# <u>The Geometry of Heartbreak</u>

## On Topology, Continuous Deformation, and Why Some Things Cannot Be Undone

## Tom Rocks Maths Essay Competition 2026

My friend Meera has this habit of being accidentally precise. Last year, when she was going through something difficult — the kind of ending that arrives not loudly but inslow, awful increments — she said something that I have not been able to stop thinking about. "It doesn't feel broken," she said. "It feels torn. Broken things you can glue back.

Torn things leave a different shape behind."

I nodded. I told her I understood. I didn't, not really , I just knew she needed me to say something. But weeks later, sitting with a topology textbook I had borrowed on a whim from my school library (mostly because the cover had a donut and a coffee mug on it, and I was curious why), I kept hearing her words back. Because topology, it turns out, is almost entirely a subject about the difference between bending and tearing. And Meera, without knowing it, had landed on one of its most important distinctions.

Topology is the branch of mathematics that studies shape in the most stripped-back sense possible. It does not care about distances or angles the way geometry does. What it cares about — almost obsessively is this: which properties of a shape survive when you actually stretch it, squish it, bend it, but never cut it and never fuse two separate points together? Tearing is not allowed. Gluing is not allowed. Everything else is fair game.

That single constraint — no tearing — turns out to have enormous mathematical consequences. And once you understand why, Meera's description stops sounding like poetry and starts sounding like a theorem.

### Homeomorphisms and the Mathematics of "Same"

Topologists have a formal way of saying two shapes are the same. It is called a homeomorphism — a continuous, invertible function between two spaces whose inverseis also continuous. If such a function exists, the two shapes are topologically identical.

They are the same object, just wearing different clothes.A circle and a square are homeomorphic. So are a sphere and the surface of a cube. You can stretch one continuously into the other. But a circle and a figure-of-eight are not homeomorphic — because creating that crossing point requires picking up one part of the circle and gluing it to another. Not allowed. The moment you do that, you have created something structurally different.

Now think of a close friendship as a topological space. Not a metaphor, exactly more like a model. The space is connected: from any point in it (a Tuesday, a habit, an inside joke that takes three words to tell) you can reach any other point through a continuous path. You know how the other person takes their tea. You finish sentences. The days have a shape you did not design but have come to rely on.

When things change slowly — people drift, grow in different directions , a topologist might model that as a gradual deformation. The space is stretching. But it is still connected, still homeomorphic to what it was. Then there are endings that are not deformations. They are cuts.

**Connectedness and What a Single Cut Destroys**

In topology, a space is called connected if it cannot be split into two separate, non-empty open sets. Informally: it is all in one piece. You cannot pull it apart without tearing something.

Consider the closed interval [0, 1] — a connected space. Now remove the single point at 0.5. What remains? Two disconnected pieces: [0, 0.5) and (0.5, 1]. One point gone, and the whole space fractures. This is not an analogy being forced. In one dimension, removing a single point genuinely destroys connectedness. The space before and the space after are not homeomorphic. They are, in a rigorous and unambiguous sense, different objects.

I thought about Meera when I read this. The shape of her daily life had been connected where you could draw a continuous path from Monday morning to Saturday evening and it would pass through another person, reliably. Then it did not anymore. The path between those two points now went through empty space where something used to be. That is not the same as gradual change. That is a change in topological type.

This is also, I think, why people always say it comes in waves. A disconnected space still has its own internal structure — each piece is still connected within itself. Lifecontinues, and much of it works fine. But certain paths are now different. Certain routes that used to be continuous now pass through absence.

**The Fundamental Group and What You Carry**

Here is where topology offers its most quietly devastating concept: the fundamental group. For any topological space, you can ask — what happens when you draw loops?

Start at a point, trace a path, return where you started. Two loops are considered the same if you can continuously deform one into the other without leaving the space. In a simply connected space — like the surface of a sphere — every loop can be shrunk to a point. There are no holes to get caught on. But in a space with holes, loops that wind around those holes are stuck. They are topologically distinct from loops that do not wind at all. The fundamental group, π1(X), classifies all the essentially different loops in a space.

For the circle S1, the fundamental group is ■ — the integers. Each integer is how many times a loop winds around (positive in one direction, negative in the other). This winding number is a topological invariant. It cannot change under continuous deformation. You cannot unwind a loop that has gone around twice without cutting it.

I think this is what it means to carry something. After a loss — of a friendship, a relationship, any version of a world you built with someone — there are loops in your life that wind around an absence. You are going somewhere you used to go together. You are hearing a certain song. You are having the kind of day you would have told them about.

Those loops have non-trivial winding numbers. They cannot be deformed away. The hole is part of the topology now. But — and this is the part I keep coming back to — a topological invariant is not a defect. It is information. It is the thing that survives transformation, the thing that tells you something real about the shape of a space. Carrying something is not the same as being stuck.

**Manifolds, Seams, and the Geometry of Healing**

There is a construction in topology I find strangely hopeful. When you cut a smooth manifold — a space that locally looks like ordinary flat space — you create boundaries. The object becomes a manifold with boundary, a different category of thing. But here is what you can do: take two manifolds with boundary and glue them along their edges. Theresult is a new closed manifold. Smooth again, in a sense. Whole.

The seam is still there, mathematically. The gluing site is detectable if you look carefully. But the object is no longer broken open. It has incorporated the cut into its structure and become something new — not a restoration of the original, but a genuinely different manifold that includes the history of what happened to it.

I think healing is more like this than we usually admit. We talk about it as return —getting back to normal, going back to yourself. But topologically, that is not really possible once a cut has been made. The original object is gone. What you can do is become a new object that contains the cut in its structure. That is not lesser. It is genuinely richer — a more complex topological space than the one that existed before it was ever deformed.

I told Meera this, badly, over a phone call where I probably used the word "homeomorphism" at least four times and she told me I needed to go outside more. But she also said — after a pause — "so it's like I'm a different shape now." Yes. Exactly that. A different shape. One that has been through something and come out the other side not restored, but transformed.

**Why I Think This Matters**

I want to be upfront: I did not come to topology looking for a framework for grief. I came to it sideways, the way I come to most things I end up caring about — through a strange cover, a borrowed book, a question I could not let go of. But mathematics has this habit of showing up in places you did not invite it, and saying something more precise than the words you were already using.

The language we usually give people for heartbreak is all about force and fracture: crushed, shattered, broken. These words imply that healing means returning to the original form — reassembling the pieces, gluing them back. But topology offers different words. Continuous. Connected. Invariant. Transformed. These words do not ask you to return. They ask you to understand the shape you have become.

There is a result called the invariance of domain — a theorem that says, roughly, that if you map a space continuously and injectively into another of the same dimension, the structure is automatically preserved. Continuity has consequences. Being careful and gentle and consistent is not just a feeling — in certain spaces, it is a mathematical force.Meera is doing well now, by the way. She would want me to say that. She has the particular energy of someone who has gone through something genuinely hard and come out curious instead of closed. She still describes things with that same accidental precision. Last week she said, "I feel like I've been rearranged." I didn't say anything this time. But I thought: yes. Rearranged. Homeomorphic to nothing that came before. A new space, with new loops, and new paths through it. That is what topology taught me — not that loss is fine, or that time fixes things, or any of the comfortable things people say. It taught me something more interesting and more honest than that.

It taught me that you are not trying to go back to the shape you were. You are trying to discover what kind of shape you have become — and that is not a smaller thing to be. Every great topological space has holes in it. The ones without any are just called contractible. They collapse to a point. They have no interesting loops at all.