Infinity seems impossible - so how can it be useful?

By Lenny Sangwa

1. Introduction

Imagine a man standing at one end of an empty room. To leave he must first walk half the distance to the door, then he must walk half the remaining distance, then half of that increment. He then takes a moment and thinks about whether it's even possible for you to leave the room, if its length can be infinitely divided by two. He starts walking once again and a few seconds later he finally reaches the door and leaves the room. If the distance between him and the door could be infinitely divided by two how could that distance ever be covered?

2. A Solution To Zeno's Paradox

The idea I've presented is an iteration of Zeno's dichotomy paradox. Proposed by the ancient Greek philosopher Zeno of Elea, this puzzle was designed to defend his teacher, Parmenides' idea of reality which was monism, the philosophical belief that reality is ultimately a part of one substance rather than fundamentally separate (Palmer, 2025). However in modern mathematics, understanding the solution to this problem serves as the perfect introduction to using infinity. The first step in finding a solution is to form a mathematical equation that we can then solve to come to a final solution. In the paradox the distance between the man and the door continuously halves meaning it can be represented by the following expression:

$$\frac{1}{2^n}$$

$n \in \mathbb{N}$

In the expression '*n'* is defined as a natural number (Bourbaki, 1968) which represents each relevant stage in the journey to the door. For example, when '*n = 1'* the distance from you to the door is ½ , then when you cover half of the remaining distance after that '*n = 2'* and the distance between you and the door is ¼ (½^2). Understanding '*n'* is very important as it is the reason the problem is a paradox. Since *'n'* is a natural number the set of possible values of *'n'* is infinitely long, ∴ there is an infinite number of values for the expression shown above. You could also think of each value of the expression as a task for the man to complete as he must walk that distance. This then means that the man has an infinite number of tasks to complete before he can leave the room. You can then logically conclude that it is impossible for the man to leave the room as he cannot complete an infinite amount of tasks in a finite amount of time. The contradiction between the conclusion of logical reasoning and the conclusion from reality is what made this paradox so famous.

The next step in finding a solution is to create an infinite geometric series. We can do this because in the paradox the total distance the man travels is the sum of all the increments given by the expression shown above. We have already established that there is an infinite number of values of the expressions $\therefore$ the upper limit of the summation would be ∞ and because *'n'* is a defined as a natural number the lower limit of the summation is 1. This means our summation is:

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

What this infinity series shows us is that the summation of infinite values doesn't necessarily go on forever. In mathematics, 'a *series in which the sum of the series tends to a definite value as the number of terms tends to infinity'* is called a convergent series (Fox, Bolton 2002).To solve this summation we need to first identify the first term and the common ratio between terms. The first term , a, is ½ and the common ratio , r, is also ½. We can then input these values into the formula for the summation of an infinite series which is

$$S_\infty = \frac{a}{1 - r}$$

and the value of the summation comes out as 1.
This solution demonstrates that even though infinity may seem impossible, with mathematics it can be broken down and understood.

3. Why Mathematics Needs Infinity

The methods used in solving Zeno's paradox aren't just for finding the answer to century old problems, but are in fact the foundation of an entire bridge of mathematics. The ability to sum an infinite number of values is the fundamental logic behind calculus, *'a branch of mathematics concerned with the calculation of instantaneous rates of change and the summation of infinitely many small factors to determine some whole'* (Berggren, 2026). In the late 17th century, before Sir Isaac Newton and Leibniz had invented calculus, the world of mathematics was incredibly static.

For example, around 300 BCE Euclid of Alexandria, wrote 'The Elements' which was a mathematical textbook on '*all the known mathematics of his time, including the work of Pythagoras, Hippocrates, Theudius, Theaetetus and Eudoxus*' (Mastin L, 2010). This creation led to him being known as the 'Father of Geometry' and the book contained 450 theorems and proofs. This was widely considered as the pinnacle of mathematics for the next 2000 years until calculus was introduced. The problem is that Euclid's mathematics couldn't solve problems involving infinitely small quantities, like finding the area under a curve. Mathematicians were almost fully limited to solving problems with motionless objects. They could find the average speed of an object over a certain distance (distance/time) but they could not find an object's instantaneous speed at an exact point in time. Mathematicians before Newton and Leibniz failed to treat the concept of infinity as a tool to push mathematics forward rather than a philosophical paradox and this is why it took so long for infinity to be understood and utilised. This then limited the rest of society because mathematics is the backbone of innovation.

