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How do patterns in nature relate to mathematical laws

A pattern may be present, not just as a regularity in design with a mathematical law underlying it, but also as an outlier to this definition within an abstract idea: centre of gravity being one, as proposed by scientist Daniel Dennett (Dennett, D.,1991), or perhaps patterns in terms of their processes such as the pattern of earth's rotation. For the purposes of this essay, we shall only discuss the mathematical laws governing visible, structural patterns present in and around us.

"The universe cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language……"

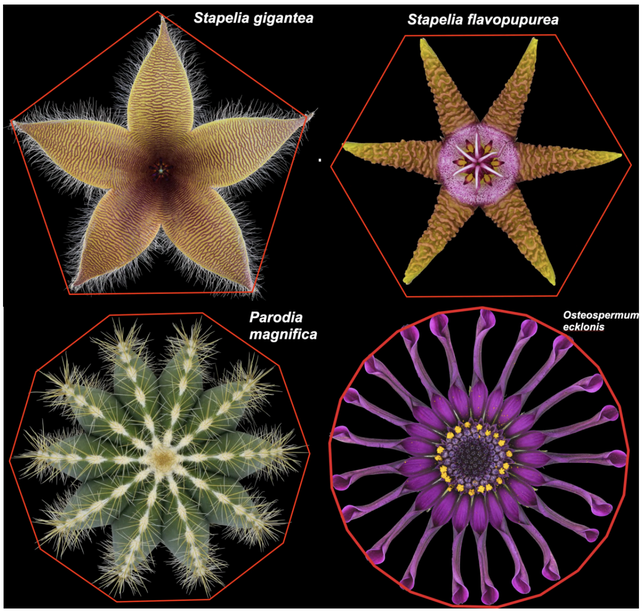

Fig 1: Regularity in plants

As Galileo suggested, if the natural world speaks, it seems to do so in mathematics. One of the first and most easily recognisable mathematical elements in nature would be that of shapes. From triangles and pentagons in Oxalis Triangularis and Stapelia Gigantea to spheres in Dahlia and Hoya Pubicalyx and spirals in Rebutia Krainziana. But what could possibly be nature's mathematical magnum opus is the Romanesco Broccoli, featuring not only spirals, but also fractals, which are never ending patterns and have the property of self similarity i.e. any part of the plant if compared to the plant as a whole retains the same shape.

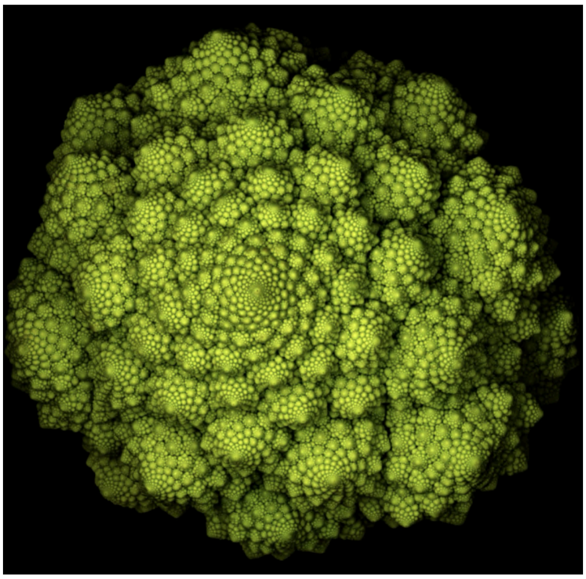

Fig 2: Romanesco Broccoli

These patterns can be seen not only in the flower arrangement but also in the phyllotaxis, or arrangement of leaves on the stem of the plant. The leaf arrangement might be whorled, like in Nerium, spiral, like in sunflowers, or distichious, like in Strelitziaceae or maize, where the leaves are at 180-degree angles to one another or Dechusate where the leaves are at 90 degree angles to each other as in Choleus or Aizoaceae, . It's interesting to note that additional examination of a sunflower's primordia may reveal recurring patterns of clockwise and anticlockwise spirals. If counted separately, the pair of numbers of clockwise and anticlockwise numbers are together referred to as parastichy numbers. For a sunflower, they are 21 and 34 respectively. Following the same trend, most succulents have the number 3 and 5, pineapples, pinecones and romanesco broccoli have 8 and 13 and so on. Such a pattern can also be found on artichokes or the aloe plant.

