The Still Points in Math

(Writer's Notes: Wanted to walk about interesting real world examples where maths is used, love the yt videos!)

Mathematics could be considered being about hidden points of convergence: an unexpected triangle that can always be made geometrically no matter the shape you're dealing with, solving for "**x**" in an equation, or how "π" is universally associated with circle equations. In a system where everything is moving, ranging from fluid-mechanics and Navier Stokes where in the flow of a river, there will always be a stagnation point where the water is not moving amidst everything that's happening, to the question of ancient time, in which position should Josephus stay so that he doesn't end up dying, here I would love to talk through the most unique examples of "Still points in Math"that I've found interesting through my career in maths!

I : Josephus

To test the theory of inevitable stillness, we must first look back to 67 AD, to a cave in Jotapata where Flavius Josephus found himself in a predicament. Him and 40 others surrounded by Roman soldiers, and instead of being captured alive, the others would rather die by each other's hand, by standing in a circle the 1st person would kill the other on his left and this would repeated until the last person would be left and commit suicide. Unfortunately Josephus wanted to live and would have preferred being captured.Consequently the question arises, in which position should he place himself in?

The Josephus problem is an incredible example of "stillness". When every 2nd person is removed, the system effectively "halves" itself with every rotation. If there are exactly 2^n people in the circle, the math collapses inward with perfect symmetry, and the person in the 1st position is inevitably the last one standing. But when the number of people **n** is not a perfect power of two, the "still point" shifts and Josephus would have to think a little bit more on where he should stand.

When the number of people (**n**) is not a perfect power of two, the system develops an "offset." We can express any number of people as **n = 2^a + ℓ**, where **2^a** is the largest power of two contained within **n**, and **ℓ** is the number of "extra" people. For every extra person **ℓ** added beyond the power of two, the starting point of the "reset" cycle shifts by two positions. This leads us to the inevitable formula for the survivor:

W(n) = 2ℓ + 1

With n=41, we find the largest power of two is 32 (2^5), leaving us with a remainder of ℓ = 9. Plugging this into our formula, we get 2(9) + 1 = 19. Therefore, Position 19 is the "still point", the only position where the logic of the circle allows life.

The beauty of this inevitability is hidden in how we represent these numbers. While the **2ℓ + 1** method provides the proof, the binary trick provides the "reflection." If we write 41 in binary (101001), the leading 1 represents the 2^a (the power of two that "resets" the circle), and the remaining digits represent ℓ. By performing a "left shift" taking that leading 1 and moving it to the end (010011) we are essentially performing the $\mathbf{2\ell + 1}$ calculation in a single motion. Josephus simply identified the only point in the circle where the "flow" was forced to zero - which could be considered a "point of stillness" and due to the equation and the nature of the problem this point will always exist no matter what **n** is therefore making it inevitable.

II : Convergence of Markov Chains

Named after Andrey Markov, these mathematical models describe a world of transitions, where the future depends on the present, not the sequence of events that preceded it. The best way that I imagine it is continuously throwing dice and adding the score that you just rolled to your sum, the next roll isn't determined by what you previously just got but instead by the probabilities of what you're about to throw next.

In a Markov system, we track an almost probability-like cloud. Imagine the soldiers in the cave from the previous system were not following a strict rule that they had to capture every living person, but instead the ratio of people captured to killed on the spot had a varying probability. Initially, this looks like a mess. .However, even in this "mess of a flow", the math still shows a point of convergence. Most Markov systems possess what is known as a Stationary Distribution (represented by π). As the system evolves over time continuing through infinite iterations the probabilities eventually settle. The system reaches a Steady State where:

$$\boldsymbol{\pi P = \pi}$$

In this equation, **P** is the transition matrix (the "rules" on the probability of each event). An interesting point to touch upon is that even though the individual probabilities are still technically moving, the overall state of the system no longer changes. The "flow" has reached an equilibrium.

