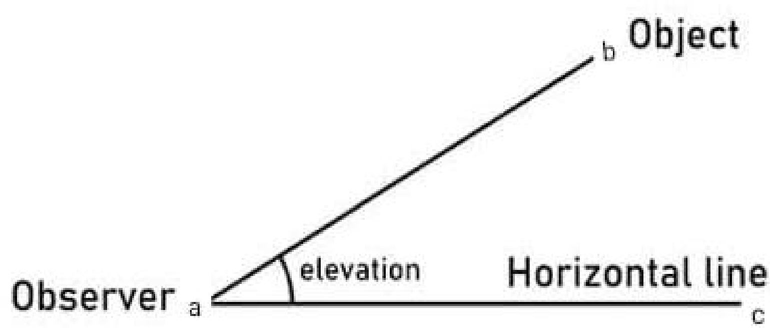
Have You Ever Thought About Measuring the Sky?

Have you ever thought about measuring the sky, from the ground up to the buildings? Have you ever seen a tall tower that seems to touch the sky, and the first question that comes to your mind is how anyone could measure its height using measuring tools?

While it might seem almost impossible to know the height of a very tall tower using standard tools, the game of mathematics brings enjoyment to finding building heights without them. It requires just a little thinking, deep observation, and calculations through angles of elevation and depression.

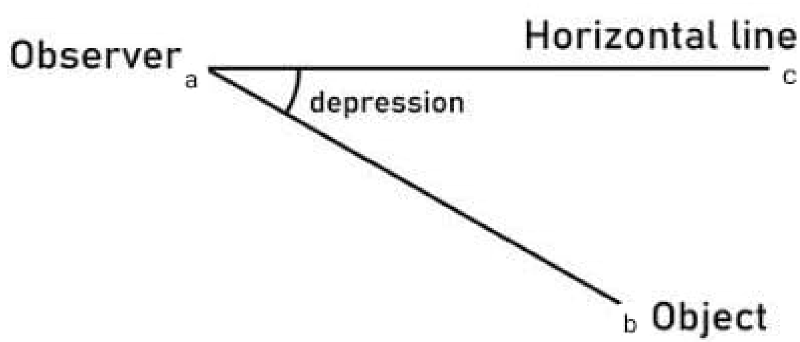
Angle of Elevation:

If we assume there is an observer at point A looking at an object at point B, which is above the level of sight, the angle between the horizontal line AC and the line AB (which connects the observer's eye to the object) is called the angle of elevation.



Angle of Depression:

If we assume there is an observer at point A looking at an object at point B, which is below the level of sight, the angle between the horizontal line AC and the line AB (which connects the observer's eye to the object) is called the angle of depression.



How Do We Draw These Angles?

- First, we draw a horizontal line, which represents the level of observation.
- We draw a point that will represent the observer.
- If the object is above, we draw a line from the observer up to the object (forming the angle of elevation).
- If the object is below, we draw a line from the observer down to the object (forming the angle of depression).
- The angle between the horizontal line and the line of sight is either the angle of elevation or depression.

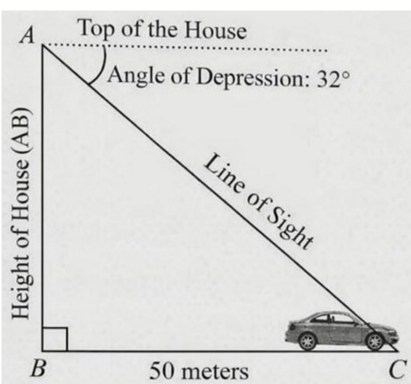
Drawing is very important because it helps determine the positions of lengths and angles before we start solving the problem.

How Do We Use Mathematics to Calculate Angles (Trigonometry)?

In reality, we sometimes cannot measure angles directly, so instead, we use trigonometric ratios: sine, cosine, and tangent. The simplest example is using the tangent of an angle if there is a right triangle that includes the height of the object. By using the horizontal distance between the observer and the base of the object, along with the angle of elevation, we can use the tangent. The formula is **tan(θ) =** $\boldsymbol{\frac{h}{d}}$, and it is used in solving most problems.

Example 1:

From the top of a house, the angle of depression to a car was measured and found to be 32°. If the car is 50 meters away from the house, find the height of the house to the nearest meter.



To understand the problem and how to solve it, we first draw a horizontal line for the observer's eye level and connect point A (the observer) to point C (the car). The angle between the top of the house and line AC is the angle of depression, and it equals 32°.

