Intro:

The alphabet is typically used as a tool for writing, typing, expressing ideas. In maths, each letter holds a place as either a constant or variable in many equations. Now, this essay could have been on any one of these formula's- each has a unique origin and story to tell when it comes to its proof. However, today I wanted to flip the script- let the letters take centre stage. Instead of being part of equations, the equations shall form the letters. How? By graphing them.

1.	**Pythagoras's Theorem**	$a^2 + b^2 = c^2$	Pythagoras,530 BC
2.	**Logarithms**	$\log xy = \log x + \log y$	John Napier, 1610
3.	**Calculus**	$\frac{\mathrm{d}f}{\mathrm{d}t} = \lim_{h \to 0} = \frac{f(t+h) - f(t)}{h}$	Newton, 1668
4.	**Law of Gravity**	$F = G\frac{m_1 m_2}{r^2}$	Newton, 1687
5.	**The Square Root of Minus One**	$i^2 = -1$	Euler, 1750
6.	**Euler's Formula for Polyhedra**	$V - E + F = 2$	Euler, 1751
7.	**Normal Distribution**	$\Phi(x) = \frac{1}{\sqrt{2\pi\rho}} e^{\frac{(x-\mu)^2}{2\rho^2}}$	C.F. Gauss, 1810
8.	**Wave Equation**	$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$	J. d'Almbert, 1746
9.	**Fourier Transform**	$f(\omega) = \int_{\infty}^{\infty} f(x) e^{-2\pi i x \omega} \mathrm{d}\,x$	J. Fourier, 1822
10.	**Navier-Stokes Equation**	$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f}$	C. Navier, G. Stokes, 1845

Figure-A

Rules:

- All letters shall be a similar size (based on size 'r' for aRbitrary size)
- All letters shall lie 'on' the x axis (to be written on the line)
- All letters to be graphed with at most two equations.

A,b,c,v?

However, you may have noted that many letters in the alphabet share similar shapes. To make graphing easier I have split the alphabet into 4 categories: 'circle' letters, 'straight line' letter, 'semi-circular' letters and 'curvy' letters.

Which is why we shall be starting with 'v'. 'v' is one of the 'straight line letters'- the simplest category to graph. But how to graph a letter v....

The equation Y= mx+c produces a line with a slope of m (we'll be ignoring 'c' (for now)). This however is just a straight line. To look like a 'v' we need a way of 'bouncing' the graph off the axis to get the signature 'v' shape. How can we do this?

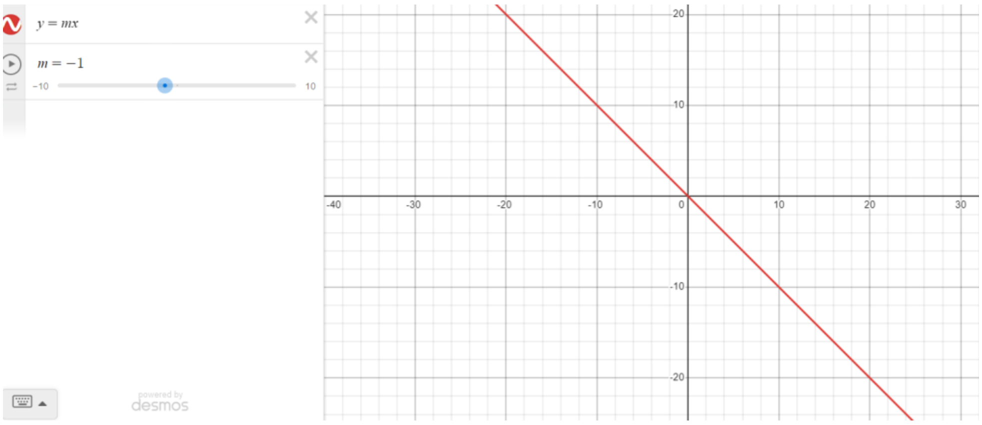


Figure1:

Let’s introduce the ‘modulus’ function (**|a|**). What this function does is it ignores the sign and only takes the ‘size’ of a number. So |-1|=1 or |5|=5. This can solve our problem for ‘v’. By writing y= |x|, we get y values that are always positive. This produces a graph as below:

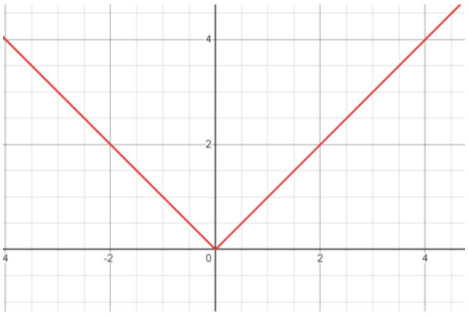


Figure2

To make, it steeper, we just make our ‘m’ 2 and there we have it :y= 2|x | forms v. Well, almost:

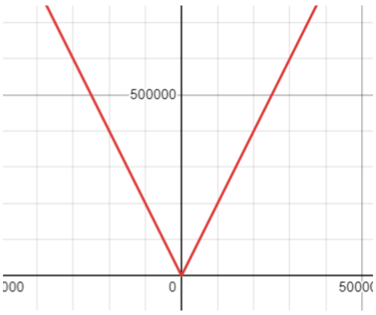


Figure3

This is far too big! Thankfully, we can limit the size of a graph by using {}. These curly brackets allow us to restrict the value of ‘x’ and ‘y’ we are using. As we wanted all letter to be of size ‘r’, we restrict so {y ≤r} and we’re done.

