What is the longest possible game of chess?

In 1989, Ivan Nikolic and Goran Arsovic played a game of chess for over 20 hours to result in a draw. 269 moves were played in the game, making it the longest recorded and rated game of chess in history. I wondered what the most inefficient game of chess could possibly be, so this essay will explore how we can find the longest game of chess possible.

To find out how many moves the longest game of chess could be, we must first set some ground rules:

1. The game cannot be infinitely long.
2. A move is defined as both black and white completing a turn.
3. Assume that the aim of both players is to waste as much time as possible and they want to maximise how many moves they make. Winning is not the objective here.
4. The 75-move rule is enforced, in which if 75 moves are completed without the movement of a pawn or without the capture of another piece, a draw is automatically declared.
5. If the same position occurs five times during the course of the game, a draw is automatically declared.
6. If there is an impossibility of checkmate because of the pieces left on the board, a draw is automatically declared.

With this, we can now begin the maths.

Due to the 75 move rule, a pawn or capture has to occur every 74.5 moves as these can be considered permanent (you cannot move a pawn backwards or uncapture a piece). Therefore, in order to delay the match as long as possible, we would have to play one of these to reset the move counter. There are 16 x 6 = 96 possible pawn moves and 30 captures available, giving a total of 126 events which we can refer to as ‘delay plies’. However, not all pawns can make all 6 moves to the end of the board. To let all 16 pawns pass, 8 captures must be made, reducing the total delay plies to 118.

So 118 x 75 = 8850

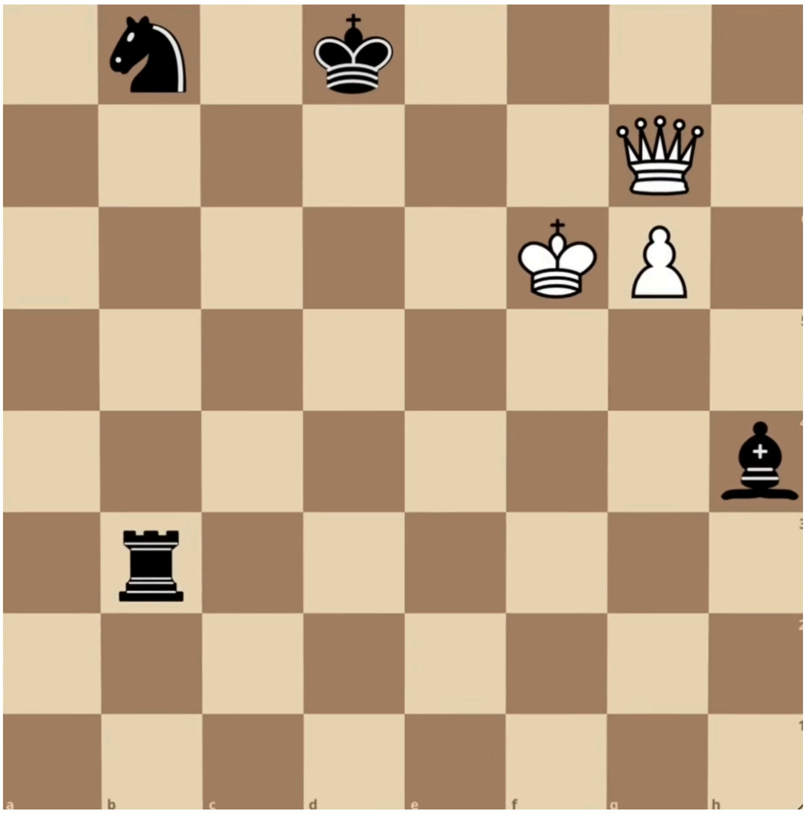
Moreover, under the 75-move rule, a player must play a delay ply every 74.5 moves, however, because one player may not be able to execute a delay ply, e.g. from running out of pawns, the other colour will have to take over. So when will these handovers occur and how many will have to happen?
1st Handover - White used up all of their possible resets, so Black must take over
2nd Handover - Black used up all of their possible resets, so White must retake control
3rd Handover - White must make the final capture as Black will be the only player with a non-king piece
Therefore for each handover, we must remove 1 ply from the total

8850 - 1.5 = 8848.5

Thus, the longest possible game of chess that there can possibly be is 17697 individual plies or 8848.5 total moves. (Fun fact: this is the same height as Mount Everest in metres!!)

What other longest possible games of chess are there?

