a curve instead of a line. From starting point I_0 to any intensity I, the total sensation S is built slowly through these stacked steps:

$$S = \int_{I_0}^{I} \frac{dI'}{kI'} = \frac{1}{k} \ln\left(\frac{I}{I_0}\right)$$

Constants fold into C, leaving **Fechner's Law**:

$$S = C \log\left(\frac{I}{I_0}\right)$$

What you feel rises only when the world pushes much harder. Double the sound and you will not double the sense. Take decibels – 80 dB feels far louder than 40 dB, though it carries ten thousand times more physical power. Logarithms explain that gap: $\log(10000) = 4$. Devices built for ears still follow this old maths. It lives inside your headphones right now.

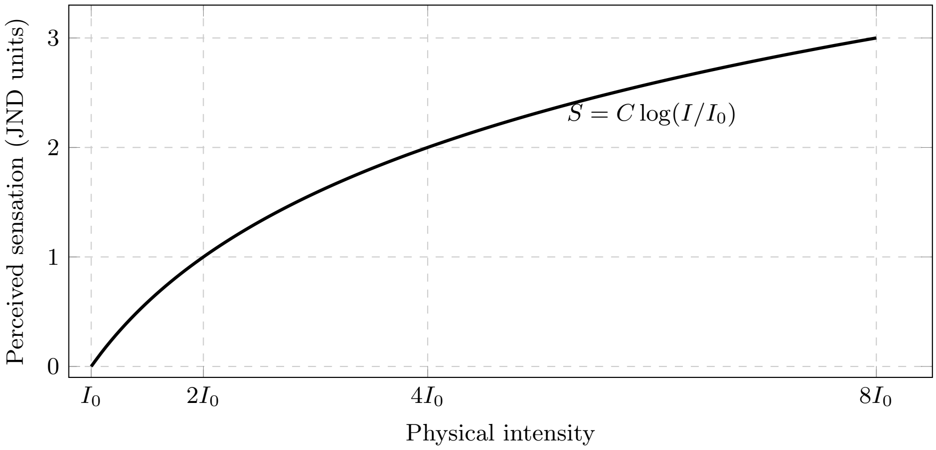

Figure 1: Fechner's Law: each doubling of physical intensity adds exactly one equal unit to perceived sensation. The grid looks orderly – but whether those units are truly *equal* in experience is precisely what the integral cannot prove.

Watch closely what builds that total sum.

Adding up dS means treating every infinitesimal sliver as feeling identical, no matter where you stand on the scale. This idea runs on a loop: sensations stack up just like marks on a measuring stick. Before writing down the integral at all, you have already accepted that crossing from 100 g to 102 g *feels* the same as crossing from 1000 g to 1020 g. Yet that very equivalence was what the experiment was supposed to establish. Seen from the outside, each step registers only as "detectable." Calling them equal in size depends on something private – the inner sense of what is being measured.

Fechner tucked the conclusion inside his starting assumption – he would have frustrated anyone grading proof-writing.[1] What strikes me most is that the flaw is not careless. The loop holds together. The maths works fine on paper. But the logic leans on an assumption at precisely the moment it was supposed to be proving that assumption true. The crack is not about wrong numbers. It is a proof whose axioms cannot be verified by the experiment that invokes them. And once you see this structure, you will see it repeat everywhere you look.

3 Stevens' Power Trip

Back in the 1950s, things shifted when the old logarithmic rule started showing cracks. Pain turned out to rise more sharply than Fechner predicted, while brightness climbed at a gentler pace. Because of this mismatch, S. S. Stevens stepped in with a broader model: **Stevens' Power Law**, $S = kI^n$, a formula meant to flex across senses. The exponent n shows whether perception stretches or compresses for each stimulus type:

[1] This is not a minor failing. The entire edifice of psychophysics rests on the assumption that JNDs are subjectively equal steps. Nobody has ever verified this from the inside, because the inside is precisely what is being measured.

Sense	Stimulus	Exponent n
Brightness	Light intensity	0.42
Loudness	Sound pressure	0.67
Visual length	Line length	1.00
Heaviness	Lifted weight	1.45
Electric shock	Current	3.50

Table 1: Selected exponents from Stevens [3]. The range from 0.42 to 3.50 traces survival maths across ages – an imprint left by past dangers. Higher values signal urgency once tied to risk; lower ones let perception handle wide input swings without overload.