Moving onto w, we now know how to make the shape. However, we need the lines to ‘shift’ to form a w.

This can be done by subtracting from the x or y. Why? Well, by subtracting, we are essentially 'dragging' across the value that was the amount we took away across. By using this trick, we can shift the graph, adjust the gradient, change the restrictions (S.G.R for short) and get 'w' as below.

$$y = 3\left|x + \frac{r}{4}\right| \{x < 0\} \{y < r\}$$

$$y = 3\left|x - \frac{r}{4}\right| \{x > 0\} \{y < r\}$$

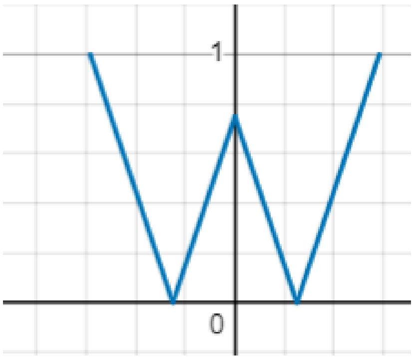
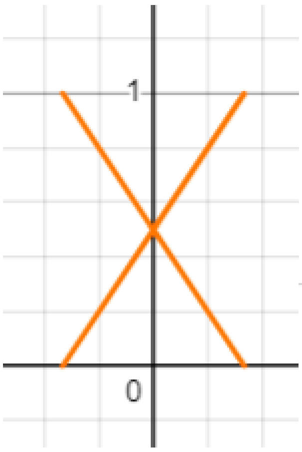


Figure4&5

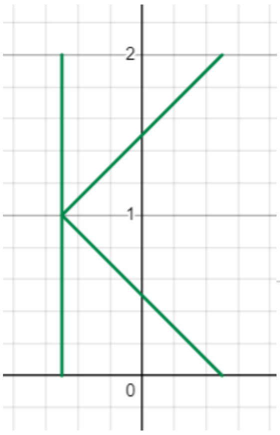
Now, letter x has a similar shape to v, but we want the lines to extend both ways. By repeating the modulus function on both sides, we can get a double 'v' (not 'w'- we just graphed that!). Doing another S.G.R produces a graph as below:



$$\left|y - \frac{r}{2}\right| = \frac{3}{2}|x| \{0 < y < r\}$$

Figure6&7

For k, we take the same principes of 'v' but flip it 90 degrees. This can be done by replacing the x's with y's. We apply another S.G.R...

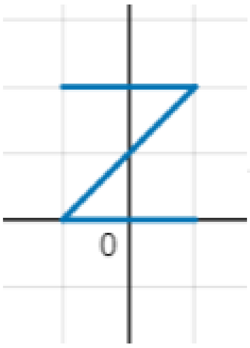


$$x = -\frac{1}{2}r\{0 < y < 2r\}$$

$$x + \frac{1}{2}r = |y - r|\left\{x < \frac{r}{2}\right\}$$

Figure8&9

Finally, for letter Z it becomes even easier. By applying the modulus function to ‘y’ for set values, we get a pair of parallel lines. Shifting up by the required amount gives us the equation as below:



$$y = x + \frac{1}{2}r\left\{-\frac{1}{2}r < x < \frac{1}{2}r\right\}$$

$$\left|y - \frac{1}{2}r\right| = \frac{1}{2}r\left\{-\frac{1}{2}r < x < \frac{1}{2}r\right\}$$

Figure10&11

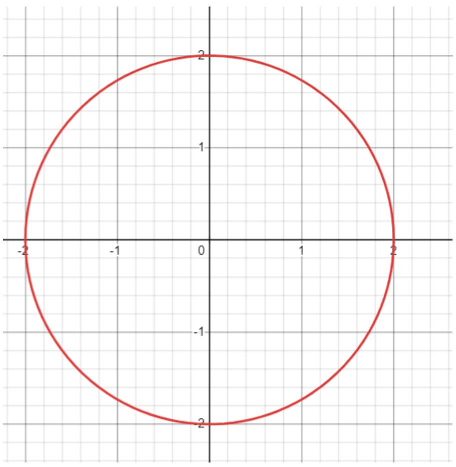
‘b is for… circular?’

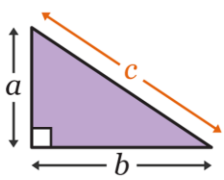
Next, we shall move on to the circle-based letters. We’ll leave ‘a’ till later and will start with b.

