<u>Maths in Forensic Science</u>

By Ava Mewawalla

Introduction

Forensic science is the fascinating study of how we can scientifically ascertain evidence in relation to a crime, such as murder. It is founded upon Locard's exchange principle, which states that "every contact leaves a trace". This means that all criminal behaviour leaves a trace behind, from DNA to microscopic fibres, which is used by forensic scientists to identify the criminal. Although this may seem like a field that primarily focuses on biological, chemical, and physical factors, mathematics is required to actually interpret the data. This essay will explore how maths can be employed in the analysis of blood splatters, the estimation of time of death, and lastly matching up fingerprints.

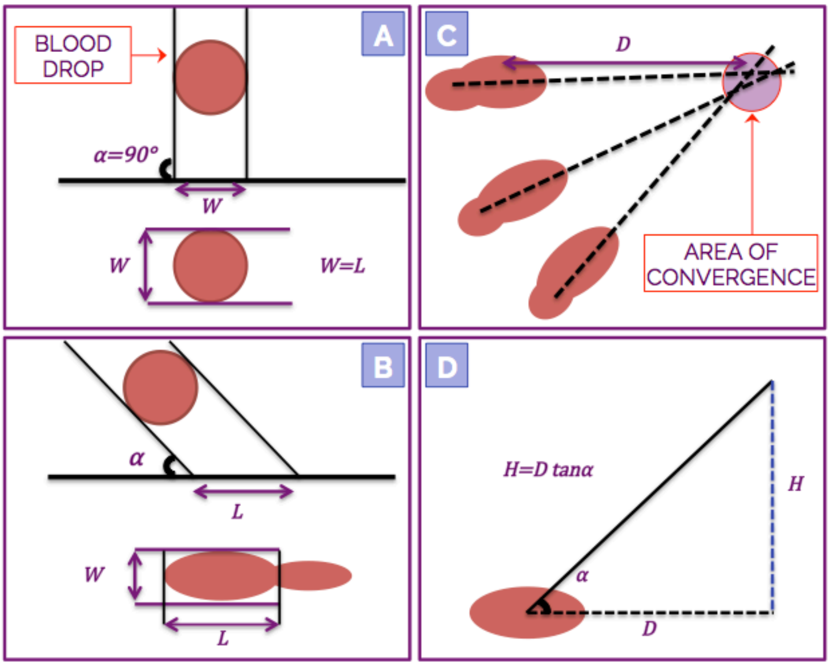
Bloodstain Pattern Analysis - Trigonometry

Bloodstain Pattern Analysis (BPA) uses trigonometry, as well as the properties of ellipses, in order to reconstruct crime scenes. This is important in order to answer questions that help police put together the puzzle pieces of a crime. What caused the blood wounds? How were the victim and offender positioned? How many people were present?

Bloodstains are often comprised of an elliptical shape, with a small extended drip (the 'tail') caused from gravity, the force of the weapon, and the momentum as the droplet hit the surface. Therefore, the tail is ignored in BPA. In order to find the angle of impact, meaning the angle between the surface and the trajectory of the bloodstain, there is a simple trigonometric formula:

$$angle\ of\ impact\ =\ arcsin\left(\frac{width\ of\ bloodstain}{length\ of\ bloodstain}\right)$$

$$\alpha\ =\ arcsin\left(\frac{W}{L}\right)$$



(Sánchez et al., 2016) [6]

In the diagram above, boxes A and B display the width and length used in the formula. A key observation is that as the bloodstain becomes more elongated, the angle of impact decreases. In box C, the major axis of each ellipse is extended and the point where they intersect is known as the 'area of convergence'. Then, the distance (D) between the centre of each bloodstain and the area of convergence is measured. After this, as shown in box D, a second formula is used:

$$height = tan(angle\ of\ impact) \bullet distance$$

$$H = Dtan(\alpha)$$

The height indicates the 'point of origin' of the bloodstain, which should correspond with the bloodstains that originated from the same occurrence. The process of determining the point of origin is called 'stringing', and this data is useful to identify whether the bloodstain came from the victim or the perpetrator.

Overall, BPA is a surprisingly crucial element of forensic science, helping to create a 3D reconstruction of a crime scene, discover the potential cause of death, and how it all went down. Trigonometry and geometrical mathematics are also useful at interpreting spatial evidence in other forensic scenarios, such as bullet trajectory analysis and the examination of bite marks.

Time of Death Estimation - Calculus

Another essential piece of data is the time of death in a murder case. This is because it can exonerate suspects with an alibi, build evidence against those without, and in general helps to form a chronological timeline of the events leading up to and following the murder. To estimate this value, forensic scientists use Newton's law of cooling, as well as differential equations.

There are 3 stages of death: rigor mortis, which describes the stiffness of death; livor mortis, meaning discolouration; and algor mortis, the rate at which the body cools after death. Analysing these factors allows scientists to approximate the post-mortem interval (PMI), meaning the length of time lapsed since death.

