How A Human Raced AI's Perfect Lap and Won.

Saturday 15th September 2018, reigning F1 champions Mercedes-AMG look to extend their lead over Ferrari's Sebastian Vettel during the Singapore grand prix. The AMG team models the perfect lap; perfect turns, flawless braking, superhuman reaction speeds: 1:36.150. Lewis Hamiton returned in 1:36.015. Is the model faulted, or did Hamilton defy physics? Did an engineer slip up, or are computers simply not able to keep up with us?

A good place to start our investigation would be to investigate what affects a lap time. First of all the physical conditions, from the moisture on the track to the engine itself. Any change in the car will change our 'perfect' lap time. A trivial escape would be to blame inaccuracies in the data available to the statisticians. However, in a sport where brake discs are manufactured with a tolerance of 0.02mm, precision can be taken for granted. Secondly, this modeled lap time I have discussed is the figure Mercedes generated themselves, so a lack of information can be ruled out. So where did Lewis find this time?

We've established that both Lewis Hamilton and his virtual rival were able to race in equal machinery, on the same track (be it physically or not) on the same hot and humid Saturday in Singapore. At this point, with no unrealistic assumptions, we can isolate one independent variable: the driving.

Firstly, I'd like to explore the nature of the model itself. Formula teams are able to predict almost every element of the race, and crucially so. To know when a tyre will need replacing means pit stops can be planned, but predicting a lap time is, in my opinion, fascinating. The mathematical model begins simply as a summation of times, each time being the result of an advanced piecewise function. The track (in our case: 4940 metres including 19 turns) is modelled as a series of functions of time. Exactly how Mercedes would choose to divide the track isn't known, but we could deduce logically that the simplest model would be to divide the track into 19 areas of deceleration (entering corners) and 20 areas of acceleration (race start and exiting corners).

In the interest of simplicity, let's isolate this first section: roughly 178 metres from the start grid to the braking zone of the first corner. Given a distance, sufficient information about the car, and an initial velocity, in this case zero, we can mathematically quantify the forces applied to the car, find an acceleration, and subsequently a time, and a final velocity, allowing us to do the same again for the proceeding section of track. At this point, Newton's second law can be used to find an acceleration. Once again Mercedes exact method is a mystery, however I will assume mass to be constant for each section. Over the 178 metres of acceleration, I've estimated the mass lost as combusted fuel vapours to be roughly 70 grams given the 2018 V6 statistics. Not negligible, however using a median value for each section seems to suffice given each car has an enforced minimum mass of 733kg. Next, given the acceleration of the car is equal to the sum of the forces divided by the mass, we would need to quantify each non-trivial force acting upon the car: thrust, drag and friction.

First of all, resolving in the direction the car is travelling, thrust can be calculated by multiplying the output torque of the engine (camshaft) and the gear ratio and finally dividing by the wheel's radius. Both the torque of the engine and the gear ratio will vary massively during our 170 metres, this problem can be solved using graphical methods and calculus, so I will try to avoid looking into this, safe in the knowledge it isn’t the source of Lewis Hamilton’s miracle. Given the torque of the vehicle is found experimentally using a dynamometer, and will be known by the Mercedes team, given enough information, we too could calculate the thrust experienced by the racecar.

Secondly , we will tackle friction. This is calculated by multiplying the coefficient of friction with the normal reaction force of the car. In mechanics, the normal reaction force is often simply assumed to be exactly equal to the weight of the car. We won’t make this assumption in order to avoid inaccuracies due to the speed the car travels, and the vast quantities of downforce produced. This downforce is proportional to speed, adding further complications, but is all completely possible given reasonable data of the car, such as the frontal area, and reasonable information of the track on the day, such as the air density. At first glance, this air density measurement appears to be an obvious source of inaccuracy, however this too can simply be measured prior to the race. Finally, the values for drag are proportional to speed, but can be eloquently calculated using data from wind tunnel experiments.

At this point we have all the values to apply Newton’s second law of motion to calculate an acceleration for a section of track, and therefore calculate a time, however we have not yet encountered an obvious source of uncertainty. However, at this point the Mercedes team likely wouldn’t simply have calculated a single time, but rather have employed Stochastic modelling, or colloquially the ‘Monte Carlo method’. Hereby, statisticians and powerful computers vary values slightly, considering values that may change during the race such as air density. Exploiting varied values and probability models, an array of results (times) are produced. This is almost certainly where Lewis got ahead. We cannot say for certain, however, given that Lewis beat the model, we can assume that the final time used by the Mercedes-AMG team was not simply the fastest time predicted by the model, but rather a slightly conservative estimate, perhaps still assuming perfect driving, but slightly sub-optimal conditions.

To conclude, Lewis Hamilton miraculously beating a modelled ‘perfect lap’ was a fascinating display of performance, however, we can safely conclude that Lewis did not simply outpace a perfect lap, or that Mercedes’ statisticians blundered, but rather that the application of Stochastic modelling introduced a level of variance. Statisticians are left to decide which fraction of the very fastest results produced seem unfeasible, and should be excluded from tactical planning within the Mercedes team. On the 15th of September 2018, Sir Lewis Hamilton reduced human error to such an extent that Mercedes’ statistical methods did not predict, forcing a change in the methods used since, and an increase in consideration when discarding seemingly impossible results.