Modeling the Door Hinge Effect Using the Flag Region Function

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Word count $\approx$ 1800 (excluding formula, headings and captions)

0.1 Introduction

0.1.1 Background

I enjoy climbing since it mixes problem solving with physical activity. In climbing people are constantly manipulating their body so that they are in equilibrium.

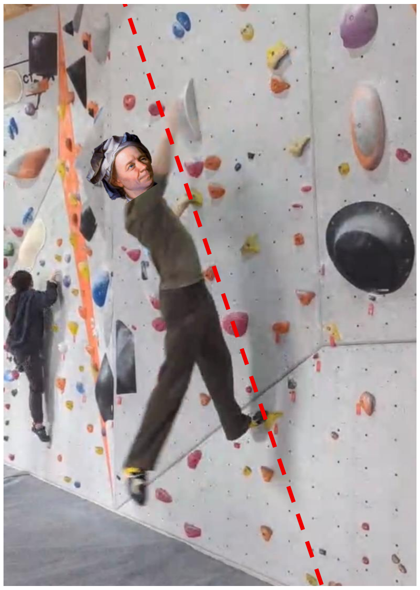
When I climb I often find myself falling off due to improper distribution of my weight over the holds which I am using. One of the results of this is known as the door hinge effect.

0.1.2 What is the door hinge effect

The door hinge effect describes the moment experienced by a climber who is in equilibrium using three holds and then lets go of one hold in order to grab a new hold, however they end up pivoting around the two other holds like a door opening.

Here is an example of what the door hinge effect might look like,

(a) Before door hinge effect



(b) After door hinge effect

Figure 1: Door hinge example

0.1.3 What is flagging

The normal solution used is to press the climbers leg against the wall so that they can use their body's tension to counteract their weight.

This is called flagging due to the shape that the climber sometimes makes with their legs, however it technically describes any form of pressing against the wall.

Figure 2: Flagging example — Source: Gekco.uk

0.1.4 How I want maths to help

I often get stuck on particular sections of climbs due to the door hinge effect. Before experiencing the door hinge effect I will let go of a hold in order to grab the next hold.

I want to calculate the optimum region that I can put my foot to flag so that I don't experience the door hinge effect

0.1.5 Disclaimer

In order to model a climber I am going to need to make some assumptions which simplify the conditions and therefore it **does not** guarantee that a certain point in the flag region is safe. It therefore shouldn't be relied on in real life without testing (and some well positioned crash mats!)

0.1.6 Assumptions

- The climbers center of mass is the centroid of the holds used
- The climber is in static equilibrium on the wall.

- Climber is permanently attached to holds A and B
- The wall is perfectly flat
- Only two forces contribute to the climbers moment
- Climber pivots exactly about AC
- Force components perpendicular to the pivot plane cancel
- $|MD| \leq |MA|, |MB|, |MC|$

Some of these assumptions will become clear later on in the paper

0.2 The Wall

0.2.1 Non-specialist audience

If you are a non specialist reader then feel free to glaze over the derivation. I have tried my best to explain the tools that are used but I didn't have enough space to write about the dot and cross product.

There are some good videos on youtube from 3blue and 1brown which explain these concepts.

0.2.2 Setting up the co-ordinate system

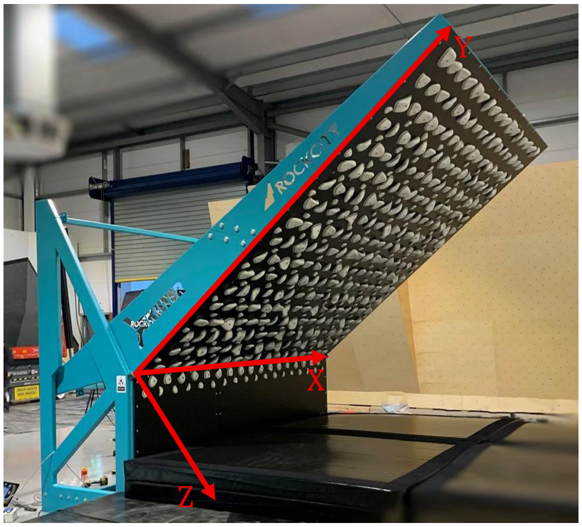


Figure 3: Training board with co-ordinate axis

Now that we know what I want to achieve lets consider a co-ordinate system in order to locate the holds.

Since we assume that the face of the climbing wall is flat we can create a xy plane that sits flush with the wall.

This plane will allow us to take the x,y the co-ordinates of the holds.

We will then set a z axis perpendicular from both the x and y axis. As shown in figure 3 most climbing walls are at an angle to the horizontal.

Using this new co-ordinate system we can generalise the first 3 holds (figure 4).

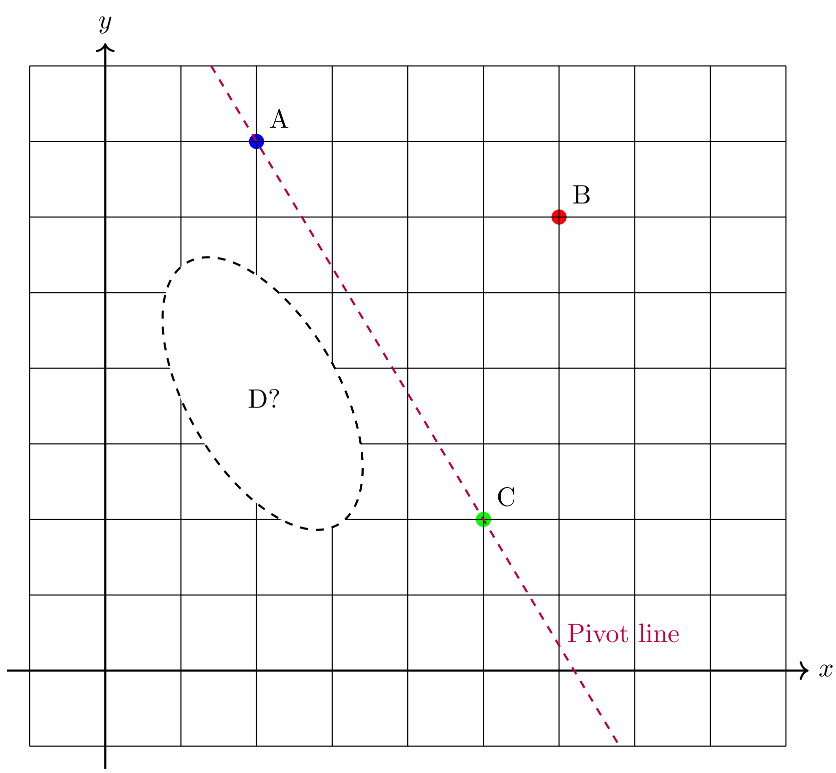


Figure 4: x,y Holds

Since the holds are modeled as points their z co-ordinates are 0

These points are:

$$A : (x_a, y_a) \quad B : (x_b, y_b) \quad C : (x_c, y_c) \quad D : (x_d, y_d)$$

Where x_d, y_d are unknown.