First, however, we need the equation of a circle:

$(x-h)^2+(y-k)^2=r^2$

This is the equation of a circle with centre (h,k) and radius r. This centre comes from our ‘shifting’ principles above, but what about our radius:



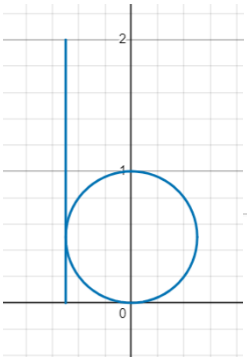


$a^2 + b^2 = c^2$

Figure12&13

It's Pythagoras' theorem! The square of the 'long side' is equal to the sum of the squares of the shorter sides. For the circle, we can think of the lengths 'a' and 'b' being the x and y coordinates of the point- if they form a right-angled triangle with the centre of the circle. If these x and y points squared are equal to the radius squared then the point will be plotted.

For 'b', we add the necessary lines, S.G.R and we're done.

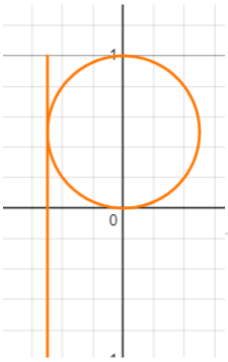


$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

$$x = -\frac{r}{2}\left\{0 < y < 2r\right\}$$

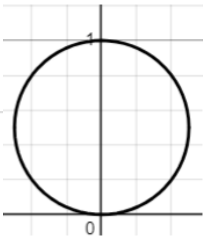
Figure14&15

Using this same process we can get 'p' too. O is even easier!



$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

$$x = -\frac{r}{2}\left\{-r < y < r\right\}$$

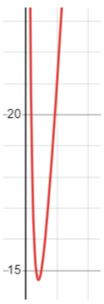


$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

Figure16,17,18&19

Unfortunately, q is a little trickier. To get the shape of the flick, we shall graph use the graph y=x+1/x, producing the shape below. Adjusting the gradients we can get a slightly steeper flick.

$$y = x + \frac{1}{x}$$



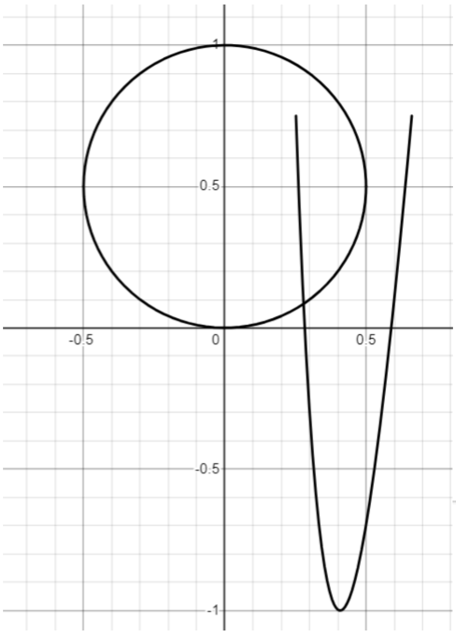
$$y = 18x + \frac{3}{x}$$

Figure20,21,22&23

However, the tail of the q should lie beneath the line. This equation doesn't. So, we must shift it down! But, by how much? How do we know where the 'flick' ends? Well, we can find this by differentiating. All this involves is finding the gradient at a point on the graph. The bottom of the flick (turning point) will have a ‘slope’ of zero, so we want to differentiate and find the ‘x’ where the gradient is zero.

To differentiate, we multiply the ‘x’ by the power and subtract 1. So, for x^2 dy/dx=2x. Applying this to our formula gives ‘dy/dx=18-3x^{-2} If we set this equal to 0 and solve, we get that x= $\sqrt{6}/6$. Now we know the x co-ordinate, we can substitute back in to find our y and shift…

However, this tail still doesn’t ‘touch’ the circle at the right point



$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

$$y + r + 6\sqrt{6} = 18(x) + \frac{3}{(x)}$$

<u>Figure24&25</u>

To do this, we need to move the ‘x’

If we want the circle to touch halfway yp the circle, then we know that the ‘y’ value there will be equal to ½ r . Substituting this value back into the equation, then we can find our x value. After this, we just shift across! Solving for the x value gives us this:

$$\frac{\left(\frac{3}{2}r + 6\sqrt{6} - \sqrt{\left(\left(\frac{3}{2}r + 6\sqrt{6}\right)^2 - 216\right)}\right)}{36}$$

<u>Figure26</u>

How did we get this? Well, by multiplying through by ‘x’ this turns it into a quadratic equation. And, these equations have a ‘simple’ solution given by the ‘quadratic formula’:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$ax^2 + bx + c = 0$$

