How is Mathematics present in The Traitors?

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1 Introduction

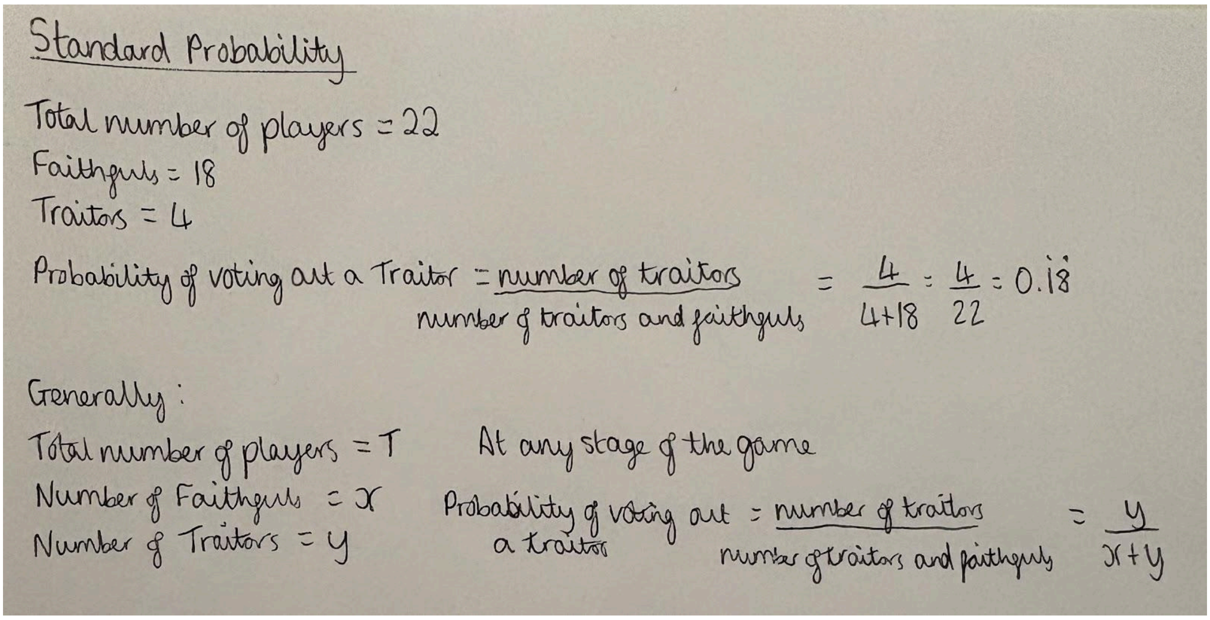
Like the 9.4 million others who tuned into the Finale of the Traitors Season 4, I was hooked by the recent UK series. However, as a Mathematician, I was not convinced that this gameshow is simply the psychological thriller it appears to be. Through probability trees, Bayes' Theorem and Game Theory, this essay aims to explore the variety of mathematical concepts underpinning the Traitors. Whether you are a superfan or have never heard of the show, we will answer the question 'How is Mathematics present in the Traitors?'.

However, for those of you that have never come across the Traitors before, here is a brief summary. In the past seasons, between 20 and 22 players have started the game, with 3-4 assigned as Traitors, whose objective is to 'betray' the Faithfuls through murdering (when a Faithful is murdered, they are eliminated) and banishing (voting off) all of them. On the other hand, 17-18 players, who are assigned as Faithful, aim to banish all of the Traitors. The winner(s) of the game split the prize money (more on this later) and the prize pot is built through players working as a team in missions to win money.

