# Modelling sunlight exposure for my School's Secondary Building

**Number of Pages:** 22

## 1 Introduction:

School in peak summer feels great in the morning, however in the afternoon, when the sun's beams get through the classroom windows; the temperature and glare from the sunlight makes it quite a challenge for students near the window to focus in class. As a student, I noticed that classrooms on the shaded side of the building were always cooler and more pleasant to work in, especially during warm months. However ending up with such a classroom was challenging due to lack of research conducted in this field. This is what made me curious to research and base my exploration on the Suns trajectory for the month of May[1]. By doing so, my aim is to figure out which classrooms were least susceptible to the Sun during school hours for the month of May.

Due to my school being located in the high northern latitudes, the amount of sunlight exposure varies hugely throughout the year with 1-2 hours in December compared to 7-8 hours in the month of May[2]. This means some classrooms in peak summer get almost 8 hours of sunlight exposure contributing to overheating of classrooms which make the environment super humid and uncomfortable for students to study in. This also increases energy consumption as such classrooms often have electronic coolers such as fans.

In contrast to this, there are classrooms that are dim throughout school hours and rely on artificial lighting. Understanding Sunlight exposure and trajectory is not only relevant for student comfort but also reduces power consumption for lighting and cooling appliances. This investigation could also help our Climate action group to figure out which roof spaces will expose the Solar panels to the most sunlight during the month of May.

To track the path of the sun I require 3 key measurements: **solar position, hour angle** and **location coordinates**.

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[1] Being the only complete month in our school timetable that falls during the summer season and has the least amount of holidays making it relevant for me to track the whole month.

[2] Weather2Travel.com. "Weather2Travel.com." *Weather2Travel.com*, 2025, www.weather2travel.com/germany/berlin/climate/. Accessed 27 Aug. 2025.

**Solar position** is measured via two angles. These are Solar Altitude and Solar Azimuth. The Solar Altitude measures the angle of inclination of the Sun from the horizon. This means during Sunset and Sunrise, the Solar Altitude is approximately zero. The Solar Azimuth is the horizontal bearing (starting from the North) of the sun from the observer's point of view. These angles together track the movement of the Sun on the spherical coordinate system . Both these angles

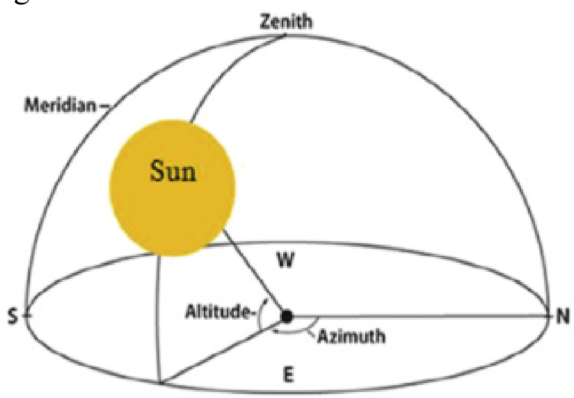


*Fig 1:* Solar Altitude and Azimuth[3]

are indicated in figure 1. The Sun's relative location in the sky is not constant from day to day at the same clock time, but the variation is relatively small over days compared to the variation during a single day. Hence through **Systematic Sampling technique**, **4 days (1st, 11th, 21st and 31st)** are chosen, with each day being *10 days* apart. This makes the data collection simpler with measurable variations between days.

The **hour angle** is the angle by which the sun moves away from the meridian. The meridian is

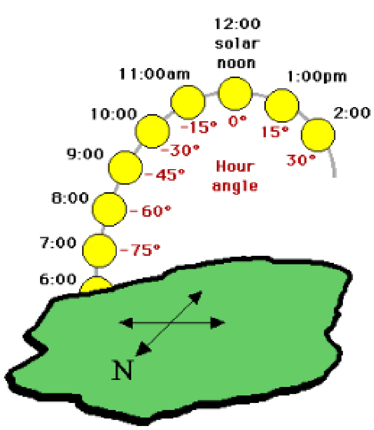


*Fig 2:* Hour Angle of the Sun[4]

the central longitude on which the hour angle is 0.0° and the sun is at its solar noon. Because the earth is rotating on its own axis with respect to time, the sun moves in a positive hour angle for hours beyond 12 pm and negative hour angle for hours before 12 pm noon. Based on the earth's rotation, there is a 15° angle change per hour . Represented by angle $H$ in the exploration.