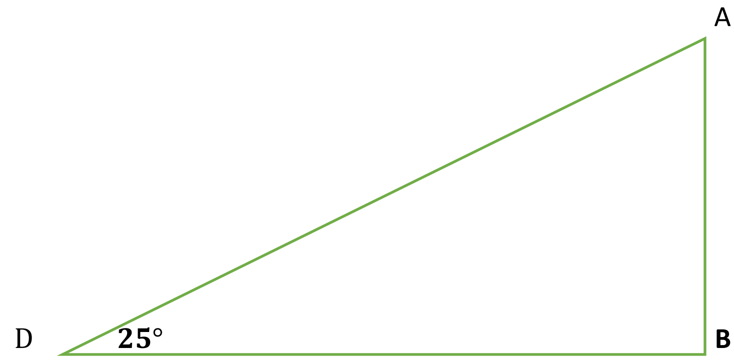
In ▲ ABC, which is right-angled at B: **tan(C)** = $\frac{\boldsymbol{BA}}{\boldsymbol{BC}}$.

Therefore, **tan (32°)** = $\frac{\boldsymbol{BA}}{\boldsymbol{50}}$, which means **BA = 50 * tan (32°) ≈ 31 m**. Therefore, the height of the house is approximately 31 meters.

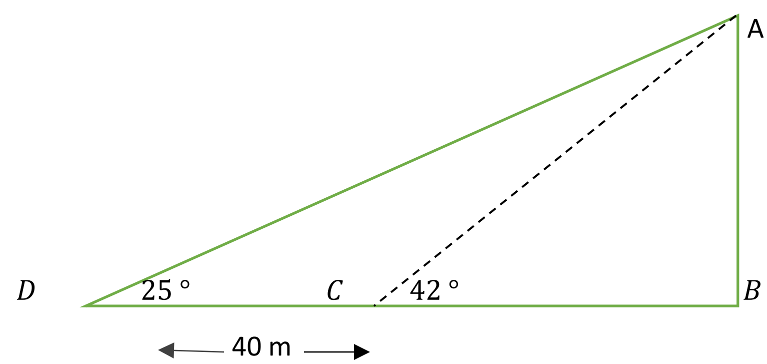
This is a great example of how to find the height of buildings using the angle of depression and the tangent law.

Example 2 (Using Sine and Cosine Laws):

Imagine you are standing and observing the angle of elevation to the top of a tower from a point on the ground, and you find that it measures 25°. First, we draw a horizontal line DB and mark the observer's point D. Then, we draw a line from the observer D to the object A (the top of the tower) and mark the elevation as 25°.



Next, you walk along a horizontal path toward the base of the tower for a distance of 40 meters. And you observed the angle of elevation to the top of the tower again and find it measures 42°. What do you expect the height of the tower to be, to the nearest meter?



In ▲ADC: $(A\hat{C}D)$ = 138°, and $(D\hat{A}C)$ = 17°.

Using the sine rule: $\frac{AC}{sin(25)} = \frac{40}{sin(17)}$ which gives **AC = 57.8 m**.

In ▲ ABC: $\frac{AB}{sin(42)} = \frac{57{,}8}{sin(90)}$ which gives **AB = 38.7 m**.

As seen in the last example, when you move closer to the tower, it appears larger because the angle of elevation becomes bigger (or increases) as we get closer to the observed object.

Practical Application: Make Your Own Tool!

There is a simple and easy-to-make tool for measuring angles. One of the fun ways to make understanding angles of elevation and depression easier is to build a device using basic tools.

To make this device, we need a protractor, a hollow pen or tube, a thread, and a circular metal object like a coin. First, we place the protractor on a flat surface such as paper or cardboard. Then, we place the pen on the straight edge of the protractor at 180°. After that, we tie the thread to the protractor's center point and tie the coin to the other end of the thread. We leave the thread free so gravity can pull it down.

When using it, we hold the device at eye level, look through the pen, and direct it toward the building or observed object. We then observe where the thread intersects the protractor; the angle that appears is the angle of elevation. Likewise, if you want to measure the angle of depression, you simply look down. In this way, you can use everyday items to understand mathematics easily and make it more fun.

Real-Life Applications

- **Engineering:** They are used to determine the height of buildings without the need for difficult measuring tools.
- **Aviation:** Pilots use them to determine the angle of ascent or descent for the plane.

- **Problem Solving:** They allow you to calculate the height of a building without a ladder or heavy equipment, using only the angle of elevation from a known point on the ground.
- **Distance Measurement:** They are used to determine the distance of an object from a specific point.
- **Video Games:** In 3D games, developers use angles of elevation and depression to accurately calculate building heights and control the camera view when a player looks up or down, making the game more realistic.

Many engineers rely on concepts that may seem simple but are actually crucial to our daily lives. Angles of elevation and depression are highly important and have countless real-world applications.

I have shown how to measure the angles of elevation and depression in a simple way, which makes understanding the concept much easier. This is what makes mathematics fun; there is no real complexity to it, despite what many people think!

By trying one simple, hands-on experiment, a lesson can become much more interesting and enjoyable, significantly helping with our studies.

Perhaps, the next time you look at a tall building, maybe you will not just see a normal structure, but a mathematical problem waiting to be solved.