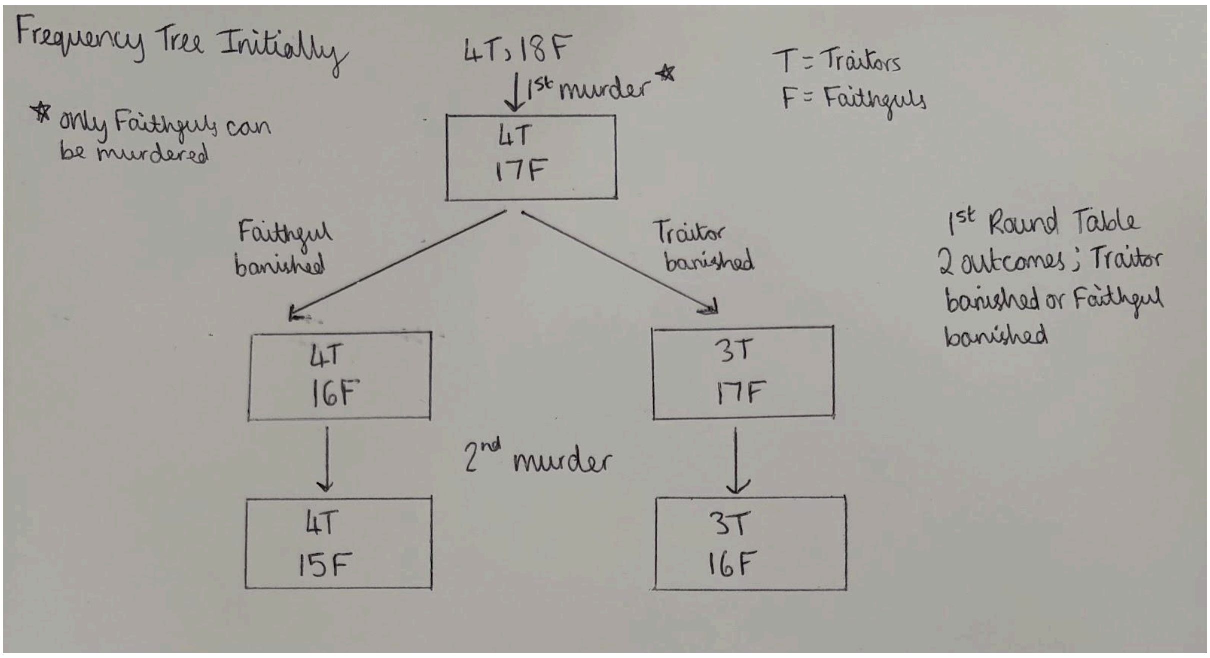
But how can Mathematics possibly help you in The Traitors? It is vital to understand the initial statistics in the game.

2 Probability

One key feature of the show is the round table, where all players sit at a round table each night and discuss their suspicions with the group. Each player can vote for who they want to eliminate and the person with the most votes must leave the game, but before leaving, shares their identity with the group. In terms of the game, the Faithful want to vote off the Traitors and the Traitors want to frame the Faithful and vote them off instead! However, before I get too excited, for mathematics to come into play, we should firstly ignore any external factors, such as players' opinions making them more likely to vote for a certain individual. In doing this, we can calculate the probability of voting a Traitor out at any one round table. This is similar to the probability you may be familiar with and is simply:

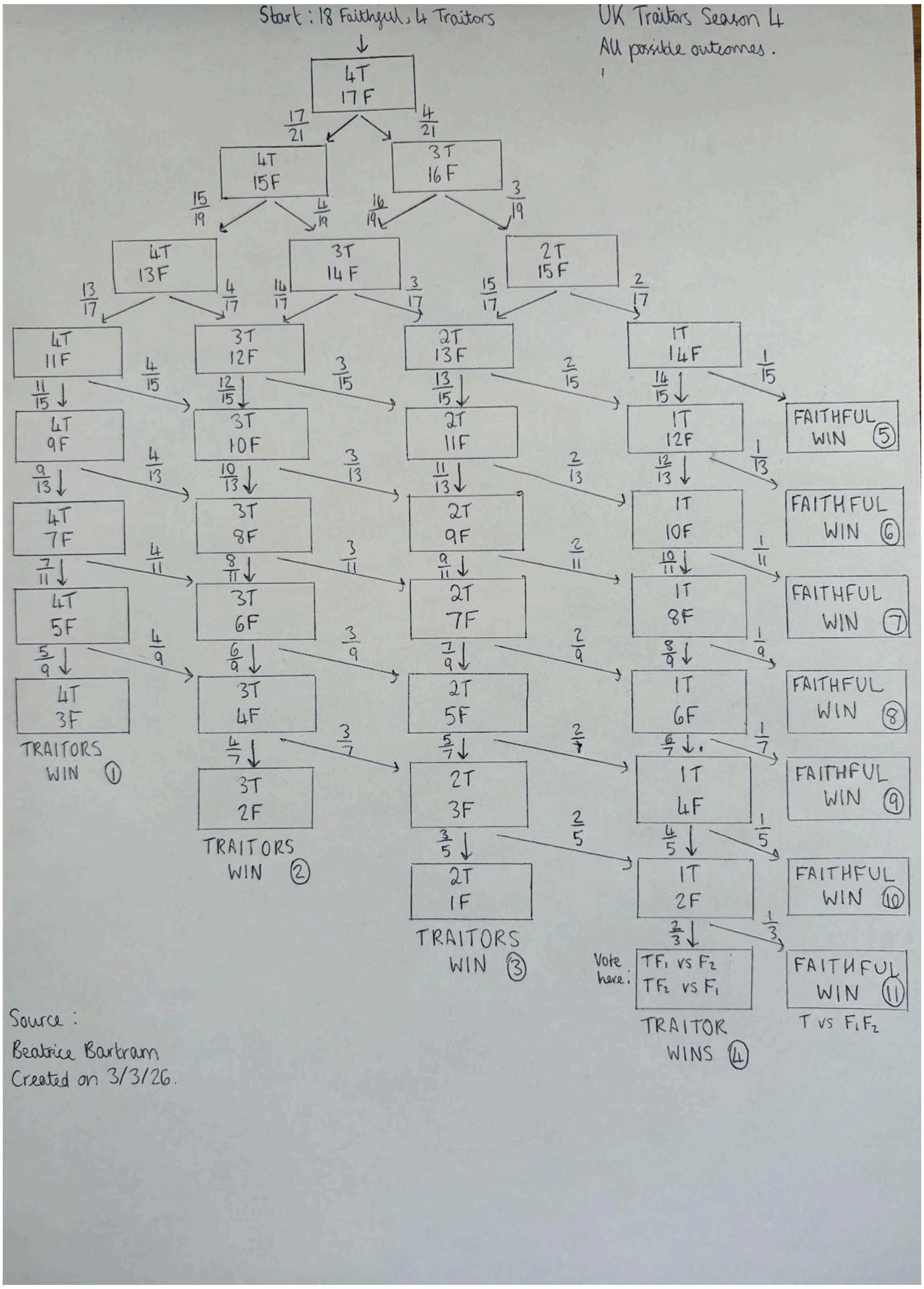


Therefore, at the start of the game the probability of a Traitor being voted off is 4/22 and the probability of a Faithful being voted off is 18/22. In the case of the show, a player is murdered first, so this reduces to 17 Faithfuls and 4 Traitors, then a round table occurs and one player is banished, before the Traitors commit their second murder that night. This means, at this point there are 2 possible outcomes when relating the probability to the actual gameplay, either 4 Traitors and 15 Faithfuls remain or 3 Traitors and 16 Faithfuls.



However, before long, the game will have accelerated past this initial stage and as a player, you would want to know how the probabilities change over time. Or perhaps, you want to gain an overview of all possible scenarios, to track how the game is unfolding.

Probability can be utilised through the whole show, and enables us to find out all possible outcomes of the game. This must consider every round table, so uses conditional probability. This results in a probability tree, where the difference in the total number of players between each branch is 2, given that one player is banished at a round table and one is murdered that night. Presuming that the game involves no unexpected twists, where there is a no murder night or a Traitor is recruited, this can be applied to the duration of a season. Each final outcome of the tree is a possible outcome of the game. In the Faithfuls' case, they want to vote all of the Traitors out, but in the Traitors' case, they want to reach the final episode of the show and outnumber the Faithful to vote against them and win. This results in a complex probability tree, with 11 possible outcomes, as pictured below.