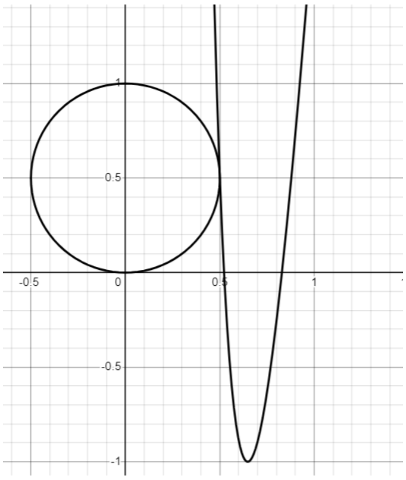
Figure27&28

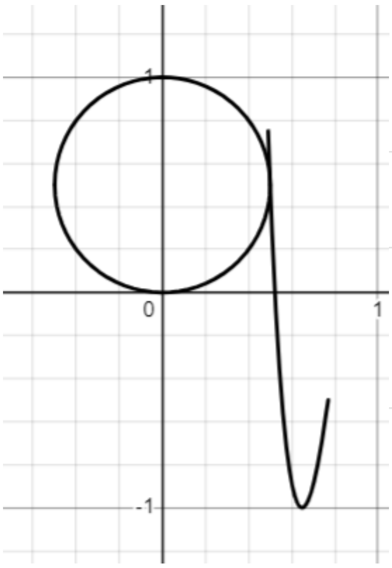
This formula solves any equation of this form, by substituting the terms as required. We re-arranged to fit and then took the lowest value solution (as we wanted the lower x value). Now we know the x co-ordinate, we want to shift across by ½ r- this x co-ordinate- as this gives us the distance away from the required point. Adding restrictions to our now correct shape produces q as below:

$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

$$y + r + 6\sqrt{6} = 18(x-a) + \frac{3}{(x-a)}$$

$$a = \frac{r}{2} - \frac{\left(\frac{3}{2}r + 6\sqrt{6} - \sqrt{\left(\left(\frac{3}{2}r + 6\sqrt{6}\right)^2 - 216\right)}\right)}{36}$$





$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

$$y + r + 6\sqrt{6} = 18(x-a) + \frac{3}{(x-a)} \left\{y < \frac{6r}{8}\right\}\{x < b\}$$

$$b = \frac{\left(36a + 6\sqrt{6} + \frac{r}{2} + \sqrt{\left(\left(36a + 6\sqrt{6} + \frac{r}{2}\right)^2 - 4(18)\left(18a^2 + a\left(6\sqrt{6} + \frac{r}{2}\right) + 3\right)\right)}\right)}{36}$$

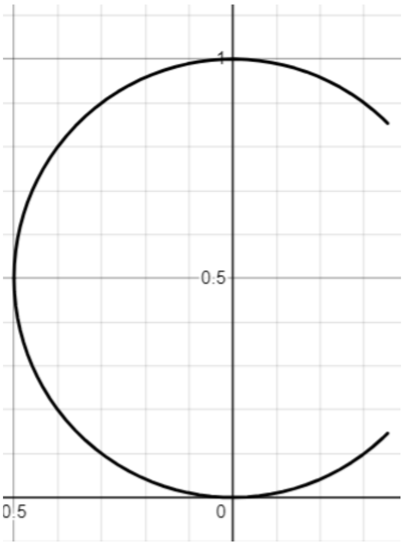
$$a = \frac{r}{2} - \frac{\left(\frac{3}{2}r + 6\sqrt{6} - \sqrt{\left(\left(\frac{3}{2}r + 6\sqrt{6}\right)^2 - 216\right)}\right)}{36}$$

Figure 29,30,31&32

Semi-circular letters.

As we now know the circle equation, this section is about finding the right restriction.

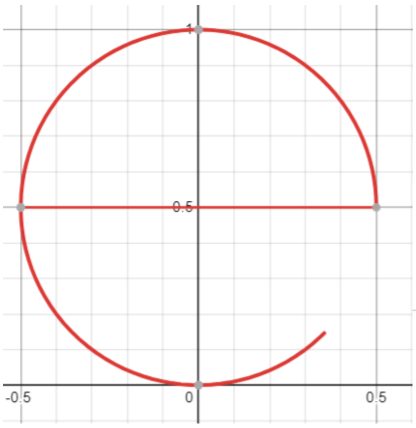
For c, we want to take a sideways wedge out of it. We've already seen this shape in our 'k'! If we want our 'y' values to be bigger/outside this section, then we restrict so |y|>x. This gives us the graph below:



$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2 \left\{ \left| y - \frac{r}{2} \right| > x \right\}$$

Figure 33&34

For e, we know what points we want to 'cut' out the circle. Finding an equation of the line is easy- we use y=mx+c. We calculate the gradient of where we cut the circle, and the 'c' does the shift. We restrict the y to be above this line giving 'e' below:



$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2 \left\{ y > \left(1 + \sqrt{2}\right)x - \frac{r\sqrt{2}}{2} \right\}$$

$$y = \frac{r}{2} \left\{ -\frac{r}{2} < x < \frac{r}{2} \right\}$$

Figure 35&36

Curved letters:

For these letters, we want to achieve a shape of a semi-circle with lines on the end (as below)