4. The Development Of Calculus

The way mathematicians understood infinity underwent a huge change in the late 17th century when Isaac Newton and Gottfried Wilhelm Leibniz redefined infinity through the creation of calculus. Newton's developments of calculus started with 'his desire to explain the motion of objects and the forces acting upon them' (***Isaac Newton and the Invention of Calculus, Science Books Blog, 2024)***. Due to mathematics being limited in terms of the understanding of infinitesimal changes, Newton had to develop his own framework. His first being the 'Method of fluxions', which described how quantities changed continuously over time. Through this Newton was able to model dynamic systems, 'systems whose state is uniquely specified by a set of variables and whose behavior is described by predefined rules.' (*Hiroki Sayama, n.d*). On the other hand, Leibniz developed calculus in a symbolic manner, introducing core notation such as 'dy/dx' for differentiation and '∫ f(x) dx' for integration. The combination of the efforts of both Newton and Leibniz led to the development of modern calculus and took infinity from being an avoided concept to being a core part of mathematics and could be applied to the rest of the world.

5. How Calculus Changed The World

As of April 1st 2026, NASA launched Artemis Ⅱ. It was the first crewed mission around the moon in over 50 years and it was only possible due to the understanding of infinity humanity was developed. The rocket's path was curved as it left Earth's atmosphere due to its constant change in path due to gravity. Differential equations can be used to model the effect of gravity on a rocket which allows NASA to accurately predict the rocket's pathway. But calculus and its use of infinity haven't only been useful in the space industry. Although calculus was developed in the 17th century it became significant to pharmacokinetics in the 20th century. Before calculus the doctors relied on trial and error and approximation to find the correct drug dosage for a drug which led to under or over dosing which is possibly lethal. Fortunately scientists found that 'a drug course over time can be calculated using a differential equation' (ndj8585, 2019). This led to much more optimal drug testing and precise dosages being found. As this was a step forward in the medical world, it saved a lot of lives.

6. Conclusion

Zeno's Paradox suggests that infinity is paradoxical, yet modern mathematics reveals the opposite. Due to influential mathematicians such as Newton and Leibniz, infinity became a widely understood and explored mathematical concept which has ended up showing itself and being useful in various parts of our lives from rockets to medicine.

Bibliography

Palmer, John, "Zeno of Elea", *The Stanford Encyclopedia of Philosophy* (Summer 2025 Edition), Edward N. Zalta & Uri Nodelman (eds.), URL = <https://plato.stanford.edu/archives/sum2025/entries/zeno-elea/>.

Bourbaki, N. *Elements of Mathematics: Theory of Sets.* Paris, France: Hermann, 1968.

Huw Fox, Bill Bolton - IIE Core Textbooks Series pages 235 - 269
https://www.sciencedirect.com/science/chapter/monograph/abs/pii/B9780750655446500084

John L. Berggren
https://www.britannica.com/science/calculus-mathematics

Mastin, L. (2010) *The story of mathematics*. Available at:
http://www.storyofmathematics.com/hellenistic_euclid.html

Isaac Newton and the Invention of Calculus, Science Books Blog, 2024
https://www.kroneckerwallis.com/isaac-newton-and-the-invention-of-calculus/#:~:text=Isaac%20Newton%20began%20developing%20calculus,aspects%20of%20the%20natural%20world.

Hiroki Sayama, n.d
3.1: What are Dynamical Systems? - Mathematics LibreTexts

Ndj8585, 2019
https://sites.northwestern.edu/recalculated/2019/05/05/calculus-in-medicine/