The climbers center of mass (M) is modeled as the centroid of the 4 points:

$$M : \left(\frac{x_a + x_b + x_c + x_d}{4}, \frac{y_a + y_b + y_c + y_d}{4}\right)$$

0.2.3 Pivoting

When the climber experiences the door hinge effect they pivot around the line $\overline{AC}$ since they are invariant points.

The gradient of $\overline{AC}$ will be,

$$\frac{y_a - y_c}{x_a - x_c}$$

The y intercept is,

$$y_a - \frac{y_a - y_c}{x_a - x_c} x_a$$

Putting this into the equation of a line we get:

$$y = \frac{y_a - y_c}{x_a - x_c} x + y_a - \frac{y_a - y_c}{x_a - x_c} x_a \implies y = \frac{y_a - y_c}{x_a - x_c}(x - x_a) + y_a$$

I am going to make a substitution to help keep the equation tidy,

$$\boxed{\mathcal{A} = y_a - y_c} \qquad \boxed{\mathcal{B} = x_a - x_c}$$

therefore,

$$\boxed{\overline{AC} : \quad y = \frac{\mathcal{A}}{\mathcal{B}}(x - x_a) + y_a}$$

0.2.4 Identifying forces

When pivoting around two climbing holds there are two main forces at play. The climbers weight is the force which induces a moment around the holds, then there is also the tension/compression($\overrightarrow{T}$) from the climbers free leg if they are flagging.

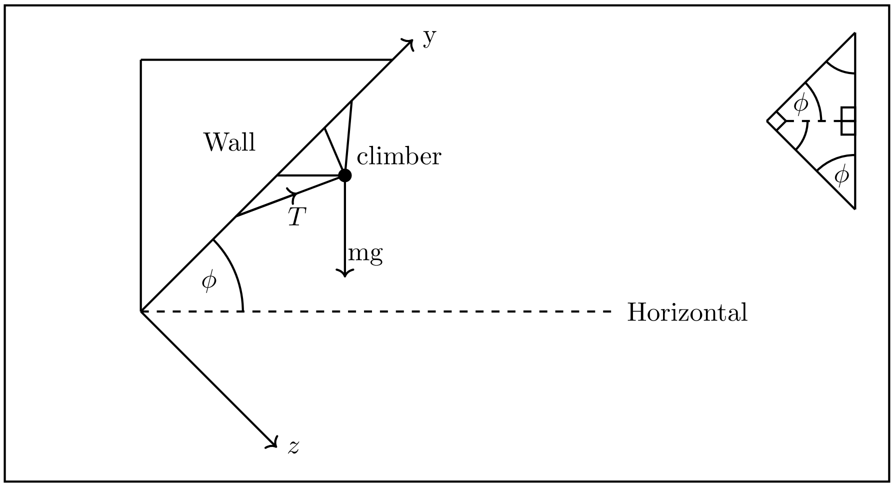


Figure 5: Side view of wall

The other forces either act at the pivot or they keep the climber fixed to the wall. In order to quantify the weight of the climber we can resolve its components in the x,y,z directions.

Since the weight points straight down it has no x-component. $\overrightarrow{W}_x = 0$.

Using the right angle triangle in the top right of figure 5 we can see that the weight vector can be written as,

$$\boxed{\overrightarrow{W} = mg \begin{bmatrix} 0 \\ -\sin(\phi) \\ \cos(\phi) \end{bmatrix}}$$

Since the tension vector acts through the climbers leg then the tension vector is a scaled version of the vector from the point of contact (D) of the climbers leg to the climbers center of mass (M).

$$\overrightarrow{T} = k\overrightarrow{DM}$$

Since,

$$\overrightarrow{DM} = \overrightarrow{DO} + \overrightarrow{OM}$$

substituting in,

$$\overrightarrow{DO} = -\overrightarrow{OD} = \begin{bmatrix} -x_d \\ -y_d \\ 0 \end{bmatrix}$$

and

$$\overrightarrow{OM} = \begin{bmatrix} \frac{x_a+x_b+x_c+x_d}{4} \\ \frac{y_a+y_b+y_c+y_d}{4} \\ z_m \end{bmatrix}$$

Where z_m is the distance from the climbers center of mass to the wall

this implies that,

$$\overrightarrow{DM} = \begin{bmatrix} \frac{x_a+x_b+x_c+x_d}{4} - x_d \\ \frac{y_a+y_b+y_c+y_d}{4} - y_d \\ z_m \end{bmatrix} = \frac{1}{4} \begin{bmatrix} x_a + x_b + x_c - 3x_d \\ y_a + y_b + y_c - 3y_d \\ 4z_m \end{bmatrix}$$

I am going to use two substitutions to help simplify $\vec{T}$:

$$\boxed{\mathcal{C} = y_a + y_b + y_c} \qquad \boxed{\mathcal{D} = x_a + x_b + x_c}$$

so,

$$\boxed{\vec{T} = \frac{k}{4} \begin{bmatrix} \mathcal{D} - 3x_d \\ \mathcal{C} - 3y_d \\ 4z_m \end{bmatrix}}$$

0.2.5 Plane of rotation

When the climber pivots around $\overline{AC}$ their center of mass will move through a single plane. This means that only the forces that are parallel to that plane will affect the moment.