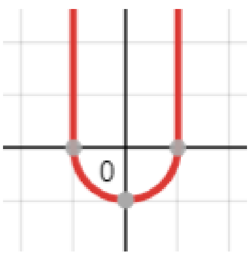


Figure37

But how can we get that shape? To do this, we must use the modulus function as in figure 10 and the circle equation simultaneously

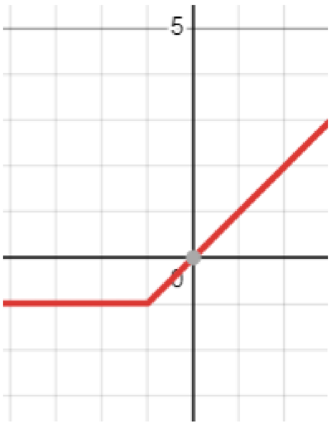
As we know the equation of a circle requires 'x^2' and 'y^2', we need a way to get the modulus function in terms of x^2. Squaring a positive or negative always gives us a positive (the negatives cancel out). We are left with just a magnitude of x squared. The modulus function works on just magnitude- we have our solution. Of course, this square value is formed by squaring, so we must square the other side. Hey presto, $x^2 = (r/2)^2$ is the same graph as the graph of $|x| = r/2$

Even better, this result is in terms of $(r/2)^2$ and x^2 -which we need for the circle

However, plotting $x^2 + y^2 = (r/2)^2$ isn't going to get us to $x^2 = (r/2)^2$; we need a way of subtracting the 'y' term for some values.

Thankfully, there is a function for this...

The 'max' function is a function that always takes the largest value. So, if I said max (0,5) it would spit out a 5. If I was to graph y=max(-1,x) then I would get a line at –1 or at the value of x:



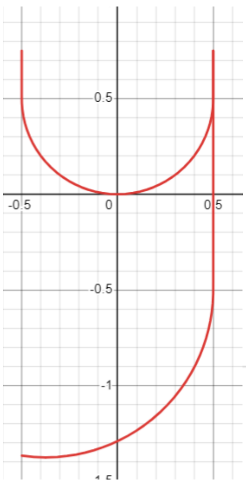
$y = \max(-1, x)$

Figure38&39

To achieve our curve, we want the top half of the circle to 'disappear' and just become straight lines. So, we want to subtract y^2 when $y > 0$. If we use the max function for max(0,y) then we can subtract only when y is greater than zero. so, if we set the equation $x^2 + y^2 - \max(0,y)^2 = (r/2)^2$ then we get the shape.

Now we have the principle of the max function, we have all of the ingredients for the remaining letters:

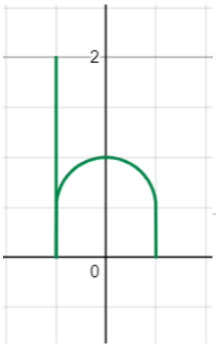
'Curved' letters:



$$x^2 + \left(y - \frac{1}{2}r\right)^2 - \max\left(0, y - \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \left\{y < \frac{6}{8}r\right\}$$

$$\left(x + \frac{3r}{8}\right)^2 + \left(y + \frac{r}{2}\right)^2 - \max\left(0, \left(y + \frac{r}{2}\right)\right)^2 = \left(\frac{7r}{8}\right)^2 \left\{x > -\frac{1}{2}r\right\} \left\{y < \frac{6}{8}r\right\}$$

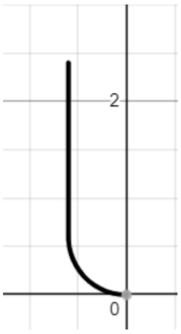
Figure40&41



$$x^2 + \left(-y + \frac{1}{2}r\right)^2 - \max\left(0, -y + \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \{y > 0\}$$

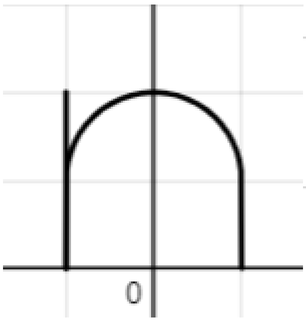
$$x = -\frac{1}{2}r\{0 < y < 2r\}$$

Figure42&43



$$(x)^2 + \left(y - \frac{1}{2}r\right)^2 - \max\left(0, y - \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \{x < 0\}\left\{y < \frac{16r}{8}\right\}$$

