

Is Mathematics discovered or invented?

Mathematics seems to describe universal truths suggesting it is discovered, yet also depends on systems and assumptions created by humans. To understand this, we need to understand what mathematics is, why we use it and where it comes from. On one hand, mathematics appears to be discovered, as different civilisations independently developed similar ideas and reached similar conclusions. On the other hand, the systems, notation and ways of thinking about mathematics clearly show human invention. So is mathematics something we uncover and discover or something if it is something we create or invent? It makes sense to think of Mathematics as both.

Maths aims to find meaning in the world around us by identifying patterns and structures that we constantly interact with. It allows us to predict outcomes, describe relationships, and use logical thought to interpret both abstract systems and the natural world. It starts with counting. This suggests mathematics may be discovered as it appears to describe patterns that already exist in the world around us.

Civilisations through history, for example the Mayans, Babylonians and ancient Egyptians, used mathematics as a tool for practical purposes including building economies, tracking time, measuring land, building to cities, even tracking the stars and predicting astronomical events. Although the different civilisations had different number systems, but they had similar goals, using maths for solving problems such as equations. Babylonians used a base 60 system in an algebraic approach to solve quadratic and cubic equations, while the Maya used a base 20 system and developed symbols for zero so they could calculate astronomical cycles. This shows us that to these people maths not abstract or a theoretical concept but had obvious practical application. The fact different civilisations, separated by time and geography developed similar ideas also points to discovery rather than invention.

It was the Ancient Greeks who developed a more abstract understanding of Mathematics. Figures such as Pythagoras and Euclid introduced the idea of theorems and proofs. Euclid's thirteen volume work, 'Elements' is often referred to as the most fundamental textbook ever created. It helped shape how we think about Mathematics. Elements, which focused on number theory, ratios and proportions and solid and plane geometry, was used for centuries, surviving today as a historic educational text, rather than a textbook for practicing modern maths. In Phaedo, Plato argued that the changing world we experience is not true reality, and that beyond it lies a realm of unchanging forms. According to this theory, everything we see is an imperfect version of these ideal forms. Plato illustrates this idea using objects such as beds, but the principle applies more generally: We recognise objects as chairs not because they are identical, but because we share an understanding of what the concept of a chair is.

This can be applied to mathematics. A drawn circle is never perfectly round, and a measured line is never exact, yet we understand both as precise concepts. Mathematical ideas appear to be exact, unchanging and true everywhere. For instance,

$2 + 1 = 3$ regardless of time or place. This suggests that mathematics exists to be discovered, as if these truths exist independently and we are simply uncovering them.

However, history complicates this view. For example, concepts like negative numbers and imaginary numbers were rejected for long periods of history because they did not seem real or useful. So, if mathematical truths already exist waiting to be discovered, it is difficult to understand or explain why they are not just 'discovered' and immediately recognised, like perhaps DNA was discovered, and immediately accepted as objective truth. This suggests that human understanding and acceptance play a role in developing mathematics, which points towards an element of invention rather than pure discovery.

This becomes clearer when thinking about axioms. Axioms, sometimes called postulates, are statements accepted as true without proof. They form the foundation of mathematical systems. Euclid built geometry on five postulates, one of which, the parallel postulate, was debated for centuries. Eventually mathematicians such as Carl Friedrich Gauss and Nikolai Lobachevsky showed that rather than proving it, it was possible to develop entirely new, equally valid geometries by changing it. This suggests mathematics is not just a fixed system waiting to be discovered but something that depends on assumptions we choose, meaning there is a clear human element in shaping it.

At the same time, the underlying ideas tend to stay the same. Different cultures have used different ways of writing numbers, yet the relationships between those numbers stay the same. Roman numerals and the Hindu-Arabic system both describe quantity, just in different forms. In the same way, Isaac Newton and Gottfried Wilhelm Leibniz developed calculus independently, using different notation and approaches, but arriving at similar results. This suggests that while the language of mathematics is invented, the ideas behind it are discovered.

Pure, abstract mathematics is often viewed as meaningless 'Maths for maths sake'. However, we find real life applications for very difficult to follow, pure mathematical ideas and proofs such as Ramanujan's infinite summation. Ramanujan worked with infinite series that can lead to the surprising result that the sum of all natural numbers can be interpreted as $-1/12$. At first this seems completely illogical, as no negative or fractional numbers appear in the sum, and clearly adding numbers like this would grow larger without limit. However, this result comes from a different way of defining and handling infinite series, rather than from literally adding the numbers one by one. Although this idea cannot be applied in a straightforward, physical sense, it still has real uses, particularly in areas like string theory, where physicists work with infinite systems. In this context, a result that appears abstract or even nonsensical can help describe the structure of the universe. This suggests that mathematics can reveal truths that go beyond our immediate intuition, although the fact that we choose how to interpret and apply these ideas also shows a human element. This leans towards the idea that mathematics is discovered, as these results seem to reflect real features of the universe, even if we must invent new ways of understanding them.

Like language or maps, human inventions that enable us to describe and understand reality, mathematics can be seen as something we construct to help us interpret the real world. We decide what to focus on, what to develop further, and how to apply it. In that sense, mathematics is also shaped by invention.

Human acceptance of mathematical ideas may be guided by usefulness, but their truth is governed by logic, not utility. Gottlob Frege stated that “To discover truths is the task of all sciences; it falls to logic to discern the laws of truths”, or in other words, logic sets the laws of what’s true compared to what isn’t. Frege’s suggestion is that that logic determines what is true, not what is useful. Frege’s belief supports the idea that maths is discovered; that mathematical truths exist independently of us, even if it takes time for us to recognise them. It suggests these ideas follow the fundamental meaning about what’s true and what isn’t, i.e. If something’s true in maths, it’s 100% objectively true.

I recall a year 12 mechanics lesson. My teacher was going through a problem with the class. I solved a problem one way while my teacher used a completely different method and yet the answer at the end was the same. This the beauty of pure mathematics lies; like Plato’s idea of the Form, regardless of your method or interpretation there a seemingly infinite paths to the truth.

Despite the appeal of the idea of discovering maths, in answer to the question “*Is maths Invented or discovered?*” both are true.

The patterns and relationships mathematics describe appear consistent, recurring throughout humanity, regardless of geography and time and therefore exist independent of human thought. The building blocks of mathematics are universal truths that existed and were discovered. At the same time, the systems, notation and frameworks we use to understand those patterns are clearly human constructions. The art of Mathematics, and the practice of mathematicians, is ‘invented’; created on the foundations of discovery. It is more convincing to see mathematics as something we uncover, but also something we shape in the process of trying to understand it. The pursuit of a solution, the researching and the asking is what’s truly important. We must leave the question, enjoying the irony that despite mathematics being our most reliable judge of absolutes, it offers no such certainty when it comes to defining its own origin. When asking whether it is invented or discovered, we have no definitive answer; only the freedom as in “*Cogito, ergo sum*”, to accept that in the absence of contradiction, either view may be true.