Is Gambler’s Luck Real?

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You sit down at a roulette table. Red has come up six times in a row. Every instinct says: bet black it's due. A slot machine hasn't paid out in two hours, so surely it's about to. These beliefs feel irresistible. They are all mathematically wrong.

Let us dive into this topic after describing some mathematical backgrounds like probability and related terms.

1. Introduction

Probability is a number between 0 and 1 assigned to an event, representing how likely that event is to occur. An impossible event has probability 0 while a certain event has probability 1. Mathematically, Probability is defined and denoted as:

$$P(\text{Event}) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Independent Event: An event is independent if its outcome does not affect the outcome of another event. Example: Tossing a coin and rolling a die, the coin result doesn’t change the die result.

Dependent Event: An event is dependent if its outcome affects the outcome of another event. Example: Drawing two cards from a deck without replacement — the first card affects the probability of the second.

Since roulette is an independent event, many people fall for this fallacy, thinking the outcomes are dependent. To get 6 red in rows:

P(6 reds in a row) = (18/37)^6

= (0.4865)^6

$\approx 0.01356 \ \approx 1.36\%$

So a 6-red streak happens roughly once every 74 sequences, which is just an estimate.

It is unusual but not extraordinary. After it occurs, the probability of the next spin being red is still 48.65%, not less, not more than that.

Expected Value (EV): The expected value (EV) of a random event is the average outcome one can anticipate if the event is repeated many times. It represents the long-term "mean" result of a probabilistic scenario, taking into account both the possible outcomes and their probabilities. Mathematically, it is expressed as:

$$\text{EV} = \sum_i x_i \cdot P(x_i)$$

Where, x_i is an outcome and $P(x_i)$ is its probability.

Example: Rolling a fair six-sided die

The die has 6 faces: 1, 2, 3, 4, 5, 6. Each face has an equal probability of $\frac{1}{6}$.

Here, the outcome ,x_i is the number on the die, and $P(x_i) = \frac{1}{6}$

$$\text{Therefore, } EV = \sum_{i=1}^{6} x_i \cdot P(x_i)$$

$$\text{Or, } EV = \left(1 \cdot \frac{1}{6}\right) + \left(2 \cdot \frac{1}{6}\right) + \left(3 \cdot \frac{1}{6}\right) + \left(4 \cdot \frac{1}{6}\right) + \left(5 \cdot \frac{1}{6}\right) + \left(6 \cdot \frac{1}{6}\right)$$

$$\text{Or, } EV = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

The expected value tells us that on average, the outcome per roll is 3.5 if you roll the die many times. When I rolled a die 50 times I personally got an expected value of 3.54 which is near to the value we calculated but may vary from person to person.

Number	Frequency
1	5
2	9
3	11
4	10

Number	Frequency
5	9
6	6

2. Gamblers' Falacy

Before discussing this fallacy let us play roulette to sum up what we discussed so far**.**

Roulette is a popular casino game played with a spinning wheel and a small ball, where players place bets on where the ball will land. The wheel in European roulette has 37 pockets numbered from 1 to 36, colored red and black, along with a single green zero. In this Players can bet on color, parity of number and even specific numbers. After bets are placed, the dealer spins the wheel and rolls the ball in the opposite direction; when the ball stops in a pocket, the winning bets are paid out. Now bets like red and green may sound good and equally likely but the green pocket reduces it which over a long period of time fancies the house.

Favourable outcomes (red pockets) = 18

Total outcomes = 37

P(Red) = 18/37 ≈ 0.4865 ≈ 48.65%

P(Black) = 18/37 ≈ 0.4865 ≈ 48.65%

P(Zero) = 1/37 ≈ 0.0270 ≈ 2.70%

P(Red) + P(Black) + P(Zero) = 37/37 = 1.

Now, The probability of winning is not 50%. It becomes 48.65% and our probability of losing becomes more than 50%. Over thousands of spins, this tiny imbalance translates into a guaranteed profit for the house.

Now, **the Gambler's Fallacy** is the mistaken belief that past independent random events can influence future outcomes. Believing that past outcomes influence independent future outcomes is one of the most costly mathematical errors a person can make. It has no basis in probability theory whatsoever. The casino profits handsomely every time a player acts on this false belief. Even after 6 reds in a row, the next spin is independent. The wheel has no memory, so the probability stays the same :$P(\text{Red}) = \frac{18}{37} \approx 48.65\%$ .The streak may be rare (~1.36%), but it doesn't affect the next outcome.