To help visualize this we can look at an example if $D : (2, 3, 0)$ see figure 6

The vector $\overrightarrow{CA}$ is normal ($\vec{n}$) to the plane of rotation,

$$\vec{n} = \overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$$

This implies that

$$\vec{n} = \begin{bmatrix} x_a \\ y_a \\ 0 \end{bmatrix} + \begin{bmatrix} -x_c \\ -y_c \\ 0 \end{bmatrix} = \begin{bmatrix} x_a - x_c \\ y_a - y_c \\ 0 \end{bmatrix}$$

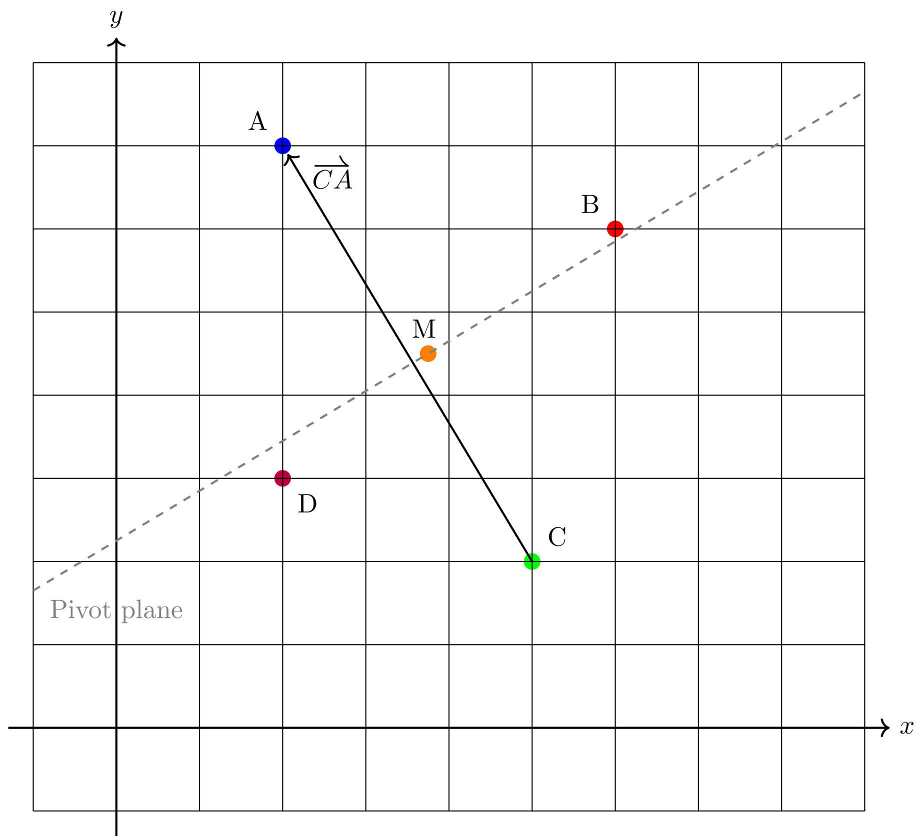


Figure 6: Pivot plane

Which can be simplified to

$$\boxed{\vec{n} = \begin{bmatrix} \mathcal{B} \\ \mathcal{A} \\ 0 \end{bmatrix}}$$

Since the point M lies on the plane, we can describe the vector to any point $(P : x, y, z)$,

$$\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OP}$$

Substituting in to get:

$$\overrightarrow{MP} = \begin{bmatrix} -\frac{\mathcal{D}+x_d}{4} \\ -\frac{\mathcal{C}+y_d}{4} \\ z_m \end{bmatrix} + \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x - \frac{\mathcal{D}+x_d}{4} \\ y - \frac{\mathcal{C}+y_d}{4} \\ z - z_m \end{bmatrix}$$

Now since the normal vector is always perpendicular to any vector that is parallel to the plane.

This implies that,

$$\vec{n} \cdot \overrightarrow{MP} = 0$$

Therefore:

$$\begin{bmatrix} \mathcal{B} \\ \mathcal{A} \\ 0 \end{bmatrix} \cdot \begin{bmatrix} x - \frac{\mathcal{D}+x_d}{4} \\ y - \frac{\mathcal{C}+y_d}{4} \\ z - z_m \end{bmatrix} = 0$$

Expanding this we get the equation:

$$\mathcal{B}\Big(x - \frac{\mathcal{D} + x_d}{4}\Big) + \mathcal{A}\Big(y - \frac{\mathcal{C} + y_d}{4}\Big) + (0)\Big(z - z_m\Big) = 0$$

Which can be expanded and simplified to:

$$\boxed{\mathcal{B}x + \mathcal{A}y = \frac{\mathcal{A}(\mathcal{C} + y_d) + \mathcal{B}(\mathcal{D} + x_d)}{4}}$$

0.2.6 Resolving

Now that we know the equation of the plane which the center of mass will rotate in, and the forces that are applied to the center of mass, these force vectors can be resolved parallel and perpendicular to the plane of rotation.

We need to resolve for the parallel force vectors since they contribute to the moment of M through the plane. If we take a vector ($\vec{v}$), then we can say that,

$$\vec{v} = \vec{v}_\perp + \vec{v}_\parallel$$

Where

$\vec{v}_\perp$ is the component perpendicular to the plane

and

$\vec{v}_\parallel$ is the component parallel to the plane

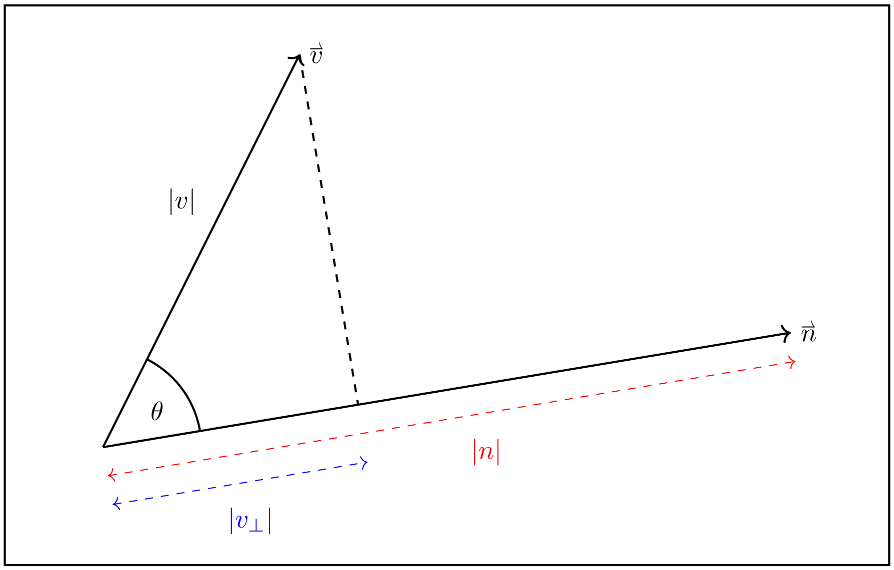


Figure 7: Vector components

Then we can re-arrange for $\vec{v}_\parallel$:

$$\vec{v}_\parallel = \vec{v} - \vec{v}_\perp$$

In order to find $\vec{v}_\parallel$ we need to find $\vec{v}_\perp$.

To do this we are going to use the normal vector ($\vec{n}$). Since the vector $\vec{v}_\perp = \lambda\vec{n}$ we can use this relationship in order to project the vector $\vec{v}$ onto $\vec{n}$

Firstly we can say that,

$$|v_\perp| = |v| \cos\theta$$

Using the dot product,

$$\vec{v} \cdot \vec{n} = |v||n| \cos\theta$$

So we can divide by $|n|$ to find the length of $\vec{v}_\perp$:

$$|v_\perp| = \frac{\vec{v} \cdot \vec{n}}{|n|}$$

Now in order to convert $|v_\perp| \to \vec{v}_\perp$ we can times the magnitude of $v_\perp$ by the unit vector in the direction of $\vec{v}_\perp$

Since $\vec{n}$ is parallel to $\vec{v}_\perp$ then we can convert $\vec{n} \to \hat{n}$ where $|\hat{n}| = 1$,

$$\hat{n} = \frac{\vec{n}}{|n|}$$

This implies that,

$$\vec{v}_\perp = \frac{\vec{v} \cdot \vec{n}}{|n|^2}\vec{n}$$

Now using this in the equation for $\vec{v}_\parallel$:

$$\vec{v}_\parallel = \vec{v} - \frac{\vec{v} \cdot \vec{n}}{|n|^2}\vec{n}$$

Now we can just apply this formula to $\overrightarrow{W}$ and $\overrightarrow{T}$,

$$\overrightarrow{W} = mg \begin{bmatrix} 0 \\ -\sin(\phi) \\ \cos(\phi) \end{bmatrix} \qquad \vec{n} = \begin{bmatrix} \mathcal{B} \\ \mathcal{A} \\ 0 \end{bmatrix}$$

Using the formula:

$$\overrightarrow{W}_\parallel = mg \begin{bmatrix} 0 \\ -\sin(\phi) \\ \cos(\phi) \end{bmatrix} - \frac{-mg\mathcal{A}\sin(\phi)}{\mathcal{A}^2 + \mathcal{B}^2} \begin{bmatrix} \mathcal{B} \\ \mathcal{A} \\ 0 \end{bmatrix}$$

Simplifying:

$$\overrightarrow{W}_\parallel = mg \begin{bmatrix} \frac{\mathcal{A}\mathcal{B}\sin(\phi)}{\mathcal{A}^2+\mathcal{B}^2} \\ -\frac{\mathcal{B}^2\sin(\phi)}{\mathcal{A}^2+\mathcal{B}^2} \\ \frac{(\mathcal{A}^2+\mathcal{B}^2)\cos(\phi)}{\mathcal{A}^2+\mathcal{B}^2} \end{bmatrix}$$

Therefore,

$$\boxed{\overrightarrow{W}_\parallel = \frac{mg}{\mathcal{A}^2 + \mathcal{B}^2} \begin{bmatrix} \mathcal{A}\mathcal{B}\sin(\phi) \\ -\mathcal{B}^2\sin(\phi) \\ (\mathcal{A}^2 + \mathcal{B}^2)\cos(\phi) \end{bmatrix}}$$

For the tension vector:

$$\overrightarrow{T} = \frac{k}{4} \begin{bmatrix} \mathcal{D} - 3x_d \\ \mathcal{C} - 3y_d \\ 4z_m \end{bmatrix} \qquad \vec{n} = \begin{bmatrix} \mathcal{B} \\ \mathcal{A} \\ 0 \end{bmatrix}$$

This implies that,

$$\overrightarrow{T}_\parallel = \frac{k}{4} \begin{bmatrix} \mathcal{D} - 3x_d \\ \mathcal{C} - 3y_d \\ 4z_m \end{bmatrix} - \frac{k\mathcal{B}(\mathcal{D} - 3x_d) + k\mathcal{A}(\mathcal{C} - 3y_d)}{4(\mathcal{A}^2 + \mathcal{B}^2)} \begin{bmatrix} \mathcal{B} \\ \mathcal{A} \\ 0 \end{bmatrix}$$

Simplifying:

$$\overrightarrow{T}_{\parallel} = \frac{k}{4(\mathcal{A}^2+\mathcal{B}^2)}\begin{bmatrix}(\mathcal{A}^2+\mathcal{B}^2)(\mathcal{D}-3x_d)-\mathcal{B}^2(\mathcal{D}-3x_d)-\mathcal{AB}(\mathcal{C}-3y_d)\\(\mathcal{A}^2+\mathcal{B}^2)(\mathcal{C}-3y_d)-\mathcal{AB}(\mathcal{D}-3x_d)-\mathcal{A}^2(\mathcal{C}-3y_d)\\4z_m(\mathcal{A}^2+\mathcal{B}^2)\end{bmatrix}$$

Therefore:

$$\boxed{\overrightarrow{T}_{\parallel} = \frac{k}{4(\mathcal{A}^2+\mathcal{B}^2)}\begin{bmatrix}\mathcal{A}^2(\mathcal{D}-3x_d)-\mathcal{AB}(\mathcal{C}-3y_d)\\\mathcal{B}^2(\mathcal{C}-3y_d)-\mathcal{AB}(\mathcal{D}-3x_d)\\4z_m(\mathcal{A}^2+\mathcal{B}^2)\end{bmatrix}}$$