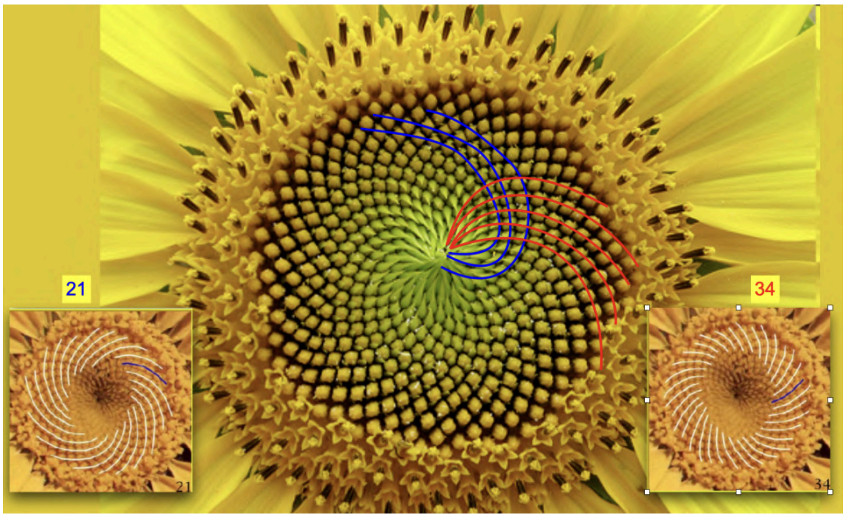

Fig 3: Spirals on Sunflower head and their parastichy numbers

Even the smaller spirals on the fractal extensions or continued "bumps" on plants like the romanesco broccoli appear in a clockwise and anticlockwise spiral of 8 and 13. These fractals aren't simply limited to plants, however, they are present everywhere from the roots and branches of the trees to our circulatory system, nervous system, respiratory system and the way lightning strikes, and so much more. All of these patterns occur as fractals as it provides the optimum and most efficient way to carry out each system's functions, from allowing the leaves to get the most amount of sunlight and water from its branches and roots, to enabling lightning to dissipate the most amount of energy into the atmosphere.

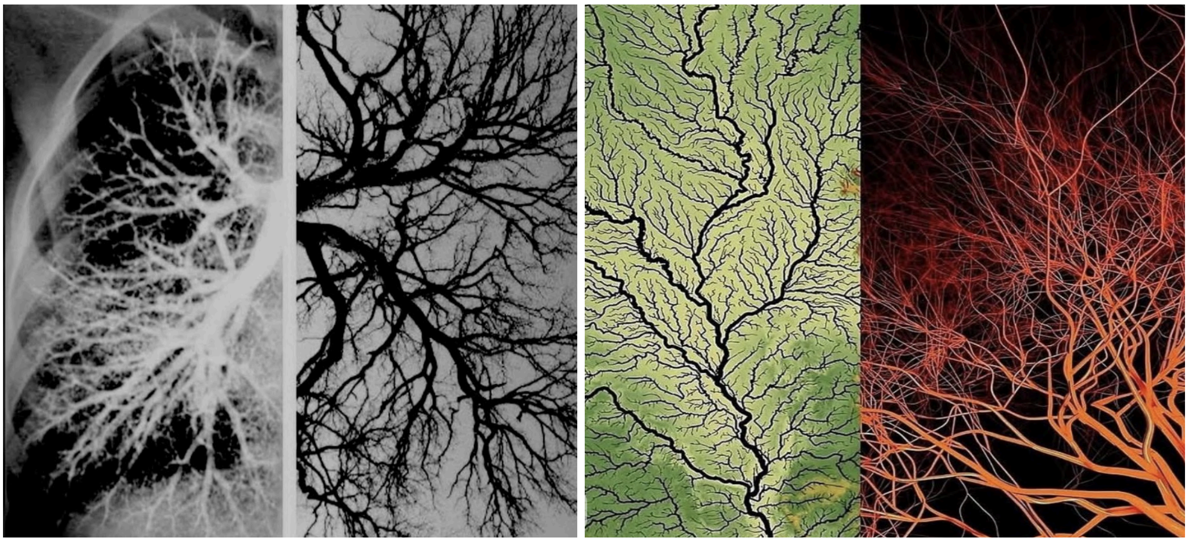

Fig 4: Fractals in nature; Branching of alveoli in lungs, Branches of a tree, Distribution of river into its tributaries, Branching of blood vessels (left to right)

