Understanding Matrices: From Simple Grids to Real-World Power Tools by Sulayman Ali Khan

Introduction

Most people think of math as just symbols and formulas, but its ideas drive so much of what happens in our daily lives. Take the matrix. At first, it's just a tidy grid of numbers—columns and rows, nothing fancy. But behind that basic look, matrices are some of math's most powerful tools, quietly running the show in everything from video games and computer graphics to engineering, economics, and even artificial intelligence.

Let's break down what a matrix is, how it connects with vectors, and why it's so useful—especially when it comes to moving things around (literally) in graphics and in a bunch of other fields.

Starting Simple: What Exactly Is a Matrix?

A matrix is just a rectangle of numbers. Imagine two rows and two columns—like this:

(3 1

2 4)

That's a 2 x 2 matrix: two rows, two columns. They come in all shapes and sizes: a single row (1 x n), a single column (n x 1), a square grid (like 3 x 3 or 4 x 4), and so on.

Matrices are really just organized storage for information. Think about a spreadsheet—that's basically a matrix of data. Or a digital photo—that's just a matrix where each number is a pixel's colour or intensity.

But matrices go far beyond storage. They aren't solely fancy tables—they also let you perform complex operations and transformations.

How Matrices and Vectors Work Together

To see how matrices "do stuff," you first need to know about vectors. A vector is a quantity having both magnitude (size) and direction, represented in 2D (x, y) or 3D (x,y,z) or as column vectors

(x

y)

That's a vector pointing to a spot on a flat surface.

Now here's where it gets interesting: when you multiply a matrix by a vector, you move that vector to a new place. It's a rule for changing points—matrix times vector equals a new vector in a new location.

(a b   (x   (ax + by)

c d)   y)   = (cx + dy)

In this case, a matrix acts as a function for moving points around.

Linear Transformations

A linear transformation moves points around in a coordinate system, whether 2D or 3D. Any transformation made to the origin will not change the location of the point.

With a matrix, you don't have to spell out what happens to every single point. One matrix lays out the rules for all points at once. That's why matrices are so efficient.

<u>Types of Linear Transformations</u>

- Scaling: Want to make something bigger? Multiply by a scaling matrix.

(2 0

0 2)

This matrix doubles both x and y, making anything twice the size.

- **Reflection**: Need to flip something across a line (like the x-axis)? Use:

(1   0

0 -1)

This turns y values into negatives, flipping the shape upside down over the x-axis.

- **Rotation**: Want to spin something? A rotation matrix does just that (it uses trigonometric functions, but the upshot is simple: every point moves in a circle around the centre).

Link Them All Up

Here's the cool part: you can chain these transformations. Say you want to rotate a shape, then scale it. Just multiply those matrices together and end up with a new, single matrix that pulls off both moves in one go. Extremely efficient, especially when speed matters (like in computer graphics).

<u>Determinants: What's Changing?</u>

Every square matrix has a special number called a determinant. For a 2 x 2 matrix,

(a b

c d)

the determinant is ad - bc.

Why does this matter? It tells you how your transformation changes area:

- determinant = 1 means nothing changes in size

- determinant = 0 means everything collapses (squished into a line). It also tells you that the matrix has singular and hence no inverse.

- negative determinant means you got a flip (reflection)

Undoing Moves: The Inverse Matrix

If you use a matrix to move something, but want to put it back, you use its inverse. It basically reverses the transformation. The inverse of a 3x3 matrix is calculated by first calculating the determinant. If the determinant is 0, the matrix is singular and has no inverse. Find the Matrix of

Minors (for each element, calculate the determinant of the matrix remaining when its row and column are removed). Find the Matrix of Cofactors**:** Apply a "checkerboard" of signs (+-+/-+-/+-+)to the matrix of minors. Transpose the Cofactor Matrix (Adjoint**):** Swap the rows and columns of the cofactor matrix to create the adjoint matrix. Finally, divide by the determinant**:** divide every element in the adjoint matrix by the determinant to get the inverse.

The Game Camera: All About Matrices

One of the coolest places you'll see matrices in action is in video games. All those realistic movements and smooth 3D animations happen thanks to matrices. When you move the camera in a game, you're just transforming the whole game world relative to where you're looking from. Matrices handle that behind the scenes.

From 3D to 2D Screens

When rendering objects in a 3D scene, we use positions and orientations to represent how an object is located. This information is used to create a mathematical matrix that can be used to transform the vertex data of rendered geometry from one space to another. The positions and orientation, which we'll call orientation for short, are specified in world space, which is also known as model-space.