0.3 Moments

0.3.1 Position Vector of M

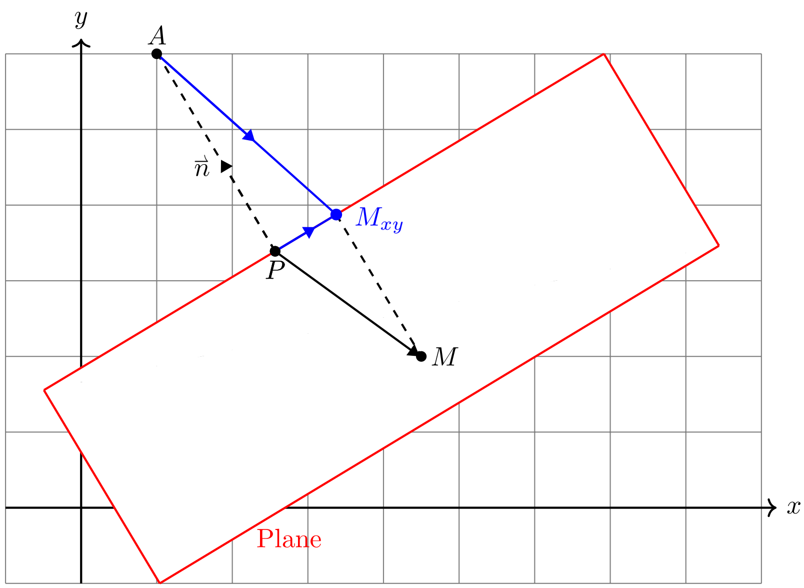


Figure 8: Position vector

In order to quantify the moment produced from the forces, we need to know the position of the climbers center of mass, therefore we need to know the vector $\overrightarrow{PM}$

Since

$$\overrightarrow{PM} = \overrightarrow{PM}_{xy} + \overrightarrow{PM}_z$$

and

$$\overrightarrow{PM}_z = \begin{bmatrix} 0 \\ 0 \\ z_m \end{bmatrix}$$

Since $\overrightarrow{PM}_{xy}$ lies on the xy plane and also lies on the pivot plane which is perpendicular to the xy plane, since it has no z term.

Therefore we can say that the vector $\overrightarrow{PM}_{xy}$ is parallel and intercepts the line,

$$y = -\frac{\mathcal{B}}{\mathcal{A}}x + constant$$

Therefore it has a direction of:

$$\begin{bmatrix} 1 \\ -\frac{\mathcal{B}}{\mathcal{A}} \\ 0 \end{bmatrix}$$

Which can be multiplied by $\mathcal{A}$ to get the direction vector $\vec{d}$:

$$\vec{d} = \begin{bmatrix} \mathcal{A} \\ -\mathcal{B} \\ 0 \end{bmatrix}$$

This implies that,

$$\overrightarrow{PM}_{xy} = \mu\vec{d} = \mu \begin{bmatrix} \mathcal{A} \\ -\mathcal{B} \\ 0 \end{bmatrix}$$

We can also use the vector $\vec{AM}_{xy}$ by resolving it into a vector that is parallel to the direction $\vec{d}$ using the formula:

$$\vec{v}_{\parallel} = \frac{\vec{v} \cdot \vec{u}}{|u|^2}\vec{u}$$

and substituting,

$$\vec{v} = \overrightarrow{AM}_{xy} \qquad \vec{u} = \vec{d}$$

We know:

$$\overrightarrow{AM}_{xy} = \overrightarrow{AO} + \overrightarrow{OM}_{xy}$$

Therefore:

$$\overrightarrow{AM}_{xy} = \begin{bmatrix} -x_a \\ -y_a \\ 0 \end{bmatrix} + \frac{1}{4}\begin{bmatrix} \mathcal{D} + x_d \\ \mathcal{C} + y_d \\ 0 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} \mathcal{D} - 4x_a + x_d \\ \mathcal{C} - 4y_a + y_d \\ 0 \end{bmatrix}$$

This implies:

$$\overrightarrow{PM}_{xy} = \frac{\mathcal{A}(\mathcal{D} - 4x_a + x_d) - \mathcal{B}(\mathcal{C} - 4y_a + y_d)}{4(\mathcal{A}^2 + \mathcal{B}^2)} \begin{bmatrix} \mathcal{A} \\ -\mathcal{B} \\ 0 \end{bmatrix}$$

Then using the equation,

$$\overrightarrow{PM} = \overrightarrow{PM}_{xy} + \overrightarrow{PM}_z$$

This implies that:

$$\overrightarrow{PM} = \frac{\mathcal{A}(\mathcal{D} - 4x_a + x_d) - \mathcal{B}(\mathcal{C} - 4y_a + y_d)}{4(\mathcal{A}^2 + \mathcal{B}^2)} \begin{bmatrix} \mathcal{A} \\ -\mathcal{B} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ z_m \end{bmatrix}$$

Therfore:

$$\boxed{\overrightarrow{PM} = \frac{1}{4(\mathcal{A}^2 + \mathcal{B}^2)} \begin{bmatrix} \mathcal{A}^2(\mathcal{D} - 4x_a + x_d) - \mathcal{A}\mathcal{B}(\mathcal{C} - 4y_a + y_d) \\ \mathcal{B}^2(\mathcal{C} - 4y_a + y_d) - \mathcal{A}\mathcal{B}(\mathcal{D} - 4x_a + x_d) \\ 4(\mathcal{A}^2 + \mathcal{B}^2) z_m \end{bmatrix}}$$

0.3.2 Solving for zero moment

The moment of the climber can be defined as:

$$\tau = \overrightarrow{PM} \times (\overrightarrow{T}_{\parallel} + \overrightarrow{W}_{\parallel})$$

Where τ is the moment. In order to find the position of M where the door hinge effect doesn't occur, we need to set $\tau = 0$