Following scenario 1, the probability of the Traitors banishing all Faithfuls in a row is;
17/21 • 15/19 • 13/17 • 11/15 • 9/13 • 7/11 • 5/9 = 0.08771929825.
Another example route is scenario 5, where the Faithful banish all of the traitors in a row;
4/21 • 3/19 • 2/17 • 1/15 = 8/33915 or 0.0002358838272.
However, these are low probabilities and the most probable outcome lies in between, where some Traitors and some Faithfuls are eliminated, as the probability tree follows the middle branching.

Frequency Tree Analysis

Outcome Number	Probability (from tree)	T/F Win?	Traitors Remaining
1	$\frac{5}{57}$	TRAITORS	4
2	0.2048890222	TRAITORS	3
3	0.2627787073	TRAITORS	2
4	0.2424456864	TRAITORS	1
5	0.0002358838272	FAITHFUL	0
6	0.0008346658502	FAITHFUL	0
7	0.003439285173	FAITHFUL	0
8	0.008709556697	FAITHFUL	0
9	0.01942487215	FAITHFUL	0
10	0.04760140356	FAITHFUL	0
11	0.1212228432	FAITHFUL	0

Example of how this has been calculated

Outcome 2 (following frequency tree) 7 possible paths

$$\frac{4}{21} \times \frac{16}{19} \times \frac{14}{17} \times \frac{12}{15} \times \frac{10}{13} \times \frac{8}{11} \times \frac{6}{9} \times \frac{4}{7} = 0.02252168196$$

$$\frac{17}{21} \times \frac{4}{19} \times \frac{14}{17} \times \frac{12}{15} \times \frac{10}{13} \times \frac{8}{11} \times \frac{6}{9} \times \frac{4}{7} = 0.02392928709$$

$$\frac{17}{21} \times \frac{15}{19} \times \frac{4}{17} \times \frac{12}{15} \times \frac{10}{13} \times \frac{8}{11} \times \frac{6}{9} \times \frac{4}{7} = 0.02563852188$$

$$\frac{17}{21} \times \frac{15}{19} \times \frac{13}{17} \times \frac{4}{15} \times \frac{10}{13} \times \frac{8}{11} \times \frac{6}{9} \times \frac{4}{7} = \frac{2560}{92169}$$

$$\frac{17}{21} \times \frac{15}{19} \times \frac{13}{17} \times \frac{11}{15} \times \frac{4}{13} \times \frac{8}{11} \times \frac{6}{9} \times \frac{4}{7} = \frac{256}{8379}$$

$$\frac{17}{21} \times \frac{15}{19} \times \frac{13}{17} \times \frac{11}{15} \times \frac{9}{13} \times \frac{4}{11} \times \frac{6}{9} \times \frac{4}{7} = \frac{32}{931}$$

$$\frac{17}{21} \times \frac{15}{19} \times \frac{13}{17} \times \frac{11}{15} \times \frac{9}{13} \times \frac{7}{11} \times \frac{4}{9} \times \frac{4}{7} = \frac{16}{399}$$

$$\sum = 0.02252168196 + 0.02392928709 + 0.02563852188 + \frac{2560}{92169} + \frac{256}{8379} + \frac{32}{931} + \frac{16}{399}$$
$$= 0.2048890222$$

Source = calculations done by me using Casio fx-83GTX based on Probability tree.

However, following this pure probability approach, the probability of the Traitors winning is 0.7978327141, significantly higher than the Faithfuls. You may think, how can the game be fair if the Traitors have such an advantage? As stated by Braverman, Etesami, Mossel (6), a small team with a large amount of information is at an advantage, compared to a large team with a very little amount of information, which reinforces our probabilities. What you may not know is that the producers of the show plan each episode meticulously to create twists that balance these probabilities, for example, by introducing a new Traitor via recruitment or having a no murder night (4). Yet, at the start, the Faithfuls are much less likely to win. Therefore, if you were a Faithful placed in the castle, knowing the odds are stacked against you, what can you do to increase your chances of winning?

One piece of information that Faithfuls can use at the start of the game, is the expected number of Traitors they will vote off over the course of the show, which can be calculated using outcomes from the probability tree (4).