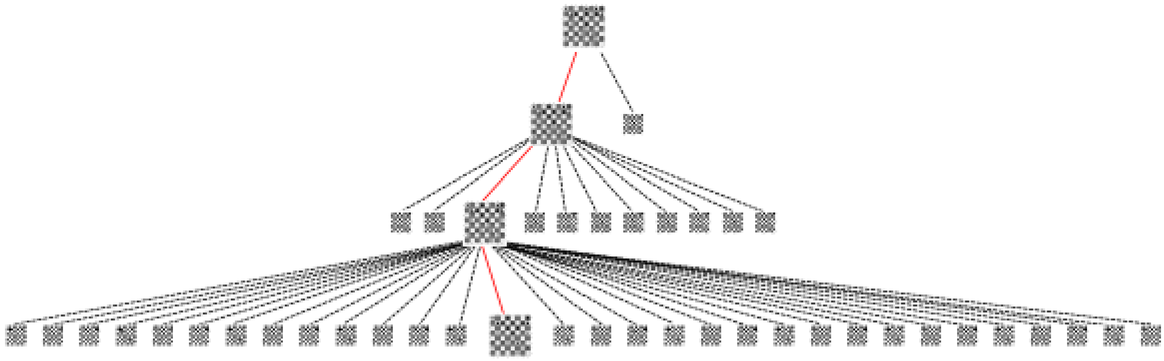
If we limited ourselves to perfect play, what is the longest forced checkmate we can construct? For this example, let's discard the 75-move rule and 5-fold repetition as it will allow us to do more interesting maths.



Source: (Naviary, 2024)

In this position, white can force checkmate in 549 moves, making it the longest checkmate that we currently know of… in finite chess.

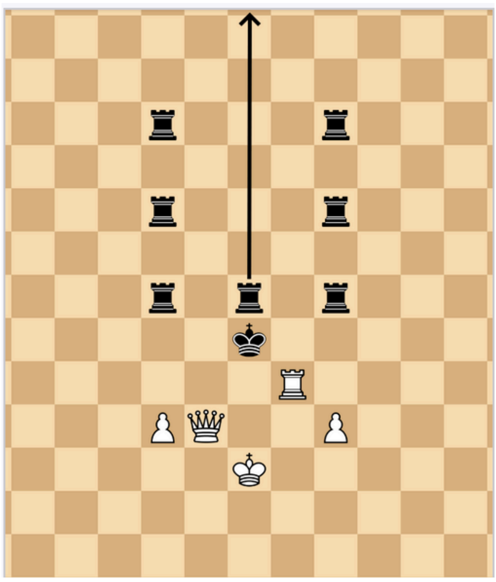
What if the chess board was infinite?



Source: (Playing Chess With a Language Model, 2018)

A game tree, as shown above, is used to represent all of the possible outcomes of moves that can arise from a given position in chess. If White has a forced checkmate, despite Black's best efforts, they will be able to execute a checkmate in *n* moves. The number of moves until an inevitable forced checkmate, assuming perfect play, is defined in a checkmate clock as '*mate-in-n*'. For example, if a position is mate-in-3 for White, Black can delay checkmate for a maximum of 3 moves. For a normal chess game and chess board, the value of *n* must always be finite as the number of possible moves is limited.

We can move this scenario to a chess board that has infinitely many squares. On this board, White can still force Black into checkmate. As we can see in the image below, Black can extend the game by moving their central rook backwards. If Black moves the rook back 10 squares, the position becomes mate-in-10; if they move it back 1000 squares, it becomes mate-in-1000. As such, Black can choose whatever number of squares they want to move the rook back to delay the checkmate. But, no matter how far they move it back, White will still eventually be able to force checkmate in a finite number of moves, so Black cannot indefinitely avoid checkmate.



Source: (Naviary, 2024)

Thus, we cannot describe this position as mate-in-infinity as this would suggest the game never ends, which it does. We must instead refer to this position as mate-in-ω (omega), as ω is used as an ordinal number which describes order and position, rather than a cardinal number which describes a quantity. The next ordinal that comes after the entire list of integers is ω, representing the first infinite sequence.

So we can conclude that mate-in-ω means that Black can delay checkmate for any finite number of moves and not forever. Even though there is no fixed upper bound on the length of the game, it will still always come to an end.

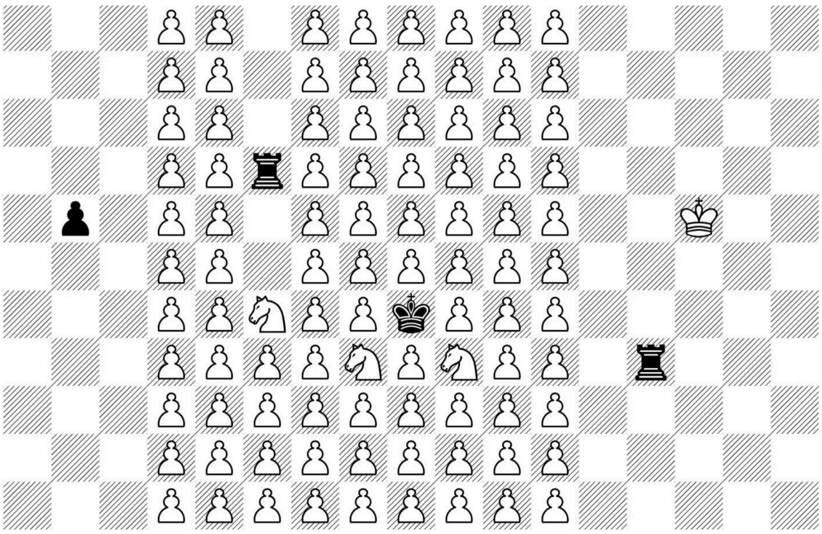
So there we have the longest possible checkmate clock - mate-in-ω.