Instead of stacking JNDs as Fechner tried, Stevens shifted ground. People judged ratios instead – not totals, but comparisons. Just name how many times louder one sound seems than another. Yet here comes another problem, sneaking in sideways. Depending on which starting numbers people see, the sequence of questions, even a fondness for tidy round digits, those ratio estimates change shape. Loudness alone – its reported exponent shifts between 0.54 and 0.80 purely by how the experiment was set up.[2] What felt like decoding the ear turned out to include the quirks of the questionnaire. In truth, the maths was measuring its own measuring instrument.

4 Colour Me Impressed

Here is where colour measurement took shape. Back in 1976, the International Commission on Illumination settled on **CIE Lab** – a system built so that equal geometric distances match equal perceived differences. Inside its maths lies a deliberate twist: the cube-root bakes Stevens' brightness compression directly into the structure:

$$L^* = 116\, f\left(\frac{Y}{Y_n}\right) - 16, \quad a^* = 500\left[f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right)\right], \quad b^* = 200\left[f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right)\right]$$

with $f(t) = t^{1/3}$. Once values are placed, colour gaps turn into straight-line distances:

$$\Delta E = \sqrt{(L_1^* - L_2^*)^2 + (a_1^* - a_2^*)^2 + (b_1^* - b_2^*)^2}$$

If $\Delta E < 1$, no person notices. Over 3, and it catches every eye. One yellow rose lands at $(52.99,\ 8.88,\ 54.53)$ in Lab space; a red rose at $(29.00,\ 52.48,\ 22.23)$. Plug in the numbers and $\Delta E \approx 59$ – the formula announces a dramatic colour difference before anyone looks at the picture. Far from a curiosity, this calculation moves through the software behind every display you have seen today, including the imaging tools surgeons rely on in operating theatres.

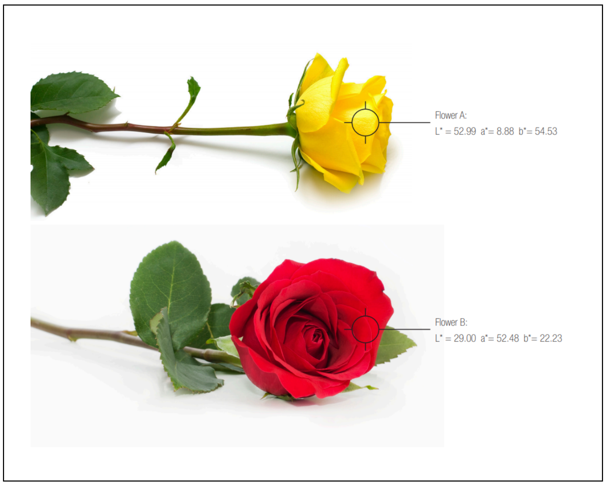

Figure 2: Two roses as points in Lab space. $\Delta E \approx 59$ correctly predicts that any human will see a dramatic colour difference. What it cannot predict is what either colour *feels like* to any of them. Source: X-Rite, `xrite.com/blog/lab-color-space`.

[2] A range of 0.54 to 0.80 means the published "constant" varies by nearly 50%. That is not a minor methodological footnote – it is the whole problem.

Yet people judged whether colour pairs matched – never how a shade actually felt to them. That is how CIE Lab was built and validated. Its strength lies in spotting differences, not capturing sensation. Agreement across viewers on a mismatch – $\Delta E \approx 59$ – means the roses appear far apart in hue. But it stays silent on the feel of crimson. Whether my red mirrors yours is totally beyond its reach. Impressive engineering. Not one bit of it touches inner seeing.

5 The Number That Ate Itself

Skipping how we sense things altogether, the most ambitious attempt linked numbers directly to awareness. In 2004, Giulio Tononi introduced **Integrated Information Theory** (IIT), giving every physical system a value Φ – how much information the whole produces beyond what each part generates independently.

Think of it this way. Snap a system in two. If the two halves, working in isolation, could predict each other's behaviour just as well as when connected, the system is not really integrated – it is just two things sitting next to each other. Φ measures how badly that prediction falls apart when you make the cut. Formally, for a bipartition into parts A and B:

$$\mathrm{ei}(A \to B) = D_{\mathrm{KL}}\Big(\, p(B_{t+1} \mid A_t, B_t) \,\Big\|\, p(B_{t+1} \mid B_t)\Big)$$

This is the KL-divergence between the system's actual behaviour and what part B would do if cut off from A. Φ is then the minimum of this quantity across every possible way of splitting the system – the weakest seam:

$$\Phi = \min_{\text{bipartitions}} \phi(\text{partition})$$

Three neurons in a loop score high on Φ – cut any link and the whole system changes. Three independent neurons score $\Phi = 0$ – cutting changes nothing, because nothing was connected to begin with. The theory earns something real: the cerebellum has more neurons than the cortex but a far more modular structure, so IIT predicts lower Φ – and cerebellar damage rarely affects consciousness. That match with reality means something.