Expectation

$E(X)$ = number of traitors expected to be banished.

Number of Traitors banished	0	1	2	3	4
Probability of this happening (from frequency tree)	$\frac{5}{57}$	0.2048890222	0.2622787073	0.2424456864	0.2021672859
Expectation	$0 \cdot \frac{5}{57}$	$1 \cdot 0.2048890222$	$2 \cdot 0.2622787073$	$3 \cdot 0.2424456864$	$4 \cdot 0.2021672859$

$$E(x) = (0 \cdot \tfrac{5}{57}) + (1 \cdot 0.2048890222) + (2 \cdot 0.2622787073) + (3 \cdot 0.2424456864) + (4 \cdot 0.2021672859)$$

$$= 2.26645264$$

In the case of season 4, you can expect the Faithful to banish 2.26645264 Traitors. Since the Faithfuls have such little information (6), it can be vital to consider the expected number of Traitors they would eliminate if the game played solely to probability, and if the Faithfuls exceed this number, their probability of winning has also increased.

3 Bayes' Theorem

However, it becomes apparent that a show like the Traitors cannot only depend on probability, it is also influenced by peoples' opinions and actions. Interestingly, each of the missions and interactions that took place impacted players' suspicions of each other. Whilst a traditional approach would argue this is the psychological nature of the game, it can be interpreted in a mathematical sense. If one event affects another, we are dealing with conditional probability. Despite players' suspicions changing over the course of the series, ultimately Rachel and Stephen won as Traitors. Whilst it can be recognised that they both played incredible games tactically, could probability have indicated this?

To investigate this, Stephen can be analysed, to uncover how suspicious we should actually be of him, given his actions in the game. It must be noted that a key turning point of season 4 was episode 4, where the players competed in a mini game known as 'the cages'. Here, 8 players were placed in cages and they were partnered up with someone who had to help prevent them from being murdered by the Traitors. Sam, who had no partner, alongside Reece, Maz and Stephen lost the game, so they were eligible for murder. Maz was murdered and later in the game, other players believed that one of the players in the cages had to be a Traitor. Despite this being a logical argument, it seems completely psychological, until we examine the probabilities.

So, should Maz being murdered increase our suspicions of Stephen? In other words, what is the probability of Stephen being a Traitor, given that Maz was murdered, which can be calculated using the formula for conditional probability: Bayes' Theorem.

<u>Bayes' Theorem Notation</u>

$P(A)$ = probability of event A occurring

$P(B)$ = probability of even B occurring

$P(A|B)$ = probability of event B occurring, given that event A has occurred

$P(B|A)$ = probability of event A occurring, given that event B has occurred

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$ conditional probability (Bayes' Theorem formula)

Bayes' Theorem Investigation

Given that Maz has been murdered, what is the probability of Stephen being a Traitor?

Event = Maz murdered

Number of players remaining = 16 ; 3 Traitors, 13 Faithful

$$P(A \mid B) = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

$P(A) = P(\text{Stephen Traitor}) = P(ST)$

$P(B) = P(\text{Maz murdered}) = P(MM)$

$P(A \mid B) = P(\text{Stephen Traitor} \mid \text{Maz murdered}) = P(ST \mid MM)$

$P(B \mid A) = P(\text{Maz murdered} \mid \text{Stephen Traitor}) = P(MM \mid ST)$

$P(ST) = \frac{3}{16}$

$$\begin{aligned} P(MM) &= P(ST) \cdot P(MM \mid ST) + P(SF) \cdot P(MM \mid SF) \\ &= \left(\frac{3}{16} \cdot \frac{1}{3}\right) + \left(\frac{13}{16} \cdot \frac{1}{4}\right) \\ &= \frac{17}{64} \end{aligned}$$

$P(MM \mid SF) = \frac{1}{4}$

$P(MM \mid ST) = \frac{1}{3}$

$$P(ST \mid MM) = \frac{P(MM \mid ST) \cdot P(ST)}{P(MM)}$$

$$= \frac{\frac{1}{3} \cdot \frac{3}{16}}{\frac{17}{64}} = \frac{4}{17} = 0.23529411765$$

Prior to Maz's murder, the probability of Stephen being a Traitor was 0.1875 (3/16, which represents the 3 Traitors that remained amongst 16 players). After, this probability was 0.23529411765, which means Maz being murdered should increase our suspicions of Stephen. Perhaps if players had used Bayesian statistics, they would have noticed that Stephen returning from the cages increased his chances of being a Traitor. Thus, analysing how certain events impact the probability of a player being a Traitor can be a valuable tool for the Faithfuls and can indicate where they should place their suspicions, especially since they have such little information (6) about other players.