Figure44&45



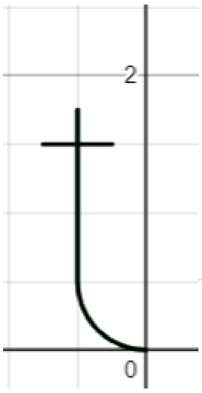
$$x^2 + \left(-y + \frac{1}{2}r\right)^2 - \max\left(0, -y + \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \{y > 0\}$$

$$x = -\frac{r}{2}\left\{0 < y < \frac{8r}{8}\right\}$$

$$\left(x - \frac{r}{4}\right)^2 + \left(-y + \frac{1}{2}r\right)^2 - \max\left(0, -y + \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \{y > 0\}\left\{x < \frac{3r}{8}\right\}$$

$$x = -\frac{r}{4}\left\{0 < y < \frac{8r}{8}\right\}$$

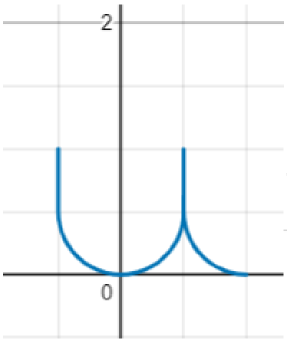
Figure46,47,48&49



$$(x)^2 + \left(y - \frac{1}{2}r\right)^2 - \max\left(0, y - \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \{x < 0\}\left\{y < \frac{14r}{8}\right\}$$

$$y = \frac{3r}{2}\left\{-\frac{3}{4}r < x < -\frac{1}{4}r\right\}$$

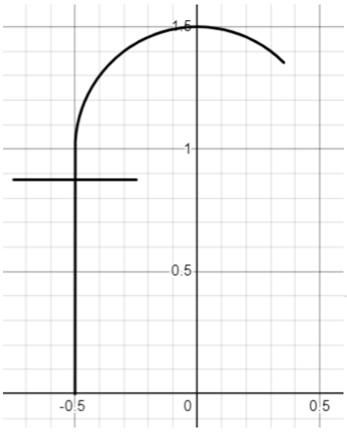
<u>Figure50&51</u>



$$x^2 + \left(y - \frac{1}{2}r\right)^2 - \max\left(0, y - \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \{y < r\}$$

$$(x - r)^2 + \left(y - \frac{1}{2}r\right)^2 - \max\left(0, y - \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \left\{y < \frac{5}{8}r\right\}\{x < r\}$$

<u>Figure52&53</u>



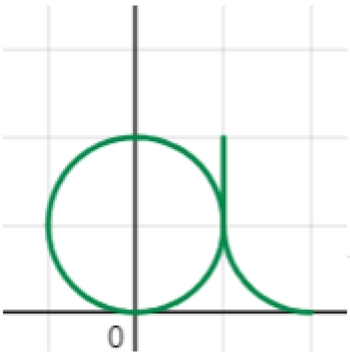
$$y = \frac{7r}{8}\left\{-\frac{r}{4} > x > -\frac{3r}{4}\right\}$$

$$x^2 + (-y + r)^2 - \max(0, -y + 1r)^2 = \left(\frac{1}{2}r\right)^2 \left\{\frac{\sqrt{2}}{4}r > x\right\}\{y > 0\}$$

<u>Figure54&55</u>

<u>Circle/curved letters:</u>

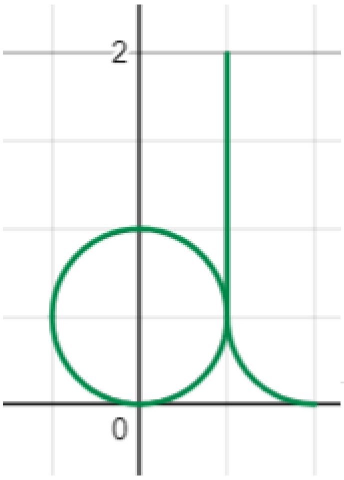
And now you see why we left ‘a’!



$$x^2 + \left(y - \frac{r}{2}\right)^2 = \left(\frac{r}{2}\right)^2$$

$$((x - r))^2 + \left(y - \frac{1}{2}r\right)^2 - \max\left(0, y - \frac{1}{2}r\right)^2 = \left(\frac{1}{2}r\right)^2 \{y < r\}\{x < r\}$$

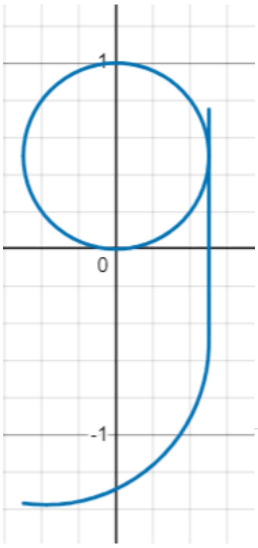
<u>Figure56&57</u>



$$x^2+\left(y-\frac{r}{2}\right)^2=\left(\frac{r}{2}\right)^2$$

$$((x-r))^2+\left(y-\frac{1}{2}r\right)^2-\max\left(0,y-\frac{1}{2}r\right)^2=\left(\frac{1}{2}r\right)^2\{y<2r\}\{x<r\}$$