On deeper analysis, it may be noted that the parastichy numbers in the vast majority of plants themselves seem to follow a pattern found in the Fibonacci Sequence, which can be defined by the recurrence relation:

$$f_n = f_{n-1} + f_{n-2}$$

Where $f_1 = 1$ and $f_2 = 1$ for all values where $n > 0$.
If one is to compare two consecutive Fibonacci numbers to obtain the following ratio:

$$r_n = f_{n+1}/f_n$$

Where n>1, which eventually converges to a value phi $\varphi=1.618$ as can be seen if a limit r is to be supposed for n as given where:

$$r = 1 + 1/r$$

$$r^2 = r + 1$$

$$r^2 - r - 1 = 0$$

$$\Rightarrow (1+\sqrt{5})/2$$

($\therefore$ positive solution upon applying quadratic formula)

$$\Rightarrow 1.618$$

This value is thus known as the Golden Ratio and if the same ratio is applied to a circle, with major arc a and minor arc b, the smaller angle α is found to be:

$$\alpha = 360*b/(a+b) = 360/\varphi^2 = 137.5077640^\circ$$

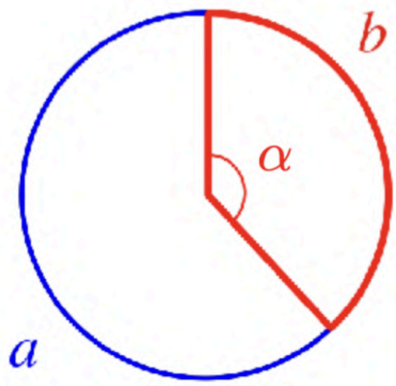

Fig 5.

Which is commonly referred to as the Golden Angle. This value plays a very significant role in the spiral phyllotaxis as well as angles separating the florets and primordia of a sunflower. At the shoot apical meristem of such plants, each new primordium seems to arise in a sequential order which may be visualised as Fermat's Spiral which is defined as:

$$r = \pm a\sqrt{\varphi}$$

Where the radius r grows as the square root of the angle does, and a represents the tightness of the spiral essentially. Thus, the patterns of primordia can be predicted if a divergence angle is known. A pattern resembling the primordia of a sunflower can be seen if the divergence angle of the golden angle is calculated. The packing pattern is completely altered by even a small variation, which isn't very common in nature. The primordial packing pattern turns out to be ideal when the golden angle is an irrational value. The golden angle is the "most irrational" thus, guaranteeing maximum utilisation of space and minimal overlap.

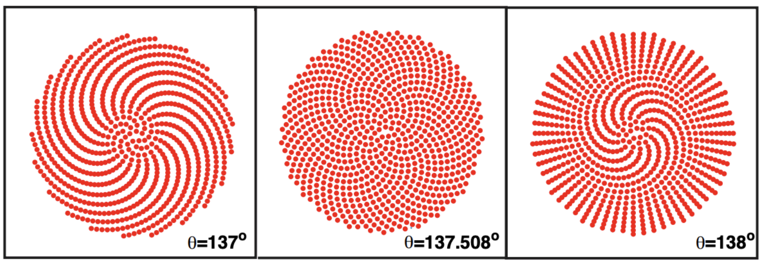

Fig 6: Effect of varying divergence angles in formation of Fermat's spirals

These primordial growths occur sequentially from the centre and are pushed outward. Existing primordia absorb the phytohormone auxin, which is necessary for their development, away from their immediate surroundings. As a result, the concentration of auxin is lowest near existing buds and gradually increases in regions further away where the new primordia develop and each growth occurs 137.5◦ away from the previous growth. (Smith et al., 2006)

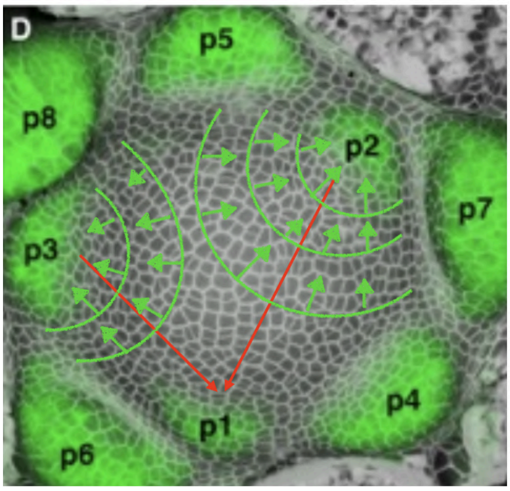

Fig 7: Formation of new primordia on shoot apical meristem of Arabidopsis Thaliana

“Eadem Mutata Resurgo”

“Although changed, I shall rise again”
Jacob Bernoulli was so inspired by the logarithmic spiral that he insisted that his tomb have the spiral engraved with the aforementioned quote. Given its prevalence in nature, specifically in horns, shells, and galaxies, he was right to have been so immensely fascinated by the logarithmic spiral. Denoted spira mirabilis, “marvelous spiral” by him, it has the notable property of being equiangular, meaning that any point on the curve keeps the same angle between the curve and the line drawn from the centre. Moreover, its shape never changes on extending it, only its size hence, they too have the property of self similarity like fractals, hence the Latin phrase. In a three-dimensional shell, this equiangular curve is maintained while its radius increases exponentially along with a linear increase in its height. (Moseley, 1838).

