How Close is close enough?

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Introduction:

Mathematics is a field that discovers and organizes methods, theories and equations that are developed and provide a response to when exact answers are either impossible or impractical. But in real life (natural sciences, engineering, medicine and finance), things often get messy - so we often can't calculate exactly.

So, mathematicians often approximate.

In the real-world a lot of the time mathematicians end up modelling, which are simplifications of reality e.g., in projectile motion, you ignore air resistance. Whereas in finance, models assume normal distributions which aren't perfectly true as they don't take into account of direct descriptions of reality rather just estimations.
So, the calculation is exact within the model, but the model itself is an approximation.

So, when does approximation become acceptable to the point we can stop calculating?

Example:

Imagine you're out on a sunny day with your mates down in town, you've just left the classroom to grab some food before you renter your lesson. As you arrive back, you're sat down at your desk you end up deciding to type in a calculation into a calculator that you've seen of the board, you end up getting an answer of 0.33333333… You think to yourself when do I stop writing the 3s? And more importantly - why is it okay to stop. So, you end up rounding it to 0.3 even though you know that's not the true value, the difference is tiny, almost invisible, yet it's still there. In most cases in mathematics, we ignore gaps like this entirely as it works well enough to be useful.
So, what happens if we keep improving an approximation over and over? You decide instead of stopping in after a few steps, imagine continuing the process indefinitely. Each step bringing in closer to the true value, but never quite the true value.

Limits and Convergence:

You look back down again still thinking about your previous result, at your desk you see the work sheet handed to you by your professor, it tells you to calculate the sum of

$$0.9 + 0.09 +0.009 + \dots = 1$$

You realise each term ends up adding a little more, bit by bit, bringing the total value closer to 1. But it still feels like your always short of the target. In mathematics, this iteration is not shown as nearly 1 - but exactly 1, this idea is called convergence. Instead, we focus in on what the sequence as a whole - approaches. When the values calculated settle towards a single number, that number becomes the result.
So, we can start of by defining the function by,

Let X = the function

$$X = 0.9 + 0.09 + 0.009 \ldots$$

At first the problem seems impossible as there's an infinite number of terms. However, in mathematics we can interpret infinity using what's called limits, which define what it means for a sequence or function to continue indefinitely.

Next, we can interpret from the question that each term is obtained by multiplying the previous one by 0.1 e.g., $0.09 = 0.9\times 0.1^2$
This means that the sum is found by repeating the process, our function can then be rewritten as

$$X = 0.9(1 + 0.1 + 0.1^2 + 0.1^3 \ldots)$$

This allows us to isolate the repeating sequence, so we can focus on a single pattern rather than infinite terms.
Since infinite addition can't be computed, we define our partial sums as

$$X_n = 0.9(1 + 0.1 + 0.1^2 + 0.1^3 \ldots 0.1^n)$$

Creating a sequence of finite approximations
This can be rewritten as

$$X = \lim_{n\to\infty} X_n$$

Which is essential as it gets rid of the image of infinite addition. Next, we evaluate the partial sum, this is done so we can understand the behaviour of the series, determining whether they approximate the total sum or converge at a specific value.

$$Z_n = 1 + x + x^2 + \cdots + x^n$$

This form allows us to study the structure of the question more easily as dependence on 0.1 is removed

We can multiply through by x:

$$xZ_n = 1 + x + x^2 + x^3 + \cdots + x^{n+1}$$

This allows us to start cancelling terms
We then subtract:

$$Z_n - xZ_n$$

Expand:

$$(1 + x + x^2 + \cdots + x^n) - (x + x^2 + \cdots + x^{n+1})$$

We can then cancel all intermediate term:

$$Z_n(1 - x) = 1 - x^{n+1}$$

This has then left us with a closed algebraic expression that can be analysed precisely:

$$Z_n = \frac{1 - x^{n+1}}{1 - x}$$

$$Z = \lim_{n \to \infty} \frac{1 - x^{n+1}}{1 - x}$$

We can then input for when x = 0.1
This causes the values to shrink, as repeated multiplications cause it to tend towards 0

$$0.1^{n+1} \to 0$$

This is convergence as the sequence approaches a fixed value as n increases

$$Z = \frac{1}{1 - x}$$

Thus:

$$T = \frac{1}{1 - 0.1} = \frac{1}{0.9}$$

So far, we've discovered that

$$1 + x + x^2 + x^3 + \cdots = \frac{1}{1 - x}$$

Now why does the function $\frac{1}{1-x}$ form an infinite pattern?
To answer the question, we write our function in term of x to the power of, due to our original question being built entirely from powers of 0.1

Our assumed structure thus is:

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$$

Which is very useful due to it translating a function into the same language as the series were looking at. Next, we determine the coefficients by setting x = 0, leaving us with the constant term. The higher functions can be determined via repeated differentiation.

e.g., $f(0) = 1$

$f'(0) = 1$

$f''(0) = 2!$

…

$$f(x) = \frac{1}{1-x}$$

This leaves us with coefficients of all 1:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

This leaves us with a Taylor-style interpretation. The Taylor series is fundamental mathematical tool used for approximations, as it's able to turn complex, non-linear functions, e.g., sin(x) and ln(x) into easy polynomials. Providing a highly accurate approximation, where each additional polynomial term reduces the error in a predictable way.

$$X = 0.9(1 + 0.1 + 0.1^2 + 0.1^3 \ldots)$$

By applying the result to the original question, we can use the derived function.

$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

And then substitute in 0.1 for X

$$X = 0.9 \times \frac{1}{1-0.1}$$

Giving us the final solution of 1

$$X = 0.9 \times \frac{1}{0.9} = 1$$

Finally, the infinite sum converges as the partial sums approach the fixed limit. The main reason this works is because of the Taylor series structure of $\frac{1}{1-x}$ which shows

the power series of a function. Linking infinite decimals, functions, and convergence into a single framework.
So how accurate or "close enough" is determined by the size of the remaining error - only becoming acceptable when sufficiently small for practical use.

Conclusion

In conclusion close enough is not a compensation rather it's a necessity. In engineering, every structure is built with, tolerances accepting that no measurements can ever be perfect yet ensuring reliability and safety in doing so. Whereas in physics uncertainty is not a limitation but rather a fundamental feature of reality, captured in numerous ideas like Heisenberg Uncertainty principle. Every single day we put our trust in systems which are most imperfect like GPS positions are only approximations, clocks no matter how accurate always drift with time.

Throughout time mathematicians have reflected the same ideology with equations like the Taylor series, we replace complex functions with simpler approximations that are accurate enough till a certain stage, but yet it still proves to me enough.

Yet despite all the imperfection and uncertainty, the world works with almost perfect consistency. Down to how computers store numbers and functions to how bridges and buildings work within tolerances and safety margins. Proves that maybe mathematics isn´t about perfection - but rather how much imperfection we know we can afford.