4 Shields

Another inaugural part of the game is a way that players can protect themselves from being murdered, by winning a shield in a mission. Does obtaining a shield reduce your chance of being a Traitor? Let's take a look at when 7 players remain in the game.

This is the information you have:

- You (Player 1) are a faithful
- Player 2 has a shield

Therefore what is the probability of each player being a Traitor?

We can label these players 1-7. Amongst these 7 players, there are 6 Faithfuls and 1 Traitor. Therefore, the probability of any one player being a Traitor is 1/7. However, that night player 7 is murdered. By updating these probabilities, given that player 7 has been murdered, the change in probability for each player being a Traitor can be calculated using Bayes' Theorem (4).

Shields Investigation

Player 2 has a shield, should we (player 1) be more or less suspicious of them?

Player Number	1	2	3	4	5	6	7
Known Status	Faithful	Shielded					
P(Traitor)	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
P(7 murdered | this player is a Traitor)	0	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	0
P(T) · P(7 murdered | this player is T)	0	$\frac{1}{36}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{30}$	0
P(T | 7 murdered) decimal	0	0.1724138	0.20690	0.20690	0.20690	0.20690	0
P(T | 7 murdered) fraction	0	$\frac{5}{29}$	$\frac{6}{29}$	$\frac{6}{29}$	$\frac{6}{29}$	$\frac{6}{29}$	0

$$\sum P(T) \cdot P(7 \text{ murdered} \mid \text{this player is } T) = \frac{1}{36} + \frac{1}{30} + \frac{1}{30} + \frac{1}{30} + \frac{1}{30}$$

$$= \frac{29}{180}$$ ∴ Total proportion is out of 29.