Yet trouble arrives fast. Finding Φ exactly means checking every possible way to divide the system, and there are $2^{n-1} - 1$ such divisions for n elements. For a brain of $\sim$86 billion neurons, this is not merely hard – it is unreachable in the lifetime of the universe.[3]

Worse still: certain grids of XOR gates can be wired to achieve arbitrarily high Φ, outranking human brains on the consciousness scale. Not due to any error in the maths – it follows directly from the definition, because XOR connections create dense interdependencies across every possible cut. The implication is uncomfortable. Φ depends on how you *describe* a system – where you draw the boundary, at what level of abstraction – not purely on what it physically is. Describe the same brain differently and you get a different Φ. At that point, Φ stops being a measurement. It becomes a philosophical position with good notation.

6 The Writing on the Wall

Something odd shows up when checking each framework one after another – the pattern feels too neat to be coincidence.

Measurement theory says any quantity worth measuring needs three things: it must rank consistently (**orderability**), combine without breaking (**additivity**), and stay fixed no matter who watches (**observer-independence**). Take heat, charge, pressure – each holds solid under outside testing. Two thermometers in the same bath agree; additivity checks out when you join two rods end to end. These starting points get verified, not merely assumed.

Weber's Law established orderability but could not touch additivity. Fechner invoked additivity in the very step that was supposed to prove it – a loop with no exit. Stevens avoided the additivity claim yet let observer-dependence slip in through the measurement procedure. CIE Lab pinned down colour differences without relying on any single viewer – yet fell short when it came to the character of experience. IIT's Φ failed observer-independence at the level of system description. Every single framework slipped in an unspoken assumption that only inner awareness could confirm.

[3] IIT proponents have proposed approximations. But approximating a number you can never verify against the true value is a fundamentally different kind of knowledge from measuring temperature.

That idea got its clearest voice in 1974, from Thomas Nagel – who gave us possibly the sharpest question ever posed in print: *What Is It Like to Be a Bat?* [6] Every computation a bat makes during echolocation fits neatly into data. Still missing is the inward sensation – which lives behind a wall no outside tool can cross. David Chalmers sharpened this in 1995 [7]: not how the brain handles information – that part we can tackle – but *why* any processing feels like anything at all.

Pay attention now, because this is the part I find most exciting.

Those frameworks did not collapse from fuzzy maths or sloppy experiments. Collapse arrived through flawless logic. The integral compiles. The Euclidean distance is exact. The KL-divergence is well-defined. Each framework broke *precisely* at the point where a measurement precondition slipped past verification – and it broke in a way the maths itself made visible. Fechner's circularity is not hidden; it sits inside the integral sign, open for inspection. Stevens' observer-dependence is not buried; it shows up plainly in the variance of the exponent. IIT's description-dependence is not a footnote; it follows directly from the definition of Φ.

Maths did not merely *fail* to measure subjective experience. It *succeeded* in identifying, with extraordinary precision, *why* subjective experience resists measurement. Qualia is the first thing we have ever pointed the machinery of quantification at for which observer-independence is not just difficult – it is impossible by definition. For three centuries, physicists assumed observer-independence without checking it, because everything they measured happened to satisfy it. Inner experience breaks that pattern at the structural level. It does not permit independent observation, even in theory.

There is no shame in that. Maths simply shows its strength here – marking exactly where knowledge ends, then sketching the boundary of what lies beyond. My red might not be your red. Our strongest tools cannot settle it. But *why* they cannot is now something we can state as a theorem – and that, I think, is its own kind of triumph.

References

[1] E. H. Weber, *De Tactu*, Leipzig, 1834.

[2] G. T. Fechner, *Elemente der Psychophysik*, Breitkopf und Härtel, Leipzig, 1860.

[3] S. S. Stevens, "On the Psychophysical Law," *Psychological Review*, 64(3):153–181, 1957.

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[5] G. Tononi, "Consciousness as Integrated Information," *Biological Bulletin*, 215(3):216–242, 2008.

[6] T. Nagel, "What Is It Like to Be a Bat?" *Philosophical Review*, 83(4):435–450, 1974.

[7] D. Chalmers, *The Conscious Mind*, Oxford University Press, 1996.

[8] S. Aaronson, "Why I Am Not An Integrated Information Theorist," *Shtetl-Optimized* (blog), 2014.