Newton's law of cooling states that the rate of heat loss in a cooling body is directly proportional to the temperature difference between the body and the surroundings. The following differential equation is used to determine the PMI, where:

- T is the body temperature
- T_s is the temperature of the surroundings
- T_0 is the initial body temperature (normally assumed to be 37°C)
- K is a constant that varies depending on the body's characteristics
- t is the time since death

$$\frac{dT}{dt} = K\left(T - T_s\right)$$

$$T(0) = T_0$$

The latter equation simply establishes that when the time since death equals 0 (so at the time of death) this is the initial body temperature, T_0. Together, the equations are known as a 'first-order initial value problem', which is essentially a differential equation that only uses the first derivative, and follows the initial condition that $y(x_0) = y_0$.

But how do forensic scientists apply these equations? They start by recording the temperature of the corpse at two separate time intervals, and plugging these into the formula to calculate a value for K. Then, we can use a second equation for Newton's law of cooling [4]:

$$T(t) = T_s + (T_0 - T_s)e^{-Kt}$$

As we've assumed that T_0 = 37°C, we've calculated K, and we can easily measure the temperature of the surroundings, this formula allows us to find t, which equates to the PMI. Subtract this from the current time, and it produces the highly crucial time of death estimation.

However, why does the corpse cool in this way? Algor mortis occurs because the body approaches the temperature of the room, to achieve thermal equilibrium. We can demonstrate this mathematically [1]:

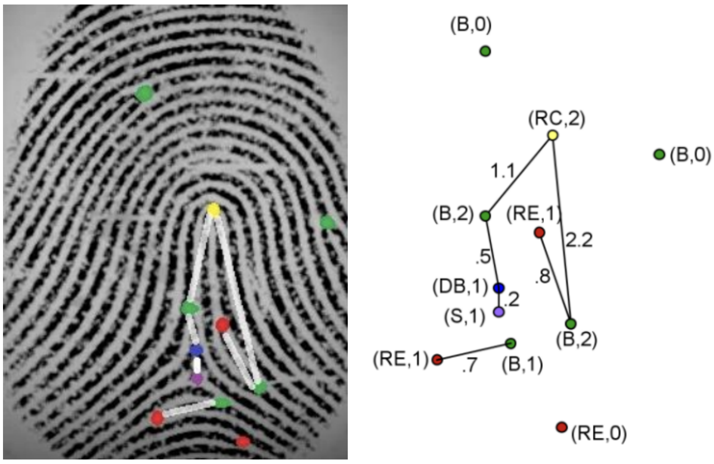
$$\lim_{t \to \infty} T(t) = T_s$$

This shows that as time tends to infinity, the body temperature will equal the temperature of the surroundings. Advanced forensic models will also include other variables, such as humidity of the surroundings, presence of insects, or the activity of microbes that may accelerate body decay.

Fingerprint Analysis - Graph Theory

Fingerprints have two properties that make them so useful when investigating a crime: they are immutable, so cannot be changed, and unique, meaning everybody has a different set. But how can forensic scientists match fingerprints to suspects? The process actually involves graph theory, from mathematics.

Fingerprints are composed of 'ridges', which are raised paths of skin on our fingertips, so they are the black lines you see in a fingerprint scan. They can adopt many different characteristics, such as bifurcation (where one ridge branches into two), ridge dots (the ridge is a small dot rather than a line), or ridge crossings (where 2 ridges intersect). A graph is simply a network of 'nodes' connected by 'edges'. In order to apply this to fingerprint analysis, we can assign each ridge characteristic as a node, and connect them with an edge if they are part of the same ridge, called a 'connected ridge flow' [3]. When a fingerprint scan is inputted to a computer software, the algorithm identifies every ridge characteristic as a node and colour codes them accordingly. An edge is formed between two characteristics on the same ridge, and is allocated a weight, which is the distance between the two nodes. Each node is then labelled with two variables: an abbreviated letter code for the type of characteristics (e.g. B for bifurcation), and the degree of the node, which in graph theory means the number of nodes it is connected to. The images below help to visualise this.



(Karim and Vasilevska, n.d.) [3]

Once the fingerprint has been converted into a graph, to check if it matches another one, we can test if the two graphs are ‘isomorphic’, meaning structurally identical. If they are isomorphic, it’s a match!

Conclusion

In conclusion, forensic science is a unique, intriguing, and yet slightly gruesome, application of mathematics. Beyond analysing bloodstains, fingerprints, and time of death, maths is also used to estimate victim's heights from only a few bones, quantify the reliability of DNA matches with Bayesian statistics, and help in drug overdose cases using pharmacokinetic equations. The table below is an overview of the vast applications of mathematics in real-world forensic cases.

Table 1: Forensic cases and applied mathematical methods.