Using the substitution:

$$\mathcal{S} = \mathcal{A}^2 + \mathcal{B}^2$$

this implies

$$\overrightarrow{PM} = \frac{1}{4\mathcal{S}} \begin{bmatrix} \mathcal{A}^2(\mathcal{D} - 4x_a + x_d) - \mathcal{A}\mathcal{B}(\mathcal{C} - 4y_a + y_d) \\ \mathcal{B}^2(\mathcal{C} - 4y_a + y_d) - \mathcal{A}\mathcal{B}(\mathcal{D} - 4x_a + x_d) \\ 4\mathcal{S} z_m \end{bmatrix}$$

and the resultant vector $\overrightarrow{R}$:

$$\overrightarrow{R} = \overrightarrow{W}_{\parallel} + \overrightarrow{T}_{\parallel}$$

which equals:

$$\vec{R} = \frac{k}{4\mathcal{S}}\begin{bmatrix}\mathcal{A}^2(\mathcal{D}-3x_d)-\mathcal{A}\mathcal{B}(\mathcal{C}-3y_d)\\ \mathcal{B}^2(\mathcal{C}-3y_d)-\mathcal{A}\mathcal{B}(\mathcal{D}-3x_d)\\ 4\mathcal{S}z_m\end{bmatrix} + \frac{4mg}{4S}\begin{bmatrix}\mathcal{A}\mathcal{B}\sin(\phi)\\ -\mathcal{B}^2\sin(\phi)\\ \mathcal{S}\cos(\phi)\end{bmatrix}$$

Using the moment equation,

$$\overrightarrow{PM} \times \vec{R} = 0$$

Since the cross product of two vectors is 0 if the two vectors are parallel, this implies that,

$$\overrightarrow{PM} = h\vec{R}$$

We can now equate the vectors components,

The z components:

$$z_m = h(kz_m + mg\cos(\phi))$$

Therefore:

$$h = \frac{z_m}{kz_m + mg\cos(\phi)}$$

Now if we look at the x components:

$$\mathcal{A}^2(\mathcal{D}-4x_a+x_d)-\mathcal{A}\mathcal{B}(\mathcal{C}-4y_a+y_d) = h\Big[k\Big(\mathcal{A}^2(\mathcal{D}-3x_d)-\mathcal{A}\mathcal{B}(\mathcal{C}-3y_d)\Big)+4mg\mathcal{A}\mathcal{B}\sin(\phi)\Big]$$

I am going to simplify this by substituting,

$$\mathcal{C}-4y_a = \mathcal{E} \qquad \mathcal{D}-4x_a = \mathcal{G} \qquad mg\sin(\phi) = \mathcal{W} \qquad mg\cos(\phi) = \mathcal{V} \qquad kz_m = \mathcal{L}$$

Substituting these in along with h to get:

$$\mathcal{A}^2(\mathcal{G}+x_d) - \mathcal{A}\mathcal{B}(\mathcal{E}+y_d) = \frac{z_m}{\mathcal{L}+\mathcal{V}}\Big[\mathcal{A}^2k(\mathcal{D}-3x_d) - \mathcal{A}\mathcal{B}k(\mathcal{C}-3y_d) + 4\mathcal{A}\mathcal{B}\mathcal{W}\Big]$$

Multiplying by $\mathcal{L}+\mathcal{V}$:

$$\Big(\mathcal{L}+\mathcal{V}\Big)\Big(\mathcal{A}^2(\mathcal{G}+x_d) - \mathcal{A}\mathcal{B}(\mathcal{E}+y_d)\Big) = z_m\Big(\mathcal{A}^2k(\mathcal{D}-3x_d) - \mathcal{A}\mathcal{B}k(\mathcal{C}-3y_d) + 4\mathcal{A}\mathcal{B}\mathcal{W}\Big)$$

Now if we take the left hand side (LHS) and expand:

$$LHS = \mathcal{A}^2\mathcal{L}(\mathcal{G}+x_d) - \mathcal{A}\mathcal{B}\mathcal{L}(\mathcal{E}+y_d) + \mathcal{V}\mathcal{A}^2(\mathcal{G}+x_d) - \mathcal{V}\mathcal{A}\mathcal{B}(\mathcal{E}+y_d)$$

Then separate for x_d and y_d

$$LHS = (\mathcal{A}^2\mathcal{L} + \mathcal{A}^2\mathcal{V})x_d - (\mathcal{A}\mathcal{B}\mathcal{L} + \mathcal{A}\mathcal{B}\mathcal{V})y_d + (\mathcal{L} + \mathcal{V})(\mathcal{A}^2\mathcal{G} - \mathcal{A}\mathcal{B}\mathcal{E})$$

Then i will call,

$$\mathcal{J} = (\mathcal{L} + \mathcal{V})(\mathcal{A}^2\mathcal{G} - \mathcal{A}\mathcal{B}\mathcal{E})$$

Therefore:

$$LHS = (\mathcal{A}^2\mathcal{L} + \mathcal{A}^2\mathcal{V})x_d - (\mathcal{A}\mathcal{B}\mathcal{V} + \mathcal{A}\mathcal{B}\mathcal{L})y_d + \mathcal{J}$$

Now doing the same to the right hand side (RHS):

$$RHS = \mathcal{A}^2\mathcal{L}(\mathcal{D} - 3x_d) - \mathcal{A}\mathcal{B}\mathcal{L}(\mathcal{C} - 3y_d) + 4\mathcal{A}\mathcal{B}\mathcal{W}z_m$$

Then separating the x_d and y_d:

$$RHS = -3\mathcal{A}^2\mathcal{L}x_d + 3\mathcal{A}\mathcal{B}\mathcal{L}y_d + 4\mathcal{A}\mathcal{B}\mathcal{W}z_m + \mathcal{A}^2\mathcal{L}\mathcal{D} - \mathcal{A}\mathcal{B}\mathcal{C}\mathcal{L}$$

Then substituting a constant, $\mathcal{K} = 4\mathcal{A}\mathcal{B}\mathcal{W}z_m + \mathcal{A}^2\mathcal{L}\mathcal{D} - \mathcal{A}\mathcal{B}\mathcal{C}\mathcal{L}$ To get:

$$RHS = -3\mathcal{A}^2\mathcal{L}x_d + 3\mathcal{A}\mathcal{B}\mathcal{L}y_d + \mathcal{K}$$