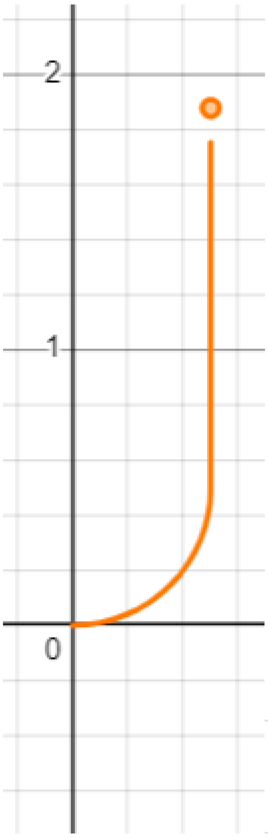
Figure58&59



$$\left(x+\frac{3r}{8}\right)^2+\left(y+\frac{r}{2}\right)^2-\max\left(0,\left(y+\frac{r}{2}\right)\right)^2=\left(\frac{7r}{8}\right)^2\left\{x>-\frac{1}{2}r\right\}\left\{y<\frac{6}{8}r\right\}$$

$$x^2+\left(y-\frac{1}{2}r\right)^2=\frac{1}{4}r^2$$

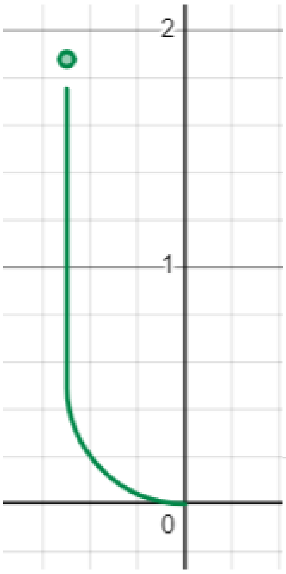
Figure60&61



$$(x)^2+\left(y-\frac{1}{2}r\right)^2-\max\left(0,y-\frac{1}{2}r\right)^2=\left(\frac{1}{2}r\right)^2\{x>0\}\left\{y<\frac{14r}{8}\right\}$$

$$\left(x-\frac{r}{2}\right)^2+\left(y-\frac{15}{8}r\right)^2\le\left(\frac{r}{32}\right)^2$$

Figure62&63

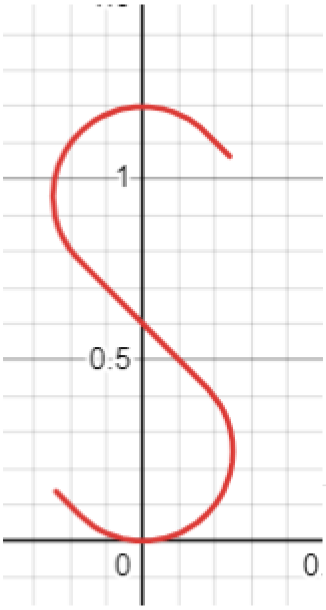


$$(x)^2+\left(y-\frac{1}{2}r\right)^2-\max\left(0,y-\frac{1}{2}r\right)^2=\left(\frac{1}{2}r\right)^2\{x<0\}\left\{y<\frac{14r}{8}\right\}$$

$$\left(x+\frac{r}{2}\right)^2+\left(y-\frac{15}{8}r\right)^2\le\left(\frac{r}{32}\right)^2$$

Figure64&65

For letter s, we take the same principle and put in on a slant. We use the ideas from letter q to adjust the positions of the curve of solving for the x values when y =0 and a r/2. This stretches the graph, so it fits and we're done!



$$(x)^2+\left(y-r+\frac{r}{2\left(1+1\sqrt{2}\right)}\right)^2-\max\left(0,\frac{\left(x-y+r-\frac{r}{2\left(1+1\sqrt{2}\right)}\right)}{\sqrt{2}}\right)^2=\left(\frac{r}{2\left(1+1\sqrt{2}\right)}\right)^2\{x<0.2r\}\{y>0.5r\}$$

$$(x)^2+\left(-y+\frac{r}{2\left(1+\sqrt{2}\right)}\right)^2-\max\left(0,\frac{\left(-x+y-\left(\frac{r}{2\left(1+\sqrt{2}\right)}\right)\right)}{\sqrt{2}}\right)^2=\left(\frac{r}{2\left(1+\sqrt{2}\right)}\right)^2\{y<0.5r\}\{x>-0.2r\}$$

Figure66&67

The end?

However, if you've been paying attention, you may notice that one letter is missing. Letter 'm'. For, without a new principle it will require 3 equations:

$$\left(x+\frac{r}{2}\right)^2+\left(-y+\frac{1}{2}r\right)^2-\max\left(0,-y+\frac{1}{2}r\right)^2=\left(\frac{1}{2}r\right)^2\{y>0\}\{x<0\}$$

$$\left(x-\frac{r}{2}\right)^2+\left(-y+\frac{1}{2}r\right)^2-\max\left(0,-y+\frac{1}{2}r\right)^2=\left(\frac{1}{2}r\right)^2\{y>0\}\{x>0\}$$

$$x=-r\{r>y>0\}$$

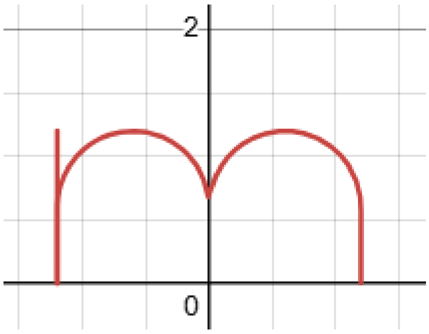
Figure68

We need a way of 'joining' lines together. How? Well, consider the expression below:

ab=0

In this expression, we know that 'a' or 'b' (or both) must be equal to zero. How does this relate to our equations of the graph?