$$\frac{1}{36} \rightarrow \frac{5}{29}$$

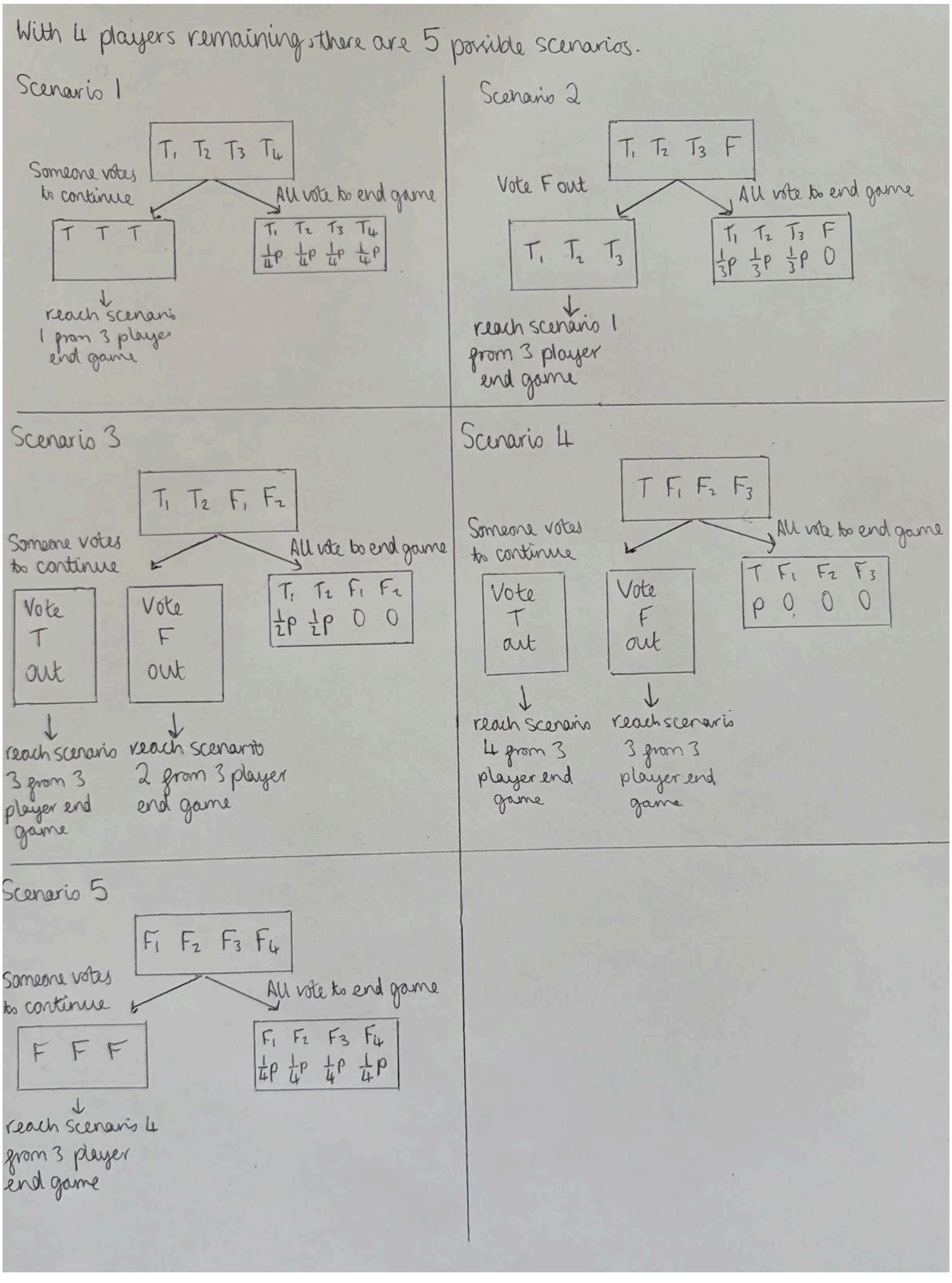
$$\frac{1}{30} \rightarrow \frac{6}{29}$$

A key difference is that P(7 murdered|this player is a Traitor) is 1/6 for player 2, as opposed to 1/5 for all other players. This is due to the fact that player 2 is shielded so cannot be murdered by other players but has all players 1,3,4,5,6,7 as options to murder if they are the Traitor themselves.