**Location coordinates** of any place on earth can be traced via the *latitude* and *longitude*. The longitude acts

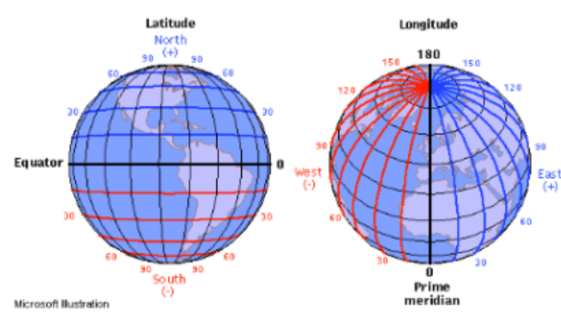


*Fig 3:* latitude and longitude system[5]

[3] Page, J. (2012). The Role of Solar-Radiation Climatology in the Design of Photovoltaic Systems. *Practical Handbook of Photovoltaics*, pp.573–643. doi:https://doi.org/10.1016/b978-0-12-385934-1.00017-9.
[4] Susdesign.com. (2025). *Sustainable By Design :: Hour Angle*. [online] Available at: https://www.susdesign.com/popups/sunangle/hour-angle.php.
[5] Journey North (2019). *Understanding Latitude and Longitude*. [online] Journeynorth.org. Available at: https://journeynorth.org/tm/LongitudeIntro.html.

as the *x*-coordinate angle east or west from the *Prime meridian* whereas the latitude works as the *y*-coordinate angle North or South from the Equator .

In our case, our Secondary building is roughly located at (52.47°N, 13.29°E).

Based on the orientation of the Secondary Building, "*the classrooms which have windows facing the south, receive the most sunlight"* (Indicated with a red line in Fig 4.) . This is made based on the fact that during the summer the Northern hemisphere tilts towards the sun and so instead of the sun traveling perfectly overhead it travels at a small angle tilted to the south. This can be visualised using Fig 4 where the classrooms facing the south are likely to be exposed to the sun much longer than the rest of the classrooms.

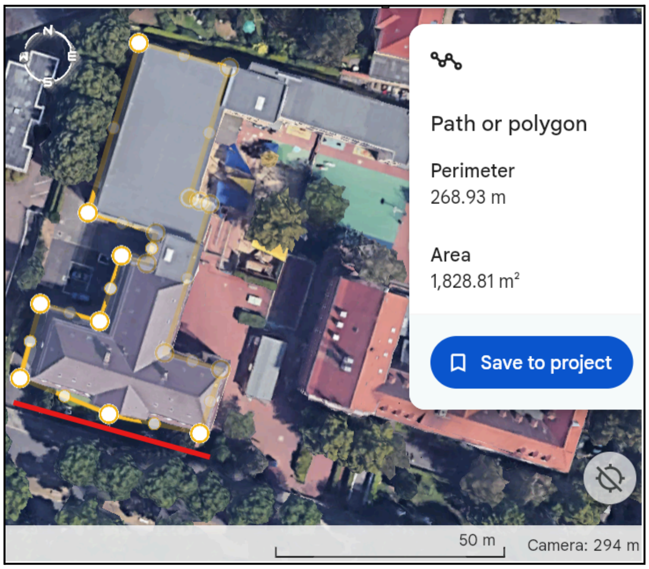


***Fig 4* :** Satellite view of the Secondary Building[6]

Instead of making the model of the secondary school from scratch, I used cadmapper[7] to get a 3D model with exact dimensions corresponding to the real building.The model was then modified by me where I added the roofs for the secondary building (T shaped building) as well as the Primary building (roughly soiree shaped). This model will act as the framework for the daylight system that will be simulated via the application 3ds max 2026[8].

