Everything is not awesome, the terrifying mathematics of Lego

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Introduction

Have you ever wondered about the mathematics that are hidden behind a small plastic brick? Lego is to be a simple toy designed for children. You place different combination of Lego bricks together to ultimately construct a spaceship, you knock it down and you repeat.However, Lego has a mathematical meaning which I will try to unpick in this essay. And by the end of this essay, I promise that will never look at a Lego brick the same way you did before.

Section 1 – Combination Confusion

Take this simple-looking LEGO brick 2×4:

Figure 1: White 2×4 brick

Have you ever considered the different distinct ways you could connect another 2x4 brick to this original one? To count the distinct different ways, we must think about 3 independent ways we could arrange these two bricks:

1. **Horizontal Offset:** Both bricks are 4 studs long and the second brick can slide left or right to the relative first stud by up to 3 studs in either direction.

2. **Vertical Offset:** Both bricks are 2 studs wide which allows for 3 vertical offsets

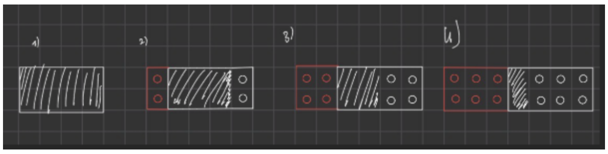
3. **Rotation:** One of the bricks can rotate by $90°, 180°$, relative to the other brick. To avoid double-counting, we must focus on parallel and perpendicular orientations which means that we must eliminate the 180 degree rotation at the end.

Before we continue, I would like to clarify what is meant by *distinct*. Two configurations are considered identical if one can be obtained from the other by rotating the entire arrangement 180°. If we place a brick at offset (x, y) in orientation θ then there will exist a twin configuration at offset $(-x, -y)$ in orientation $\theta + 180°$ that is identical. This means that we will amend our result so that we count these pairs once.

Calculating the 2×4 Case

Parallel Orientation

Let us consider the horizontal offsets.



There is a total of 4 unique spots we can place the brick. Repeating this in the negative horizontal position and not including the first orientation will leave us with a total of 7 horizontal unique spots. This can be repeated in the vertical displacement in the positive or negative direction which has 3 possible offsets. This means that for the parallel combinations we have $3{\times}7 = 21$

$$\text{Total Parallel Positions} = 21$$

Perpendicular Orientation

Let's repeat the same methodology and rotate our second brick by 90°. In the horizontal we now have 5 combinations of offsets: -1,0,1,2,3 and in the vertical offset we also have 5: -3, -2, -1,0,1. That gives a total of $5 \times 5 = 25$ perpendicular combinations

Adding these gives $21 + 25 = 46$, total positions that do not account for symmetry. However, after accounting for 180° , we have the total number of combinations as 24 distinct combinations. See figure 2 below for the full list:

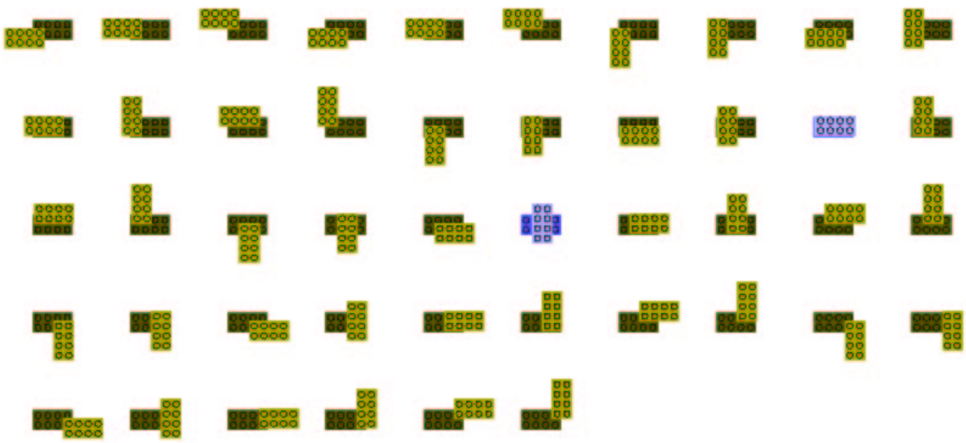


Figure 2: Total combinations of a 2x4 brick

Generalizing for $N \times M$ Bricks

Now let us consider two identical NxM bricks. How can find the total distinct combinations? First, we must generalise our Vertical, Horizontal offset both parallel and perpendicularly.

Parallel Offsets

Two bricks with width N can travel from a total offset of $+(N-1)$ to $-(N-1)$. This results in:

$$\text{Vertical Offsets} = 2N - 1$$

Using the same logic for length M:

$$\text{Horizontal Offsets} = 2M - 1$$

$$\text{Total Parallel Positions} = (2N-1)(2M-1)$$

Perpendicular Offsets

When the second brick is rotated 90°, its M dimension becomes vertical and its N dimension becomes horizontal.

$$\text{Vertical Positions} = N + M - 1$$

$$\text{Horizontal Positions} = M + N - 1$$

$$\text{Total Perpendicular Positions} = (N + M - 1)^2$$

Accounting for Symmetry

Now that we have been able to generalise the offsets of our Lego brick, we must now need to consider the symmetry. Both bricks are identical this means that if we rotate our brick 180 degrees you can count the same shape twice. This is problematic because there are some positions in which the brick is perfectly centred. This can be seen in the figure above (the bricks are highlighted in blue). To mitigate both counting twice and counting the centre brick without its twin pair, we can add one to the total positions of the parallel and perpendicular offsets leaving us with:

$$\text{Total Distinct Parallel:} = \frac{(2N-1)(2M-1)+1}{2}$$

$$\text{Total Distinct Perpendicular:} = \frac{(N+M-1)^2+1}{2}$$

Total distinct combinations:

$$R_{\text{total}} = \frac{(2N-1)(2M-1)+1}{2} + \frac{(N+M-1)^2+1}{2}$$

Sub-case: Square Bricks

Now let's consider the sub case that $N = M$ and we are dealing with a square brick. Rotating a square 90° results in the same orientation as the start. Therefore, we do not add a perpendicular term. Therefore the general distinct combination formula is:

$$S_{\text{square}} = \frac{(2N-1)(2N-1)+1}{2}$$

Conclusion

Lego has always been perceived as the idea of simple innocent pieces; however, it turns out the combinatorial complexity hidden inside these plastic bricks makes lego a delightful mathematical toy. I would also like to thank Professor Tom Crawford and the Oxford University department for continuing education for running a mathematical competition that can allow young mathematicians to see how mathematics can truly live anywhere (including plastic toys !). I hope you enjoyed my essay and if i had more time I wouldve explaored how lego could be used to estimate pi .