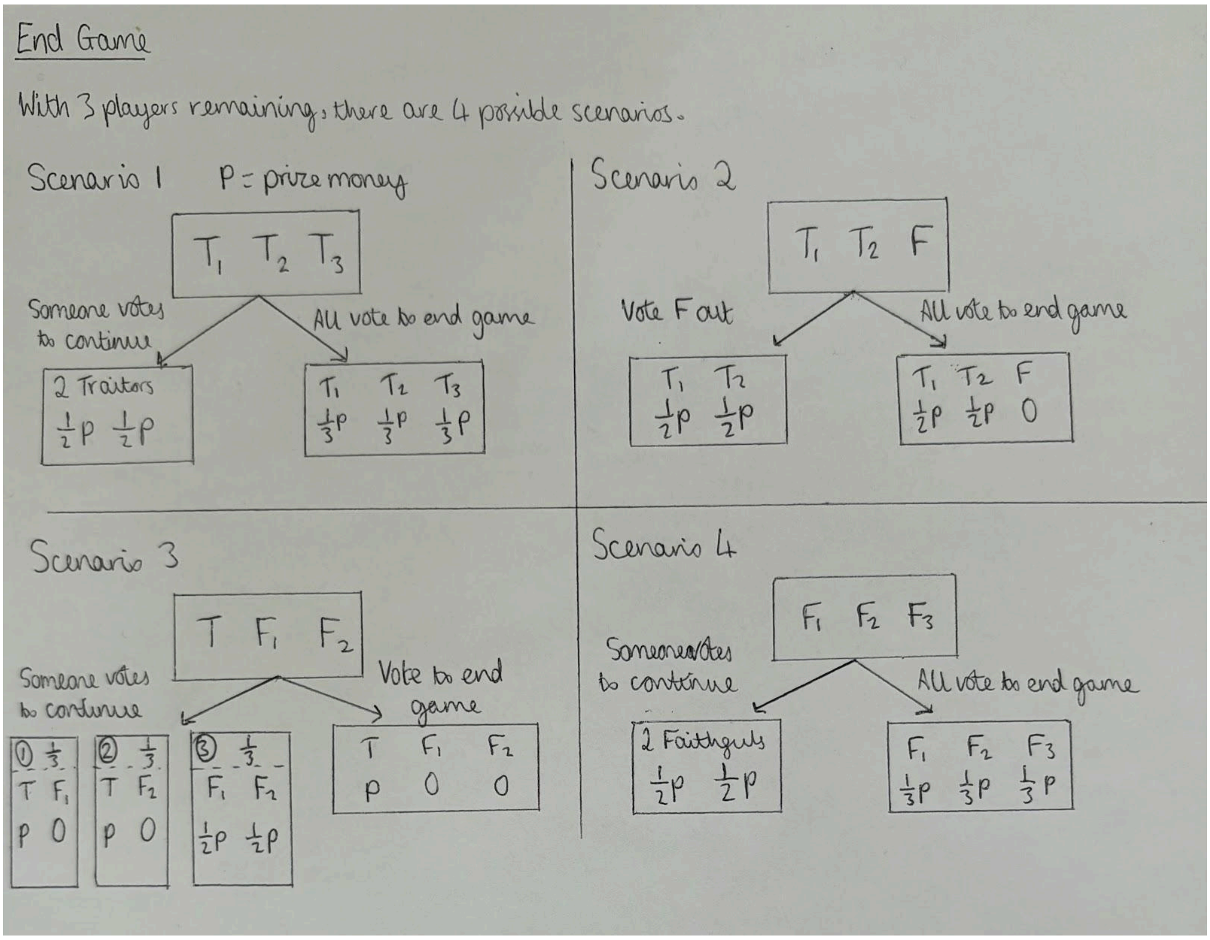
Also for player 2, $0.1724138<0.2$, which demonstrates that winning a shield reduces the chance of you being a traitor, or more logically, the chance of other players having suspicions of you. This is vital for understanding players’ motives behind wanting a shield, as Faithfuls want to protect themselves from murder, but for Traitors, a shield can be a valuable tool for proving that they are “faithful” to the other players and reducing others’ suspicions of them (4). Therefore, obtaining a shield does reduce your chance of being a Traitor.

5 End Game

However, our attention must also be turned to the final round tables, as understanding the motivations of each of the players at this point is critical for uncovering voting patterns. For the Traitor(s) to win, they must be remaining when the players unanimously decide to end the game. However, if any player votes to continue, another round of voting occurs. Focussing on an end game scenario with 4 players remaining, demonstrates there are 5 different possible combinations of players, which all have different outcomes.



Furthermore, some of these scenarios narrow the game down to 3 players, which is where the game ends (5).



Examining these scenarios informs any remaining Traitors that they need to outnumber the Faithfuls to undoubtedly win, or build strong enough relations with other players so they trust you are a Faithful and decide to end the game (5). For the Faithfuls, this indicates that they cannot simply outnumber Traitors, as the Traitors would still win. They must be certain that every other remaining player is also a Faithful, so they can be victorious and split the prize money. Therefore, analysing the different end game situations can be fundamental to creating strategies to win.

6 Conclusion

What may initially seem like an entertaining television show can actually be tactically played using mathematics. It is fundamental for all players to use the information available to them and evaluate each stage of the game, where probability can be vital for relating actions to suspicions and helping players progress through the game. It remains significant even at the end stage, as mathematics can reveal patterns in voting, as players of different roles seek to end the game in different ways to win.

References

All photographed sources are my own work. Beatrice Bartram, uploaded on 6/4/26.

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