

Can we really trust anything?
The most important Proof you've never heard of

Introduction

We often make many of assumptions quickly in our lives. Some because we believe they are trivial, and others because we have simply been told them our entire lives.

But what if we decided to challenge the most trivial part of mathematics?

Does $1+1=2$?

Part 1: Foundations

The goal of the seemingly trivial proof was to challenge everything we know, because assumptions are especially dangerous in mathematical proof as something like two odd numbers as $2n+1$ and $2n+3$ could be incorrect as it would imply the proof only works for consecutive odd numbers.

The most common starting point is the Peano axioms, which apply to natural numbers. These state that:

- 0 is a number (crazy, I know)
- Every number has a successor $S(n)$
- There is no number whose successor is 0
- If $S(a) = S(b)$ then $a = b$
- If a property holds for 0 and n towards $S(n)$ then it holds for all numbers.

We can see that two is not assumed but rather build from two uses of the successor function.

Defining Addition

Addition must also not be assumed, and rather defined recursively.

$$a + 0 = a$$

$$a + s(b) = s(a+b)$$

From this, we can create:

$$1 + 1 = 1 + S(0)$$

$$1 + S(0) = S(1+0)$$

$$1 + 0 = 1$$

$$S(1) = 2$$

$$1+1 = 2$$

Now while this proof does hold and takes very few lines it is very important to understand that this rests entirely on the definitions of $S(n)$ and the others shown earlier.

Historically: Why the long proof?

Mind bogglingly, the proof for $1+1 = 2$ took over 300 pages.

But why?

Because they wanted to avoid assumptions and have strict definitions.

Some of the parts were:

- Defining numbers in terms of sets (looked at more later)
- Delving even deeper and defining equality itself!
- Defining addition strictly.
- Avoiding Paradoxes like Russell's Paradox.

Avoiding these is extremely strenuous and difficult.

Set-Theoretic Interpretation

Here's an example using set notation which shows why definitions and any assumptions made are very important:

Let

$A = \{a\} \rightarrow$ has 1 element

$B = \{b\} \rightarrow$ has 1 element

If A and B are disjoint (no shared elements), then:

$$A \cup B = \{a, b\}$$

Number of elements = 2

Thus:

$$1 + 1 = 2$$

This interpretation uses cardinality:

- $|A| = 1$
- $|B| = 1$
- $|A \cup B| = 2$

But notice an important condition: A and B must be distinct. If instead:

$$A = \{a\}$$

$$B = \{a\}$$

Then:

$$A \cup B = \{a\} \rightarrow \text{still 1 element}$$

So in that case:

$$1 + 1 = 1$$

This shows that arithmetic depends on structure and assumptions. The statement $1 + 1 = 2$ is true within a system where objects are distinct and countable.

Polymaths

There is also a large philosophical side to mathematics too. Where many people ask the question:

“Is mathematics discovered or invented?”

In my personal opinion, I believe mathematics explains naturally occurring concepts and is therefore discovered.

But here are the other viewpoints.

Platonists argue:

- Numbers exist independently of humans
- $1 + 1 = 2$ is eternally true
- Even if no humans existed, the statement would remain valid, i.e proof is not necessary

From this perspective, “2” is a real abstract object, just like a perfect circle.

Formalists claim:

- Mathematics is a system of symbols
- $1 + 1 = 2$ is true because it follows rules
- It is no more “true” than a legal move in chess

Here, the equation is valid because of axioms, not because of any external reality.

Intuitionists believe:

- Mathematics is constructed in the human mind
- $1 + 1 = 2$ is true because we can build it step by step

They emphasize the constructive process: starting from 0, applying the successor function twice, reaching 2. This view so far seems the most correct

The Role of Identity and Distinction

This posits deeper questions which we struggle to argue for and against.

A subtle but crucial philosophical issue is identity:

If the only thing that distinguishes two identical objects is the space they occupy at the same time, why does two values of the same object sum to something completely different?

Why does $1 + 1$ equal 2 instead of 1?

Because we assume:

- The two “1”s represent distinct entities, yet the same value

And we measure:

- The cardinality, not the value assigned

Example:

- 1 apple + 1 apple = 2 apples
- But 1 pile + 1 pile may still be 1 pile

Thus, arithmetic depends on how we interpret objects.

This connects to metaphysics:

- What does it mean for two things to be “different”?
- Can identity be quantified?

In physics, even particles like electrons are indistinguishable, yet we still count them: $1 + 1 = 2$.

Symbolism and Representation

The symbols “1”, “+”, and “2” are arbitrary:

- In Roman numerals: $I + I = II$
- In binary: $1 + 1 = 10$
- In base 3: $1 + 1 = 2$

Thus:

$$1 + 1 = 2$$

$$2 = 10 \text{ (base 2)}$$

$$2 = II \text{ (Roman)}$$

The truth remains invariant, even though the symbols change.

Why This Matters

One might ask: why spend 300+ pages proving something so simple?

Because $1 + 1 = 2$ represents:

- The foundation of all arithmetic
- The reliability of logical systems
- The bridge between intuition and rigor

Without this foundation:

- Engineering (using numbers like 2, 10, 1000) collapses
- Physics (measuring quantities like 2 kg, 2 m/s) fails
- Computing (binary operations like $1 + 1 = 10$) becomes meaningless

This leads us to question everything we know about math. Could we be falling into fallacies like circular reasoning?

Does it truly matter whether there is a proof for something so trivial?

If this was just a means of counting sheep, how did we go from counting physical structures to abstracting them into ideas which have expanded into technology and physics?

Conclusion

The statement $1 + 1 = 2$ is far more than a trivial arithmetic fact. It is:

- A logical consequence of axioms (Peano system)
- A set-theoretic truth about cardinalities
- A philosophical statement about identity and existence
- A cognitive milestone in human development

From 0 to 1 to 2, we see the emergence of number, structure, and meaning. What seems simple is not always as it seems.

In the end, $1 + 1 = 2$ is both:

- Obvious (to a child counting objects)
- Profound (to a mathematician formalizing reality)