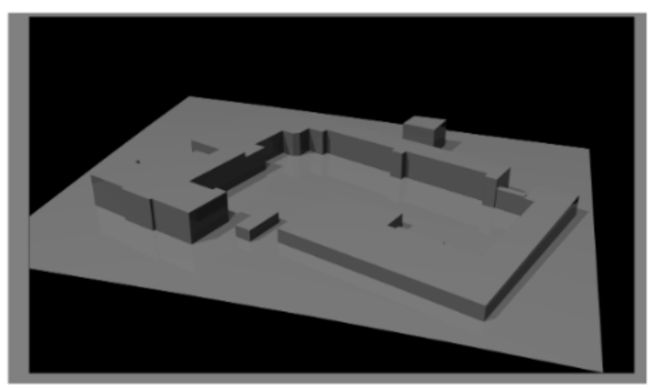

***Fig 5:*** Extraction of the 3D model of the Secondary Buildings

[6] Ghosh, A. (2025). *Secondary Building*.
[7] CadMapper (2020). *CADMAPPER - Worldwide map files for any design program*. [online] Cadmapper.com. Available at: https://cadmapper.com/pro/home.
[8] Autodesk.com. (2021). *3ds Max 2026 | Funktionen | Autodesk*. [online] Available at: https://www.autodesk.com/de/products/3ds-max/features [Accessed 24 Oct. 2025].

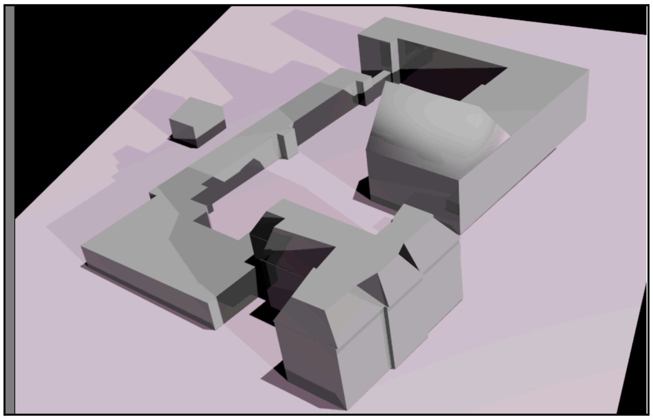

***Fig 6 :*** Final modified model with roof

The initial limitations with which I start the exploration are that I have excluded major obstructions from the school premises. This is done in order to simplify the shadows and have the sun effectively hit the building with no major obstructions (like trees, surrounding buildings that cast shadows) other than itself and the surrounding campus buildings. Other limitations include the fact that there is no atmosphere or clouds that might have reduced the exposure by making the sky partially opaque. There are also no reflections from the sunlight that occur in my calculations or simulation which might occur in real life from windows and other surfaces. All these limitations enable me to create a simplified model that calculates sunlight exposure on each facade of my Secondary School building.

My goal is to determine *how much* ($lmm^{-2}$or $lx$) and for *how long* ($mins$) Sunlight is exposed to the Southern, Eastern and Western facades of the Secondary building. Lux ($lx$) is the unit of measurement of Illuminance[9]. This is the amount of light that falls on a surface. One Lux is one lumen of light ($lm$) that is spread evenly over an area of one meter square ($m^2$). Lumen ($lm$) is the SI unit for luminous flux which measures how much light is emitted from a light source.

To achieve this aim I will, ***1)*** Determine the Solar Azimuth and Solar inclination for the 4 days ***2)*** Simulate the Heat exposure ***3)*** Take the average Illuminance by Percentage of the facade ***4)*** Graph the Illuminance against time ( $lmm^{-2}$ vs. $sec$ ) for the 3 facades ***5)*** Determine the time duration of sunlight exposure on each facade based on the graphs ***6)*** Determine the illuminance

[9] Wikipedia Contributors. "LUX." *Wikipedia*, Wikimedia Foundation, 28 Jan. 2019, en.wikipedia.org/wiki/LUX.

exposure ($lmm^{-2}$min) for each facade for the 4 days *7)* Conclude which classrooms are the most illuminated.

**2 Exploration:**

First, I would like to determine the change in solar position of the sun with respect to time ($t$) for a single day (May1st 2025). Since an average school day lasts from 8:30 to 16:30, these are the range of times I take with 30 minute increments.

The Earth does not rotate along the $Z$ axis but rather is tilted at an angle $CAI = 23.44°$ (the points are marked in white). The axis of rotation is represented by $\overline{ST}$, where $\overline{ST} \perp KLMN$. This creates the axial plane ($KLMN$) on which the earth rotates with the equator on the axial plane. The orbital plane ($RPQO$) is the plane on which the earth revolves around the Sun.