3. Why Does House Wins?

In a roulette if a player A bets $10 on Red in European roulette. If Red comes up, they win $10 (and get their $10 back) and if anything else comes up (Black or Zero), they lose their $10.

$$\text{For winning, Probability} = \frac{18}{37}$$

$$\text{If won, Net gain} = +\$10 \text{ per spin}$$

$$\text{For losing, Probability} = \frac{19}{37}$$

$$\text{So, Net gain} = -\$10 \text{ (loss)}$$

$$\text{Now, EV} = \left(\frac{18}{37}\right)(+10) + \left(\frac{19}{37}\right)(-10)$$

$$= -\frac{10}{37}$$

$$\approx -\$0.27 \text{ per spin}$$

This means for every $10 A bets on red, he loses 27 cents on average. This does not mean he loses 27 cents every spin, sometimes you win $10, sometimes you lose $10. But across many spins, his losses converges toward 27 cents per spin. The house keeps the other side: +$0.27 per spin per player.

Now lets talk about slots. Slot machines are programmed with a specific Return to Player (RTP) percentage. Slot machines advertise their Return to Player (RTP) percentage, the fraction of wagered money paid back to players over time. A machine with 95% RTP keeps 5% for the house, meaning its house edge is 5% (which is just expected value/loss in percentage).

If a person enters to play a slot machine with $\text{Bet per spin} = \$1.00$ and spins about 500 spins in a hour then:

$$\text{House edge (RTP 95\%)} = 5\%$$

$$\text{Expected loss per spin} = 1.00 \times 0.05 = \$0.05$$

$$\text{Expected loss per hour} = 0.05 \times 500 = \$25.00$$

This calculation reveals why slot machines, despite their modest appearance, are one of the most profitable casino games. The small house edge is multiplied by an enormous number of spins per hour, producing a great expected loss.

Individual gambling sessions are dominated by variance. Random swings above and below the expected value can greatly cause variance. A player can easily win $200 in a night at roulette. But over thousands of sessions, the law of large numbers guarantees that the actual average outcome converges to the expected value. But when the number of trials increases, the average outcome becomes stable and predictable, approaching the expected loss determined by the house edge.

The luck factor in a casino game can be measured using standard deviation (SD). In simple games like roulette, SD can be derived from the binomial distribution of outcomes, where a win is treated as 1 and a loss as 0. For a binomial distribution, the standard deviation is given by $\boldsymbol{\sqrt{npq}}$, where n is the number of rounds played, p is the probability of winning, and q is the probability of losing. If instead of betting 1 unit per round we flat bet b units per round, the variability of outcomes increases proportionally. As a result, the standard deviation for an even-money roulette bet becomes $\boldsymbol{2b\sqrt{npq}}$, where b is the fixed bet per round, n is the number of rounds, p is the probability of winning, and q is the probability of losing.

Consider a player who bets $10 on red, 20 times in one evening. The standard deviation comes out to be approximately $44.70 using the above formula. Now, Expected loss ± 2 SD = -$10.53 ± $89.44 i.e., anywhere from -$100 down to +$79 up is 'normal'. This is where luck lives. The expected loss is only $10.53 but the standard deviation is $44.72. Most players will finish the evening somewhere between $100 down and $79 up. A player who finishes $60 up calls it a lucky night. A player who finishes $80 down calls it an unlucky night. Both outcomes are entirely normal statistical variation.

In conclusion, casino games are driven by probability rather than everyday luck. Short-term results can vary a lot and create the illusion of winning or losing streaks. However, these changes are just natural randomness around a fixed expected value. Over many plays, the small house edge becomes steady and predictable, causing player losses to align with the expected outcome. This shows that what we often see as “luck” is really short-term statistical variation. Long-term results are based on mathematics and the law of averages. From the casino’s point of view, gambling outcomes are not random in the long run; they are structured and predictable due to the underlying probabilities of the game.

References:

[1] Gambling mathematics, Wikipedia
https://en.wikipedia.org/wiki/Gambling_mathematics

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