Modeling ECG Signals Using Gaussian Functions

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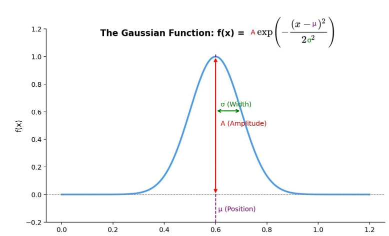
Introduction

The Gaussian function is one of the most mathematically model widely used for representing natural and scientific phenomena. Its symmetrical shape around a central mean value makes it particularly effective in describing distributions where most values cluster near the average while fewer values appear as we move away from the center for that the Gaussian function plays a fundamental role in fields such as statistics, physics, signal processing, and data science and it is commonly referred to as the bell-shaped curve.

In biomedical engineering, mathematical models are essential for analyzing and interpreting physiological signals. Among these signals, the electrocardiogram (ECG) is of great importance as it reflects the electrical activity of the heart. Each of P wave, QRS complex, and T wave representing a specific phase of cardiac activity and accurately modeling these components is very important for understanding the human heart behavior and detecting abnormalities.

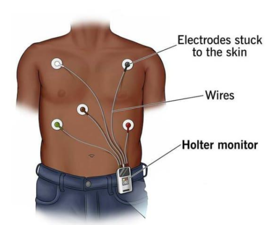
This research focuses on deriving and applying a Gaussian-based mathematical equation to model ECG signals. By approximating ECG waveforms using Gaussian functions, we aim to simplify the representation of complex biological data while maintaining accuracy.

Background



The Gaussian function was first introduced by the German mathematician Carl Friedrich Gauss in the early 19th century and has since become one of the most fundamental tools in many different sciences like applied mathematics and engineering. The general form of the Gaussian function is expressed as f(x) =$\boldsymbol{f(x)}$ = $\boldsymbol{A \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$, where A represents the amplitude of the curve, μ denotes the mean value that determines the center of the peak, and σ is a parameter that controls the width of the curve. The simplicity and flexibility of this function allow it to a wide range of applications, from probability theory to image processing and medical electrical signals like as ECG signals.

The electrocardiogram (ECG) is a non-invasive diagnostic test that records the electrical activity of the heart over time using electrodes placed on the skin.



The human heart is a hollow muscle that pumps blood through the body by contracting and relaxing in a regular pattern. This rhythmic activity isn't random because it's controlled by the heart's own electrical conduction system, which sends out electrical impulses in a very organized way with each beat. The heart has four chambers that work together in a coordinated way: the right atrium receives deoxygenated blood returning from the body; the right ventricle pumps deoxygenated blood to the lungs; the left atrium receives oxygenated blood returning from the lungs; and the left ventricle pumps oxygenated blood to the entire body. Each heartbeat follows a precise and repeatable sequence of electrical events known as the cardiac cycle.

The cardiac cycle begins at the Sinoatrial Node (SA Node), located in the upper wall of the right atrium. The SA node is the heart's natural pacemaker, spontaneously generating electrical impulses at a rate of 60 to 100 times per minute under normal resting conditions, initiating every heartbeat cycle. The impulse is then received by the Atrioventricular Node (AV Node), situated at the junction between the atria and ventricles, which introduces a brief physiological delay of approximately 120–200 ms to ensure the atria fully contract before ventricular contraction begins. From there, the Bundle of His carries the electrical impulse downward into the ventricular myocardium, splitting into right and left bundle branches. Finally, the Purkinje Fibers spread the impulse rapidly and uniformly across the entire ventricular muscle wall, ensuring a synchronized and powerful contraction. The ECG test is a simple and painless procedure that allows doctors to monitor the electrical activity of the heart and detect serious conditions such as heart attacks and arrhythmias at an early stage.