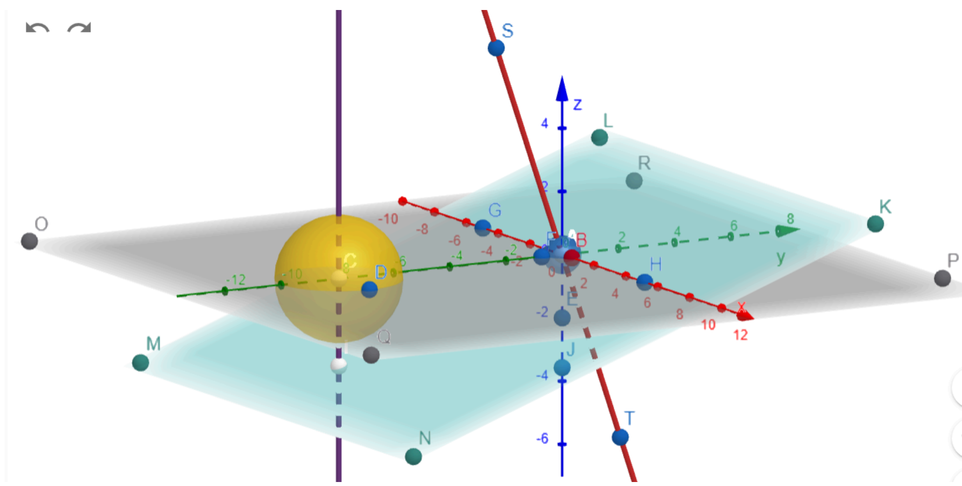


***Fig 7:*** Model of Sun and Earth on the Orbital plane (Grey) with the Axial plane of Earth (Tiel) using Geogebra[10]

Now imagine the earth rotated on the $Z$ axis and the Sun's centre is on the $RPQO$ plane, at a distance of 1u (u stands for unit) from the centre of Earth . Then the coordinates of the Sun's centre on the $XYZ$ system using the Hour angle ($\angle H$) is:

$$\vec{S} = \begin{pmatrix} X_s \\ Y_s \\ Z_s \end{pmatrix} = \begin{pmatrix} \cos(H) \\ \sin(H) \\ 0 \end{pmatrix} \begin{matrix} \text{This is the } x \text{ component as the hypothenuse is 1u} \\ \text{This is the } y \text{ component as the hypothenuse is 1u} \\ \text{As the Sun's centre in on the XY plane, Z=0} \end{matrix}$$

Now in Fig 7, the sun is directly over the Tropic of Cancer and from an observer's point of view

[10] Geogebra. "Calculator Suite - GeoGebra." *Www.geogebra.org*, 2025, www.geogebra.org/calculator.

the sun moves in a perfect arc over the observer's head as the day passes. This is because the axial plane intersects the orbital plane at point $F$. Now if I find the Sun's coordinates with the axial tilt I have to set up the diagram in such a way that all the $XYZ$ axes orient with the Axial plane. After the orientation, the solar declination is clearly seen as $\delta = \angle CAD = 23.44°$ (marked in white in Fig 8) because $\angle CAD$ is the earth's natural Axial tilt angle.