Once an object in object-space is transformed to world-space, it is transformed to screen-space, which corresponds to the X and Y axes that are aligned to the screen. Since the 3D information has depth and distance with objects that are farther from the camera, a projection is applied to the geometry to add perspective to the data. The projection matrices that are used on geometry are called homogeneous clip space matrices, which clip the geometry to the boundaries of the screen to which they are being rendered. Two types of projection are generally used in video games: orthogonal projection and perspective projection.

Orthogonal projection maps a 3D object to a 2D view, but objects remain the same size regardless of their distance from the camera. In perspective projection perspective is added to the rendered scene, which makes objects smaller as they move farther from the camera. A comparison of orthogonal and perspective projection is shown below, where both scenes use the exact same information but different projections. Perspective projection uses a horizon, which represents the vanishing point of the view.

Image Animation

In the game Assassin's Creed II, you encounter a variety of concentric ring picture puzzles, which upon successfully completing, you unlock a segment of a secret video.

Rings are connected in pairs and must be rotated together in their pairs. The aim is to form a complete picture. Different possible pairs can be selected, for example, where there just 3 rings, you could rotate A and B together, B and C together or C and A together.

Only certain pairings are available.

You can form simultaneous equations and use a matrix inverse to solve them, which can therefore tell us how many times to rotate each pair.

Another example is a character with a sword. They need to walk, turn, and swing a sword—this all happens together, thanks to stacking up matrices (i.e. combining translation (leading to character moving across screen), rotation (for the sword to rotate) and scaling (for zooming and resizing) matrices). The advantage is speed. Game engines love this: one calculation to move everything, rather than a separate one for each action.

Matrices in Economics: Crunching the Numbers

Economics is full of complicated relationships. Industries depend on one another, and a single change can ripple through the whole system. Matrices are perfect for analysing all these connections.

Modelling Whole Economies

There's a famous tool called the input-output model. Here’s the idea: each row of a matrix tells you how much one sector produces; each column tells you how much it uses from other sectors.

For example, manufacturing relies on raw materials from agriculture and power from energy. All that information goes into a matrix.

So why bother? Well, economists can use these matrix models to:

- Predict how changes in demand affect everything else

- See which industries are most “connected” (and therefore the most crucial)

- Simulate scenarios, like what happens if the demand for tech goods jumps

Solving Economic Systems

Matrices are also used to solve systems of equations that arise in economics.

Suppose you need to figure out how much each sector should produce to meet a predicted demand. You set up an equation—$AX = B$, where:

- A is the input-output matrix,

- X is the level of production you’re trying to figure out,

- B is the known demand.

You take the inverse of A on both sides of the equation to find X.

Predicting Growth and Change

Transition matrices, often used in the context of Markov chains, are square matrices where each entry represents the probability of moving from one state to another in a stochastic process

Transition matrices are extensively used by banks and regulators to model credit quality.

- **Credit Rating Migration:** They calculate the probability that a borrower’s credit rating moves from one grade (e.g., AAA, BB, Default) to another over a specific period.
- **Default Risk Analysis:** They help in estimating the probability of default for consumer loan portfolios (mortgages, credit cards) by tracking behavioural scores over time.
- **Stress Testing:** Banks use transition matrices to simulate the impact of economic downturns on credit portfolios. This involves creating "conditional" transition matrices that show higher downgrade probabilities during recession

- **Modelling Business Cycles and Labour Markets:** Transition matrices can describe the movement of economies between "recession" and "expansion" states. In labour economics, they are used to analyse transitions between labour market states, such as employed, unemployed, and inactive.

Transition matrices are used to measure the persistence of economic status, particularly intergenerational income mobility.

- **Income Mobility Analysis:** They show the probability of a child ending up in a specific income bracket (e.g., top quartile) given their parents' income bracket.
- **Decomposition of Economic Status:** Researchers decompose these matrices to understand how much of the income persistence across generations is due to family characteristics (composition effect) versus different returns on those characteristics (structure effect).

Why Economists Rely on Matrices

With matrices, economists can tackle huge, messy systems all at once, see the big picture, and make calculations that just aren't possible by hand.

Conclusion

What starts as a bunch of numbers in rows and columns quickly becomes something much bigger. Matrices let us move shapes in video games, balance entire economies, and drive all kinds of technology. They showcase the best part of math: taking a simple idea and turning it into a powerful tool that shapes the world around us. Once you understand matrices, you start to see just how much of modern life quietly relies on them.