Wrong.

Mate-in-ω^2

The half-move that Black makes when they move their rook to decide how many moves left until checkmate there will be, is referred to as an 'announcement'. If Black has to make an announcement to decide how many following announcements they will have to make, then we can call this a mate-in-ω^2**.**

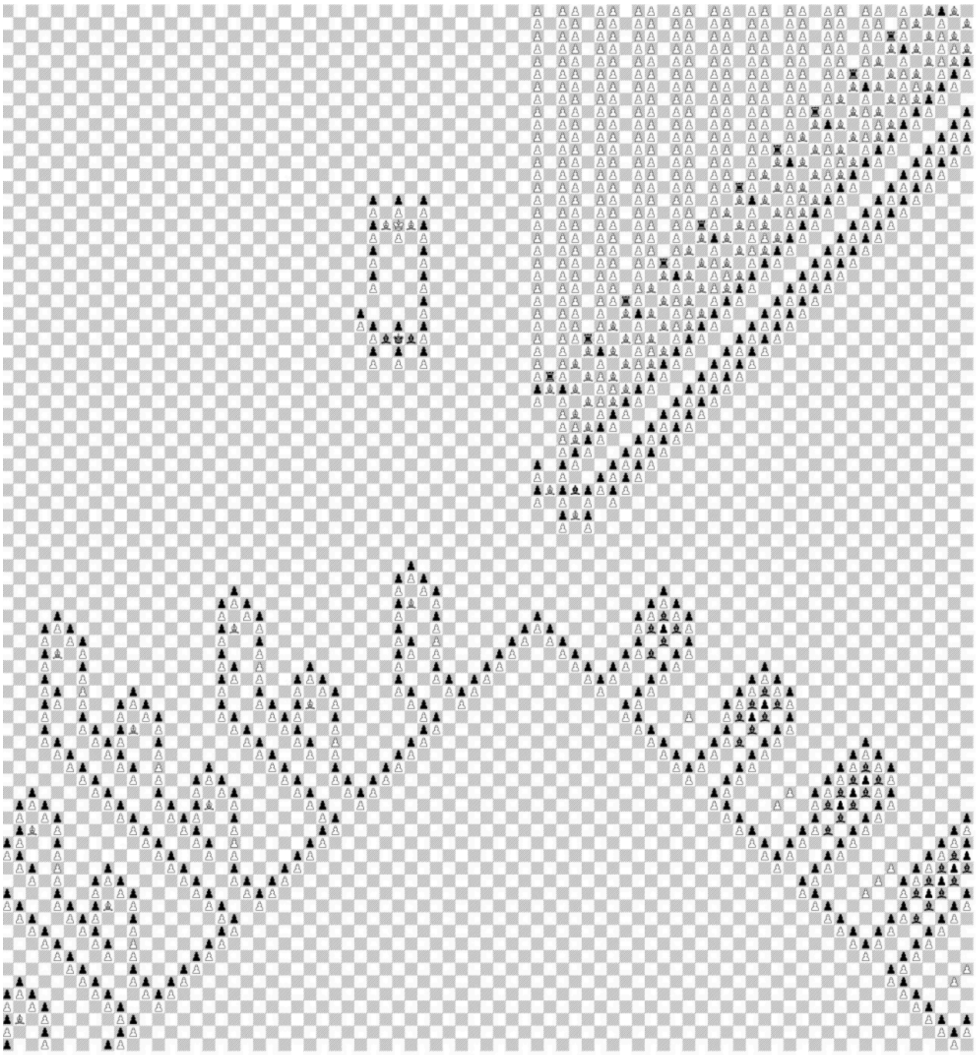
In the following position, the central Black rook, currently attacked by a pawn, may be moved up by Black arbitrarily high, where it will be captured by a White pawn, which opens a hole in the pawn column. White may systematically advance pawns below this hole in order eventually to free up the pieces at the bottom that release the mating material. But with each White pawn advance, Black can begin an arbitrarily long round of harassing checks on the White king.



Source: (Hamkins, 2013)

Whilst mate-in-ω consists of one arbitrarily long delay to checkmate, mate-in-ω^2 consists of multiple delays arranged in a sequence, which overall creates a two tiered structure to suspend the inevitable checkmate.

But it doesn't stop there. Mathematicians in combinatorial game theory have constructed increasingly complex infinite chess positions as high as ω^4, where as the power increases, an additional layer of delays is added. Below is an example of mate-in-ω^4. As you can see, it gets very complex very fast.



Source: (Hamkins, 2013)

If you, like me, are now imagining the possibilities of how long and complex a forced checkmate could be in infinite chess, beware of the rabbit hole - it is very deep; some may even call it ω moves deep. But check out the research done by Joel David Hamkins. It is very cool. Thank you for reading.

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