Well, our equations can be written to be equal to zero. This means if we swap out our 'a' for one equation and our 'b' for another, it will plot all the points where either equation equals zero- all of the points on both of the graphs! This finishes our final letter



$$\left(\left(x-\frac{r}{2}\right)^2+\left(-y+\frac{1}{2}r\right)^2-\max\left(0,-y+\frac{1}{2}r\right)^2-\left(\frac{1}{2}r\right)^2\right)\left(\left(x+\frac{r}{2}\right)^2+\left(-y+\frac{1}{2}r\right)^2-\max\left(0,-y+\frac{1}{2}r\right)^2-\left(\frac{1}{2}r\right)^2\right)=0\{y>0\}$$

$$x=-r\{0<y<r\}$$

Figure 69&70

The end end?

Now, our trick for ‘m’ can be used for all letters (but this shall be left as an exercise for the reader) ....

However, what use are letters if we cannot write with them?

The final sentence

To write words, we just need to shift our letters to the right place - the first and last letters furthest from the axis with middle letters the closest. But how much to shift by- how can we get each letter in the write place. For this, we need a ‘set’- a list of numbers

$$N=\left[-\frac{n-1}{2},\ldots,\frac{n-1}{2}\right]$$

Figure71

Let ‘n’ be the number of letters in the word and N be our set. N is a list of number increasing by 1 each time between our two end values. Now, for an ‘n’ letter word we want ½ n letters on one side of the ‘y-axis’ and the other half on the other. So, with our shifting principle, we want a set from –1/2n to 1/2n. This list allows us to shift from far to near to far again. However, the set described above also includes 0- so we have a total of n+1 numbers. To prevent this, we bring each side closer to 0 by a half.

Finally, to shift the letters to the right place, we need to define a variable ‘s’ (for shift) for each of the letters. Variable ‘s’ must pick out the kth element of N (where ‘k’ is the position of the letter in the word). Then, we multiply this element by 2r (to create a spacing of ‘2r’ between the centres of letters) , subtract it from x and that’s it. We can now write words!

$$s_1 = 2rN[2]$$

$$\left(\left(x-s_1\right)^2+\left(y-\frac{r}{2}\right)^2-\left(\frac{r^2}{4}\right)\right)=0\{x-s_1<r\}\{y<r\}$$

$$\left(\left(x-s_1-r\right)\right)^2+\left(y-\frac{1}{2}r\right)^2-\max\left(0,y-\frac{1}{2}r\right)^2-\left(\frac{1}{2}r\right)^2=0\{y<r\}\{x-s_1<r\}$$

Figure72

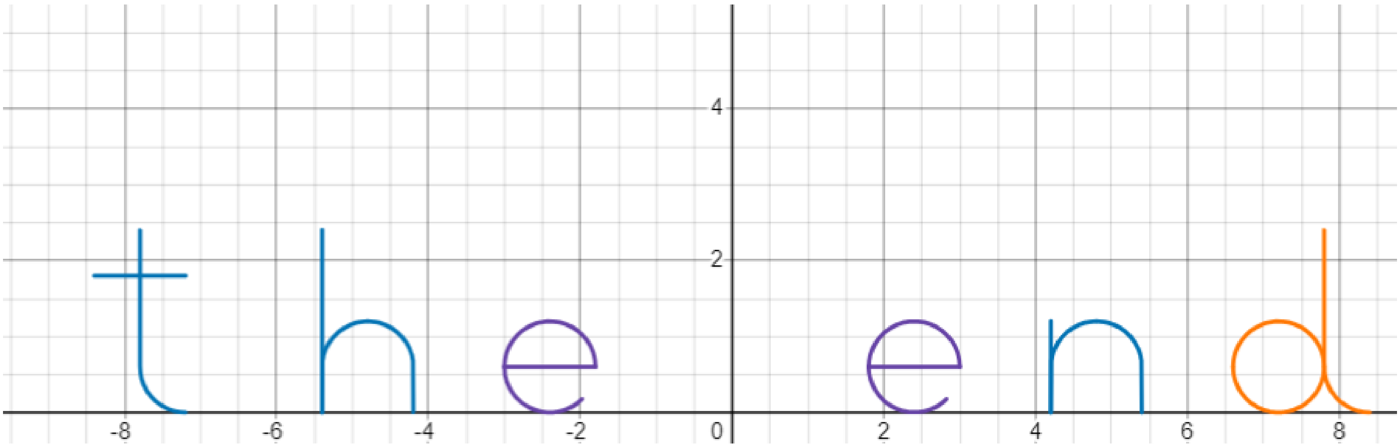


Figure73

Final product: https://www.desmos.com/calculator/2pasnusqny