This convergence isn't a lucky coincidence; it is mathematically inevitable. For a broad class of these systems (known as irreducible and aperiodic chains), the math forces the existence of this stationary state. From my view, it works very similarly to how a function does by taking an input and outputting a value, it's just that the output here is the converged "still" values of the probabilities .

III: Fluids

If probability can be thought of as a liquid that settles into a steady state, we must eventually look at the behavior of actual liquids! The realm of fluids is a field defined by the Navier-Stokes equations. These equations are essentially considered to be university-level maths and to be able to define the nature/motion of every single fluid to exist; they are so complex that proving they always have smooth, predictable solutions is one of the seven Millennium Prize Problems! They describe a world of velocity (**u**), pressure (**p**),, and viscosity (μ), where every particle is[....].

However, even within the rush of a river or lava pouring down from a volcano, the "still point" remains inevitable. In fluid dynamics, this is known as a Stagnation Point.Imagine a high-velocity stream of water hitting a solid bridge pillar. The water must go somewhere; it splits, rushing to the left and right of the obstacle. Because the fluid cannot penetrate the solid surface, the river is forced to diverge. At the exact coordinate where the flow divides, the velocity has no choice but to drop to zero. For a relatively short period, the water is perfectly still. This stagnation point is not random or caused by chance; it is forced by the conditions of the system.

In my view, this is the most visceral example of a "Point of Convergence." It is the physical manifestation of the equation $f'(x) = 0$. The entire momentum of the river hits the stationary point, where the forward pressure is perfectly balanced by the resistance of the object. Even for a turbulent fluid the math guarantees that there is always one molecule that is stationary. This can be seen in the equation, as the water approaches the solid boundary, the pressure (**p**) begins to spike. This is the "Partial Reflection" in action. The kinetic (moving) energy of the water is being converted into "pressure potential." Looking at the equation, as the pressure gradient ($-\nabla \rho$) grows strong enough to oppose the inertial "push" of the water, the velocity (**u**) is forced to respond.

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<u>IV: The Geometry of Reflection</u>

These “points of stillness” stretch much further than the natural world and can be seen in a really cool way in maths. In calculus, a stationary point occurs where the derivative of a function is zero ($f'(x) = 0$). It is the moment where the "climb" becomes a "plateau". But if we look at this through the lens of the question, a stationary point is more than just a zero-gradient; it is a point of partial reflection.

To explore this further we are going to look into cubics which possess a remarkable geometric property: they exhibit rotational symmetry around a specific point on their curve. This means that if you imagine rotating the cubic function by 180 degrees about this point, the graph maps onto itself perfectly, revealing a symmetry of order two. To understand and prove this, one can translate (shift) the cubic horizontally and vertically to obtain a simplified or "reduced" cubic, often called a depressed cubic, which doesn’t have the quadratic term and is easier to analyze. This reduced form turns out to be an odd function, meaning it satisfies the condition
$g(-x)=-g(x)$
$g(-x)=-g(x)$, showing rotational symmetry about the origin in the transformed coordinate system. The interesting link emerges when identifying this point of rotational symmetry with the cubic’s inflection point. By computing where the second derivative equals zero, the exact coordinates of the inflection point correspond to the amount by which the original cubic was shifted to achieve this symmetry. Graphically and algebraically, the inflection point acts as the pivot for this 180-degree rotational symmetry. This in fact extends beyond cubics; for example, sine functions also exhibit rotational symmetry around their inflection points (the roots). In higher-degree polynomials like quintics, this symmetry may only be local around the inflection point rather than global. Overall, this reveals that inflection points are not merely where concavity changes, but also serve as centers of rotational symmetry, a cool geometric interpretation rarely emphasized in standard mathematical education in my opinion.

V: Conclusion – The Inevitable Mirror

Is stillness an accident of nature? Who knows without concrete mathematical proof, I’m unsure how you would prove it to be true, however it can’t necessarily proved to be wrong. You can make up your opinion just based on these unique examples, I’m just happy I was able to show it through a competition essay.