Fig 9: Ammonite with Logarithmic spiral displayed

The mantle, a soft tissue layer on the surface of the organism, secretes calcium carbonate to build its shell, and hence it lengthens at the rims and thickens from the inside. It uniformly deposits the material to create a slightly larger opening each time, aiding the development of a conelike structure. It also ensures slightly more deposition on one side to cause rotation of the shell layers at the point of maximal deposition and creates a torus which twists, leading to its helico-spiral shape (Chirat *et al.*, 2021). If, however, the aperture is too small, the mantle must change to accommodate the organism within. It does so by having growth at intervals, which after repeated depositions, form spine-like structures, the number of which in a particular shell occurs as fractals. In ammonite, a similar mechanism takes place but without the "twisting." A system of growth with compressions and stretching then takes place, leading to its ribbed structure (Moulton *et al.*, 2015).

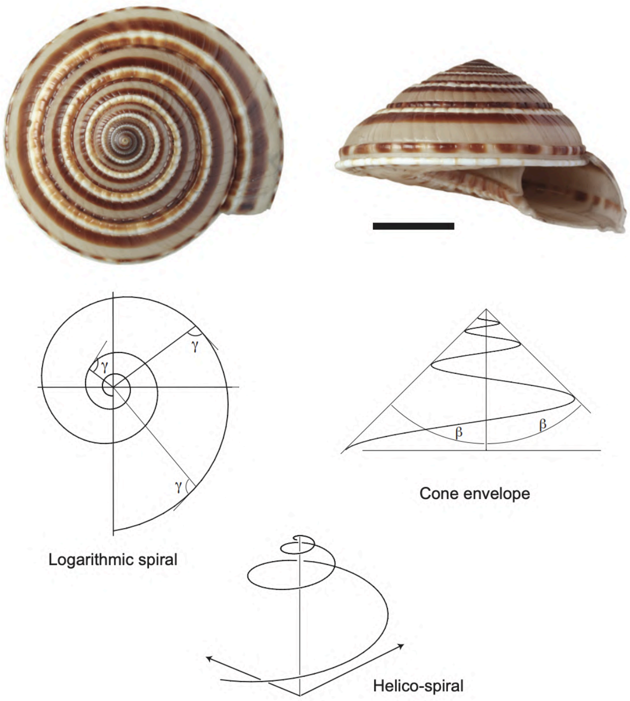

Fig 8: Architectonica as viewed from the top and side to visualise presence of logarithmic spiral and cone structure

The Chemical Basis of Morphogenesis

Most of these patterns observed on organisms are governed by a system of equations defined by Alan Turing, known as the reaction diffusion system where he essentially took the idea that two stable systems can potentially create instability causing morphogenesis. It can be displayed in its general form as :

$$\partial X/\partial t = f(X,Y) + \mu \nabla^2 X$$

$$\partial Y/\partial t = g(X,Y) + \nu \nabla^2 Y$$

Where $\partial X/\partial t$ and $\partial Y/\partial t$ represent the rate of change of the morphogen concentrations over time, f(X, Y) and g(X, Y) represent the chemical reactions functions and $\mu \nabla^2 X$ and

$\nu\nabla^{2}Y$ represent the diffusion functions with μ and ν acting as the diffusabilities of the morphogens (Turing, A., 1952).

This works on the idea that morphogens like proteins, hormones, or genes, act as activators and inhibitors to create patterns like stripes, spots, or waves depending on the size of the surface area on which they are acting and the variables within the equation.

Fig 10: Various patterns found on living creatures as predicted by Turing's formula

This further applies to the ridges on the upper surface of a rat's mouth,the toothlike scales on a shark, the auxin distribution pattern as previously discussed, the spacing of feathers on a bird or hair on a human's body(Widelitz, R., Baker, R., et al, 2006). Most surprisingly, perhaps, it even plays a role in the development of human digits, and if the number of morphogens are more than two, they form fingerprints(Alvarado,D., Martinez, A., 2011)!

The Unreasonable Effectiveness of Mathematics

To pivot to a different perspective: Eugene Wigner defines mathematics not as a discovery of nature, but as something devised primarily to satisfy a mathematician's sense of formal beauty and "ingenious logical operations" thus, questioning why mathematics is the right language for the universe to abide by. The evolutionary development of nature mirrors mathematical patterns as we've explored. Both present a world where the rules are set, but the outcomes are manifold and unpredictable, reminding us of the bifurcation in chaos theory and its impact on determining evolutionary paths. And yet, these patterns can be traced, quantified, and often predicted through mathematics. Number and form seem not merely descriptive, but generative of the natural world itself. Then perhaps whether mathematics is the cause or a miraculous explanation of these biological phenomena remains an open question.

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