Since $RHS = LHS$, this implies that:

$$(\mathcal{A}^2\mathcal{L} + \mathcal{A}^2\mathcal{V})x_d - (\mathcal{A}\mathcal{B}\mathcal{V} + \mathcal{A}\mathcal{B}\mathcal{L})y_d + \mathcal{J} = -3\mathcal{A}^2\mathcal{L}x_d + 3\mathcal{A}\mathcal{B}\mathcal{L}y_d + \mathcal{K}$$

Moving the y_d terms to one side and factorising:

$$(\mathcal{V} + 4\mathcal{L})\mathcal{A}\mathcal{B}y_d = (\mathcal{V} + 4\mathcal{L})\mathcal{A}^2x_d + \mathcal{J} - \mathcal{K}$$

Then dividing both sides by $\mathcal{A}\mathcal{B}(\mathcal{V} + 4\mathcal{L})$:

$$y_d = \frac{\mathcal{A}}{\mathcal{B}}x_d + \frac{\mathcal{J} - \mathcal{K}}{\mathcal{A}\mathcal{B}(\mathcal{V} + 4\mathcal{L})}$$

and this is the line which the point d can be located so that the climber doesn't pivot around AC.

We can fully expand the constant term:

$$\frac{\mathcal{A}\big(\mathcal{D}mg\cos\phi - 4x_a(kz_m + mg\cos\phi)\big) + \mathcal{B}\big(-\mathcal{C}mg\cos\phi + 4y_a(kz_m + mg\cos\phi) - 4mgz_m\sin\phi\big)}{\mathcal{B}(mg\cos\phi + 4kz_m)}$$

0.3.3 Restricting flag region

One restriction that could be added to the function is that $|MD| \leq |MA|$, $|MB|$, $|MC|$, assuming that my other limbs are fully extended.

Since the distance from M to any point $V \in A, B, C$ is

$$|MV| = \sqrt{(x_{a,b,c} - x_m)^2 + (y_{a,b,c} - y_m)^2 + (z_{a,b,c} - z_m)^2}$$

Since points A,B,C have a z co-ordinate of zero the expression becomes

$$|MV| = \sqrt{(x_{a,b,c} - x_m)^2 + (y_{a,b,c} - y_m)^2 + (z_m)^2}$$

Since $|MD| \leq |MV|$ this implies that:

$$\sqrt{(x_d - x_m)^2 + (y_d - y_m)^2 + (z_m)^2} \leq \sqrt{(x_{a,b,c} - x_m)^2 + (y_{a,b,c} - y_m)^2 + (z_m)^2}$$

Therefore:

$$(x_d - x_m)^2 + (y_d - y_m)^2 \leq (x_{a,b,c} - x_m)^2 + (y_{a,b,c} - y_m)^2$$

Then substituting $x_m = \frac{\mathcal{D}+x_d}{4}$ and $y_m = \frac{\mathcal{C}+y_d}{4}$

$$\left(\frac{3x_d - \mathcal{D}}{4}\right)^2 + \left(\frac{3y_d - \mathcal{C}}{4}\right)^2 \leq \left(\frac{4x_{a,b,c} - x_d - \mathcal{D}}{4}\right)^2 + \left(\frac{4y_{a,b,c} - y_d - \mathcal{C}}{4}\right)^2$$

which simplifys to:

$$\boxed{\left(x_d - \frac{\mathcal{D} - x_{a,b,c}}{2}\right)^2 + \left(y_d - \frac{\mathcal{C} - y_{a,b,c}}{2}\right)^2 \leq \frac{(\mathcal{D} - 3x_{a,b,c})^2 + (\mathcal{C} - 3y_{a,b,c})^2}{4}}$$

0.4 Final Model

The flag region function is defined as,

$$y_d = \mathcal{F}(x_d) : x_d \in \mathbb{R}$$

restricted by

$$\left(x_d - \frac{\mathcal{D} - x_{a,b,c}}{2}\right)^2 + \left(y_d - \frac{\mathcal{C} - y_{a,b,c}}{2}\right)^2 \leq \frac{(\mathcal{D} - 3x_{a,b,c})^2 + (\mathcal{C} - 3y_{a,b,c})^2}{4}$$

and

$$k > 0 \qquad \mathcal{B} \neq 0$$

where

$$\boxed{\mathcal{F}(x_d) = \frac{\mathcal{A}}{\mathcal{B}}x_d + \frac{mg\cos\phi\big(\mathcal{A}\mathcal{P} - \mathcal{B}\mathcal{Q}\big) - 4z_m\big(\mathcal{U}k + mg\mathcal{B}\sin\phi\big)}{\mathcal{B}\big(mg\cos\phi + 4kz_m\big)}}$$

and

$$\mathcal{A} = y_a - y_c \qquad \mathcal{B} = x_a - x_c \qquad \mathcal{C} = y_a + y_b + y_c \qquad \mathcal{D} = x_a + x_b + x_c$$

$$\mathcal{P} = x_b + x_c - 3x_a \qquad \mathcal{Q} = y_b + y_c - 3y_a \qquad \mathcal{U} = (y_a - y_c)x_a - (x_a - x_c)y_a$$

Using the inputs:

ϕ = Angle of the wall from the horizontal

$\mathbf{mg}$ = The climbers weight

$\mathbf{z_m}$ = Distance from the wall to the climbers center of mass (using same scale as the climbing holds co-ordinates)

$(\mathbf{x_a}, \mathbf{y_a})$ and $(\mathbf{x_c}, \mathbf{y_c})$ are the co-ordinates of the invariant holds

$(\mathbf{x_b}, \mathbf{y_b})$ is the co-ordinate of the hold that is being let go

0.5 Testing the model

0.5.1 Kilter Board

To test the model I used a Kilter board (training wall). It uses a mobile app to set climbs using different holds

Kilter boards are flat and set at an angle of 45^o to the horizontal.

The kilter board also features holds that are the same distance apart so I can define a unit length as the distance between two adjacent holds ($\approx 20cm$)

0.5.2 Parameters

On the kilter board I set a move which requires a flag. The green and yellow holds are invariant and the aim is to move my hand from the blue hold to the purple.