Case	Forensic Domain	Mathematical Method
Colin Pitchfork	DNA Profiling	Bayesian Statistics, Likelihood Ratios [24,25]
People v. Turner, R v. Adams	DNA Profiling	Bayesian Statistics, Likelihood Ratios [3,7,26]
Ted Bundy	Bite Mark Analysis	Geometric Morphometrics, Affine Transformations [12,13]
BTK Killer	Digital Metadata Analysis	Cryptographic Hashing, Metadata Decoding [18]
John Duffy	Geographic Profiling	Spatial Crime Pattern Modeling, Rossmo's Formula [2,21,27]
Kirk Bloodsworth	DNA Exoneration	Statistical Modeling of DNA Mismatch Probability [28,29]
JonBenét Ramsey	Text Authorship Forensics	Stylometric Analysis, Lexical Pattern Matching [16,21]
Boston Marathon Bombing	Terrorist Tracking	Trajectory Modeling, Video Analytics [27,30]
O. J. Simpson Trial	DNA Interpretation in Court	Bayesian Reasoning, Likelihood Ratio Presentation [26,31]
Brandon Mayfield	Fingerprint Analysis Error	Error Rate Estimation, ROC Curves [6,8]
Ray Krone	Bitemark Analysis	False Positive Rate Studies [13]
People v. Collins	Statistical Misuse in Court	Conditional Probability, Prosecutor's Fallacy [2,28]
Sally Clark	Double Infant Death	Multiplication Rule Error [6,29]
Lucia de Berk	Healthcare Serial Killing	Poisson Distribution, Event Clustering [5,20]
Harold Shipman	Abnormal Mortality Detection	Control Charts, Sequential Tests [30]
R v. T	Footwear Evidence	Likelihood Ratios [26,32]
Golden State Killer	Genetic Genealogy	Kinship Coefficients, Pedigree Modeling [33,34]
Karen Read	Digital Timeline Reconstruction	Time-Series & Correlation Analysis [16,18]
Victim Identification	Degraded DNA Matching	Allele Frequency Models, Likelihood Estimation [20,25]
Deepfake Detection	Digital Video Forensics	SVD, PCA, Residual Statistical Analysis [17,30]
Bosnia Mass Graves	Kinship-Based Identification	Bayesian Networks, Population Genetics [34,35]
Cold Case Investigations	Genetic Genealogy	Combinatorics, Pedigree Likelihood Models [26,33]
Amanda Knox	Low Copy Number DNA	Stochastic Threshold Modeling, Contamination Probabilities [25,29]
Jill Dando	Ballistics and Contamination	Background Particle Probability Modeling [6,36]
Interpol Investigations	Long-Term Missing Persons	Active Appearance Models, PCA [17,30]
Fire Scene Reconstruction	Arson Investigation	Heat Transfer Equations, Differential Calculus [23,37]
Overdose Analysis	Toxicological Interpretation	Pharmacokinetic Equations [ADME Models] [30,33]

(Gecioglu, 2026) [2]

Thank you so much for reading this essay, I hope that you learnt something new!

References

[1] B. Kaviya (2024). Mathematics in Forensic Science. *Louis Savenien Dupuis Journal of Multidisciplinary Research*, pp.154–158. doi:https://doi.org/10.21839/lsdjmr.2024.v3.121.

[2] Gecioglu, B. (2026). *View of Mathematics and its Applications in Forensics: Case Studies and Analytical Approaches*. [online] Clinsurggroup.com. Available at: https://www.clinsurggroup.com/gjmccr/article/view/GJMCCR-12-321/pdf

[3] Karim, M. and Vasilevska, V. (n.d.). *Having Fun with Graph Theory and Forensics: CSI Fingerprint Analysis -Whose fingerprint is this? A CCICADA Homeland Security Module*. [online] Available at: https://ccicada.org/wp-content/uploads/2015/06/Karim-and-Violeta-Module-for-Reconnect-2014-FINAL-4.pdf.

[4] MsAlbertMATH (2024). *Introduction to Math and Forensic Science Blood Splatter Lab*. [online] YouTube. Available at: https://www.youtube.com/watch?v=XSwalqF9GkM

[5] National Forensic Science Technology Center (2012). *A Simplified Guide to Bloodstain Pattern Analysis*. [online] Available at: https://www.forensicsciencesimplified.org/blood/BloodstainPatterns.pdf.

[6] Sánchez, H.C., Esparza, D.P., Curiel, R.P. and Zolghadriha, S. (2016). *Catching Criminals with Maths*. [online] Chalkdust. Available at: https://chalkdustmagazine.com/blog/mathematics-and-crime-science/.

[7] Varghese, A.M. (2025). *Mathematics in Forensic Science*. [online] Available at: http://117.239.78.102:8080/jspui/bitstream/123456789/7015/1/Anita%20m%20Varghese.pdf.