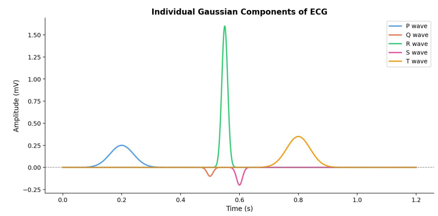
Methodology

As discussed earlier, a typical ECG signal consists of three main waveforms: the P wave, the QRS complex, and the T wave. Each of these waveforms has a smooth and localized shape that can be closely approximated using a Gaussian function. The basic idea is to represent each wave as an individual Gaussian component, where the parameters of the function are adjusted to match the shape of the real signal. Each wave is modeled using the following Gaussian equation: $\boldsymbol{f(x) = A \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$, where A controls the height of the wave, μ defines the time position of the peak, and σ determines how wide or narrow the wave is.

Since a complete heartbeat consists of multiple waves, the full ECG cycle is represented as a sum of Gaussian functions, one for each wave: ECG(x) = $\boldsymbol{ECG(x) = \sum_{i=1}^{n} A_i \cdot e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}}$, where n is the number of waves being modeled. In a standard heartbeat, this typically means using at least three Gaussian components to represent the P wave, QRS complex, and T wave respectively. The values of A, μ, and σ for each component are estimated by fitting the model to the real ECG data using mathematical optimization techniques such as the Least Squares Method. The result is a smooth mathematical representation of the heartbeat that captures the essential features of the signal while significantly simplifying its complexity, which is the core approach of this research.

The normal values used for each ECG wave are as follows: the P wave has an amplitude (A) of 0.25 mV, a position (μ) of 0.2 s, and a width (σ) of 0.04 s. The Q wave has an amplitude of −0.1 mV at 0.5 s with a width of 0.01 s. The R wave has an amplitude of 1.6 mV at 0.55 s with a width of 0.01 s. The S wave has an amplitude of −0.2 mV at 0.6 s with a width of 0.01 s. Finally, the T wave has an amplitude of 0.35 mV at 0.8 s with a width of 0.04 s. These values yield the following complete equation: ECG(x) =$ECG(x) = 0.25 \cdot e^{-\frac{(x-0.2)^2}{2(0.04)^2}} - 0.1 \cdot e^{-\frac{(x-0.5)^2}{2(0.01)^2}} + 1.6 \cdot e^{-\frac{(x-0.55)^2}{2(0.01)^2}} - 0.2 \cdot e^{-\frac{(x-0.6)^2}{2(0.01)^2}} + 0.35 \cdot e^{-\frac{(x-0.8)^2}{2(0.04)^2}}$.

In this section, the Gaussian-based model was applied to a standard ECG signal using the parameter values defined in the methodology. The goal was to evaluate how well the sum of Gaussian functions can represent the three main parts of a real heartbeat cycle. The final model was made up of five Gaussian components that represent the P wave, Q wave, R wave, S wave, and T wave respectively. The resulting curve was then compared to a reference ECG signal to check the quality of the fit. The comparison showed that the Gaussian model captured the overall shape of the heartbeat well. The R wave peak was the most accurately represented feature because of its clear and symmetric shape. The P wave and T wave were also well approximated due to their smooth and rounded profiles. The Q and S waves, despite having smaller negative amplitudes, were accurately shown as downward deflections in the modeled signal.



Conclusion

This research investigated the use of Gaussian functions as a mathematical tool for modeling electrocardiogram (ECG) signals. By representing each component of the heartbeat cycle, namely the P wave, QRS complex, and T wave, as individual Gaussian functions and combining them into a single mathematical expression, we were able to construct a smooth and accurate model of a complete heartbeat.

The results demonstrated that the Gaussian-based model successfully captured the overall morphology of cardiac waveforms, confirming that the bell-shaped nature of the Gaussian function closely matches the shape of real ECG components. This approach offers a simple yet powerful way to represent complex biological signals using a small number of interpretable parameters, namely the amplitude, position, and width of each wave.

From a biomedical engineering perspective, this model has potential applications in automated ECG analysis, cardiac abnormality detection, and synthetic signal generation for testing and research purposes. Future work could explore the extension of this model to abnormal ECG patterns such as arrhythmias, as well as the use of machine learning techniques to automatically estimate the optimal Gaussian parameters from real patient data.

In conclusion, the Gaussian function proves to be an effective and reliable mathematical framework for ECG signal modeling, bridging the gap between pure mathematics and clinical cardiology.

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