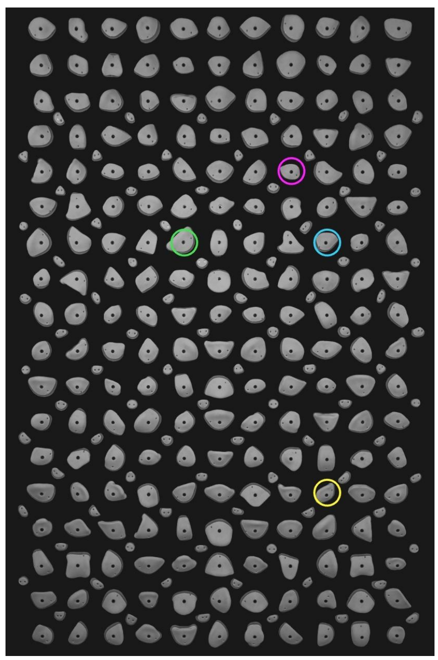


Figure 9: Kilter board

$$\phi \approx 45^o \qquad mg \approx 600N \qquad z_m \approx 70cm$$

$$(x_a, y_a) \approx (80, 220) \qquad (x_b, y_b) \approx (160, 220) \qquad (x_c, y_c) \approx (160, 80)$$

0.5.3 Flag region

Putting these into the function:

$$y_d = \mathcal{F}_k(x_d) = -\frac{7}{4}x_d + \frac{8064000\,k - 2036467.53}{22400\,k + 33941.13}$$

Bounded by:

$$(x_d - 160)^2 + (y_d - 150)^2 \leq 11300$$

$$(x_d - 120)^2 + (y_d - 150)^2 \leq 6500$$

$$(x_d - 120)^2 + (y_d - 220)^2 \leq 21200$$

If we let the value of k (N/cm) vary and assign its values to a heat scale using $\log_{10}(k)$ since there are asymptotes near the pivot line, then we can graph the flag region.

This is the region where the climber can apply a force into the wall so that they create a tension/compression in their leg in order to stop them from rotating.

Here is the x,y heat map of the flag region (figure 10) as well as overlaid kilter board for reference

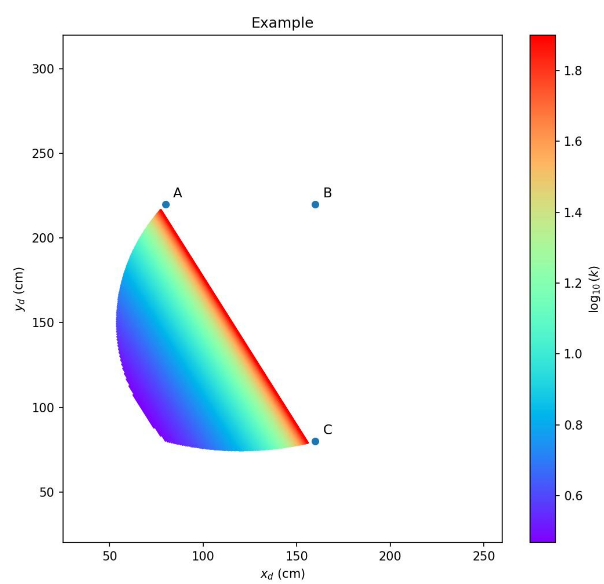


Figure 10: Flag region

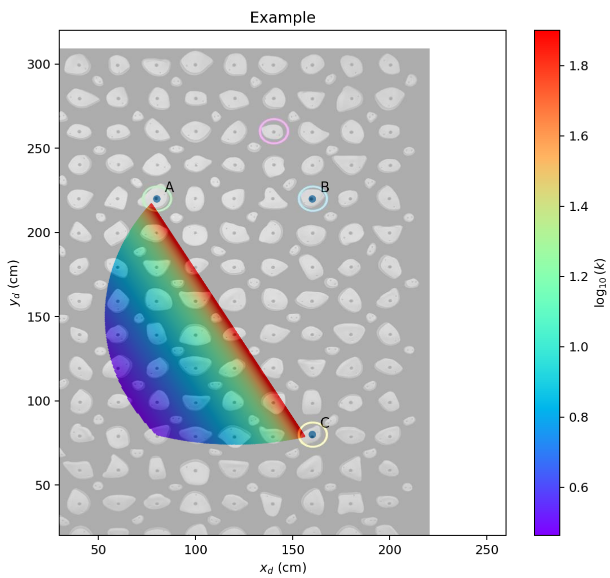


Figure 11: Kilter board overlay

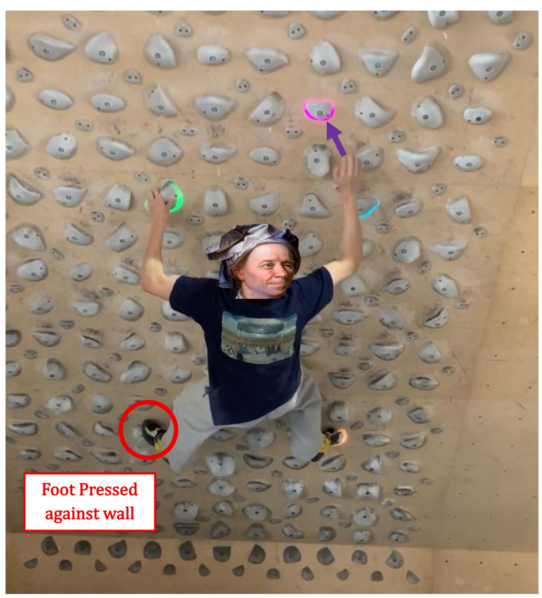


Figure 12: Euler?!? testing the model

0.6 Conclusion

0.6.1 My opinion

The model calculates the optimum region to flag with my foot with reasonable accuracy, although it can sometimes be slightly off due to the center of mass being more complicated in real life.

In the example above it shows that the further I place my foot on the left of the pivot line the less force is required.

Although it relies heavily on the modeling assumptions, it is most useful for estimating where not to put your foot and where the $\vec{T} \to \infty$

0.6.2 How could this be improved

- Using an equation to describe the shape of the wall (not flat)
- Modeling the mass of the climber in more detail.
- Accounting for certain holds that must be held from a certain side
- Using varying restrictions e.g. $0.9|MD| \leq |MA|, \ |MB|, \ |MC|$

Thank you for reading

$$\alpha \ln(e^x) : \beta \bigcup_{t=1}^{\infty} (\mathbb{R}_t)$$

March 2026