<u>How does maths relay to football in weird and wonderful ways</u>

Introduction

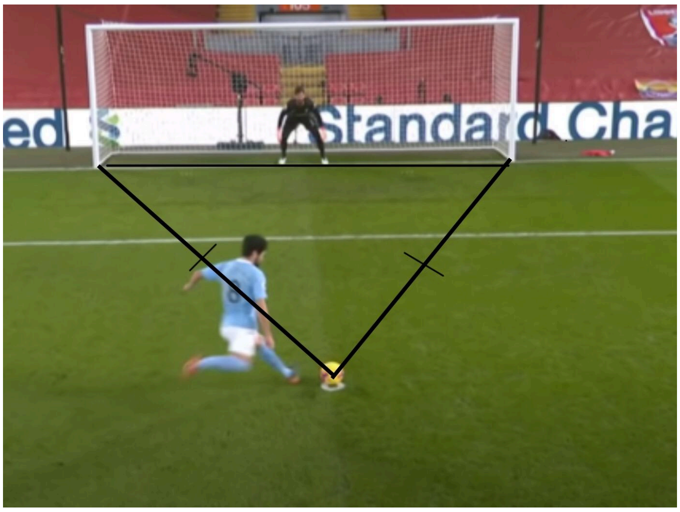
You may believe that football is a simple game without many relations to maths… but you'd be wrong. To start with I'm going to dive into how goalkeepers use cunning but simple geometry to help them save shots.

Penalties

For simplicity let's imagine a goalkeeper standing on their line during a Penalty kick. There is a rule in the game which states that a goalkeeper must stay on their line during a penalty kick. This image shows a visual of a penalty where the goalkeeper stays on their line:

Source: coachingsoccergoalkeeping.com

I will now draw lines to show the maximum possible shots to the left and right. I then joined the tips of these lines up with another line to create an isosceles triangle to get this image:



Source: coachingsoccergoalkeeping.com

How long the keeper has to react

We know that the length from post to post of the net is 24 feet, which is approximately 7.32 meters. The penalty spot is also 12 yards away from the goal, which is 10.97 meters away from the net. This means that the goalkeeper has to cover approximately 3.66 meters each side of them. This would be really hard due to the speed at which the average penalty is struck (average of 70mph which is 112.65km/h) and the goalkeeper would have to react to this shot. The shortest distance to goal is 10.97 metres (straight down the middle) so this would be the lower bound for the distance from the net. The upper bound can be either

side’s top corner. This can be found by using a^2+b^2+c^2=d^2 , where ‘a’ is the distance of the pen spot from the goal. ‘b’ is the distance from the center of the goal to the post minus 11cm as that is the radius of the football. ‘c’ is the height of the goal minus 11cm (radius of the ball). ‘d’ is the distance of the diagonal from the pen spot to the top corner. We already know ‘a’ is 10.97 meters and ‘b’ is 3.66 meters - 0.11 meters, which is 3.55 meters. We can find ‘c’ by doing the height of the goal (2.44 meters) minus 11cm, which is 2.33 meters. These can then be plugged into the formula to get 10.97^2+3.55^2+2.33^2, which is 120.3409+12.6025+5.4289. This means that d^2 equals 138.3723 so therefore d is approximately 11.76 meters.
We can use these distances to work out the range for how fast the average penalty takes to go into the net. To do this we will rearrange the formula speed(s)=distance(d)/time(t) into t=d/s. We will use distance in meters and speed in meters/second, to do this we need to convert 112.65km/h into m/s which is 31.29 m/s. Now to find the lower bound for the time a penalty takes we need to do time=10.97/31.29, which equates to around 0.35 seconds (to 2d.p). To find the upper bound we can do time=11.76/31.29, which is 0.38 seconds (to 2d.p). This means that at average penalty speed every shot on target will take between 0.35 to 0.38 seconds to reach the net. If the goalkeeper decides to react to the penalty after it has been struck then there is an even lower time they have to move to the ball. This is because we have to account for reaction time (approximately 0.23 seconds for a professional goalkeeper) so they would now only have 0.12 to 0.15 seconds to travel over to the ball. This causes it to be almost impossible for a goalkeeper to save a well placed penalty unless they take an educated guess on where the player is going to shoot. They can do this by having statistics of the penalty takers' last penalty’s and where they usually place the shot. This method was popularised by Pickford when he stuck the statistics to his bottle when he was playing in a match. For example:

Bottles like these make it so the keeper can focus on just diving to cover ground quickly instead of having to react to the penalty taker. This means that they have a higher chance to save a well placed penalty but if the taker decides to switch up where they're going to shoot then the goalkeeper could dive the complete wrong way. This means that the risk to reward ratio is high as the vast majority (around 68%) of penalties are shot in the top or bottom corners so if you can predict the correct direction then you will dive the right way 34% of the time. The percentage of penalties shot down the middle is 17% so if you don't dive then only 17% of penalties will be shot near you. This links to the style of goalkeeper that reacts to where the ball is going because most good goalkeepers will be able to save a shot, which goes straight at them.

Stepping off the line

A goalkeeper can reduce the distance that they have to dive each side, which is technically against the rules as they are gaining an unfair advantage. An exception to this rule is that if the keeper keeps one foot on the line then it won't be punished. This means the keeper can step forward just before the penalty is taken so they now have to cover less distance to save the ball. This is evident as many keepers now step off their line. For example Subasic in this image:

In this image the goalkeeper looks to have stepped around a meter out with their lead foot and around 0.2 meters with their back foot. I will use the lead foot for this example. Since the pen spot is 10.97 meters from the goal and the keeper is now 9.97 meters away from the ball the proportion is now 997/1097. As shown in the second image there is an isosceles triangle between the pen spot and the posts and the two posts are 7.32 meters apart. Now, since it is an isosceles triangle then we can use the proportion to find the new distance that the goalkeeper has to cover. You work this out by doing (997/1097)*7.32, which is approximately 6.65 (to 2d.p). This means that stepping forward reduces the distance that the goalkeeper has to cover by 9.15%. This may seem quite small but when playing a professional sport any small advantage is emphasised as they can all add up to a large advantage on the opponent. Therefore, the chance of the goalkeeper saving a penalty increases drastically. Around 76% of penalties are scored when the goalkeeper stays on their line whereas when the goalkeeper steps off the line then the conversion rate for a penalty is now 68%. This could be the difference between winning a game or losing a game and is especially useful in a penalty shootout. One downside of the stepping forward method is that if you do it wrongly and save the pen then you'll get penalised so the pen taker can have another go at the penalty. This means that they will probably score the next Penalty. Therefore, this method is high risk and high reward as if it's executed incorrectly then the other team has two chances of scoring a pen.

Conclusion

These are just some ways that goalkeepers use maths to increase the likelihood that they will save the pen. There are other more complex ways that maths relates to penalties like mind games with the taker and the unreachable areas for the goalkeeper. We have focused on how distance from the penalty spot can affect the keeper. I also looked at how the reaction time of the keeper impacts how long it takes for them to get to the ball and the statistics that some goalkeepers put on their bottles. These are all great tactics that reduce the chances of conceding a goal even though the penalty takers are always heavily advantaged by the penalty rules. Overall, each little adjustment that the keeper does adds up and in a competitive game can cause them to win especially if there is a penalty shootout.