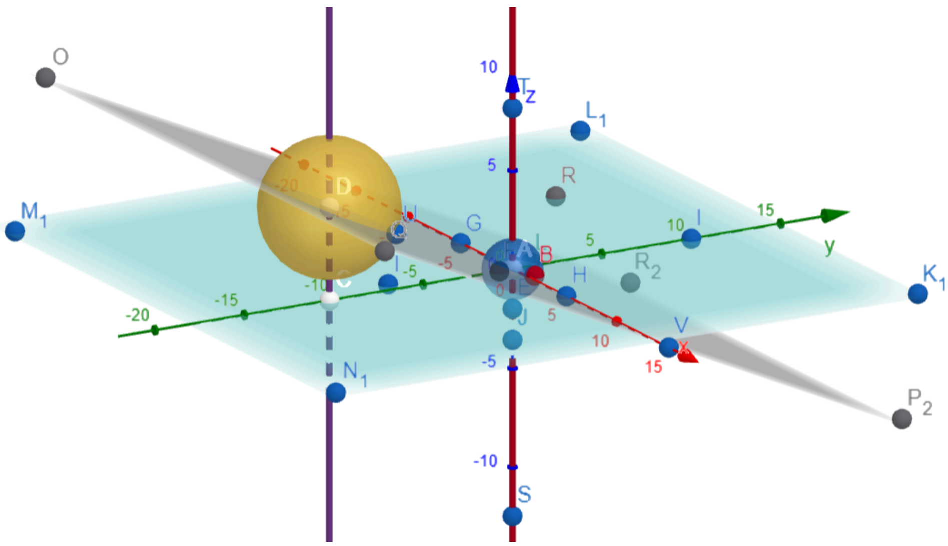


***Fig 8:*** The axis of rotation of the Earth ($\overline{ST}$) is rotated along the $X$ axis to overlap with the $Z$ axis

Now that I have modelled it, I sketched out on Fig 9 to better visualise how the XYZ components are obtained with consideration of the axial tilt of the Earth. From the figure I am trying to figure out the 3 dimensional coordinate of the sun (Point D) from the earth (Point A). This is represented as a unit position vector of length 1u.

$$\vec{S} = \begin{pmatrix} x_s \\ y_s \\ z_s \end{pmatrix} = \begin{pmatrix} \beta \\ \omega \\ h \end{pmatrix} \begin{matrix} \text{x component} \\ \text{y component} \\ \text{z component} \end{matrix}$$

$\beta$, $\omega$ and $h$ are used as notation to make Fig 9 look cleaner

$\angle DAB = \delta$ (Solar declination)

$\angle CAB = \angle ABO = H$ (Hour angle of the Sun)

$B$ is the reflection of the Sun on the XY plane

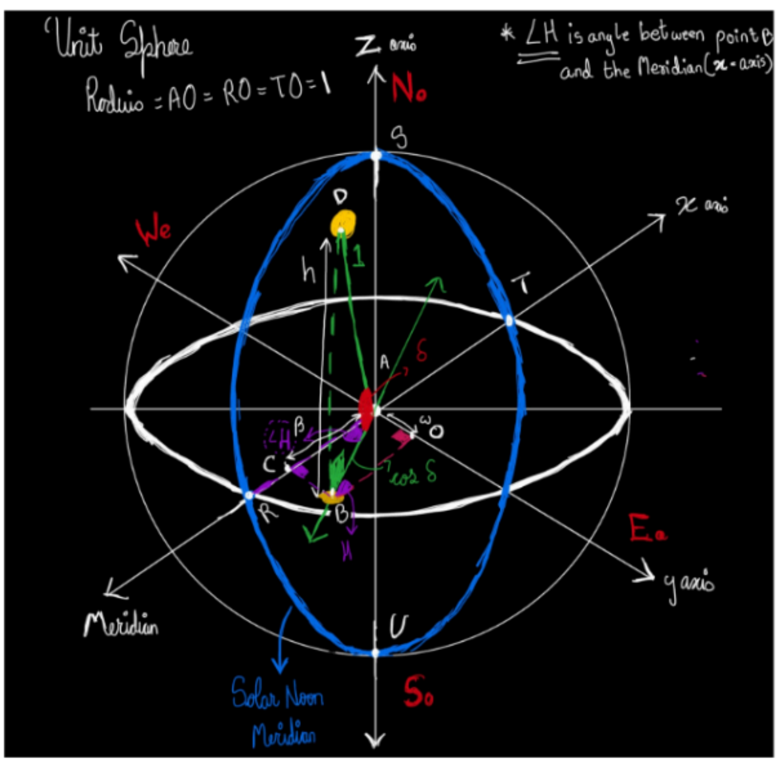


***Fig 9:*** Spherical Model for representing the Sun's Coordinate on the XYZ axes.

$\vec{S} = \begin{pmatrix} \beta \\ \omega \\ h \end{pmatrix}$, Now I derive them

To get $z_s$ it is simply the height or:

$$z_s = \sin(\delta) = \frac{h}{1} = h$$

To get $x_s$, I use point B:

$$\cos(H) = \frac{\beta}{\cos(\delta)} = \frac{x_s}{\cos(\delta)}$$

$$\therefore x_s = \cos(H) \cdot \cos(\delta)$$

To get $y_s$, I use the Alternate Angle identity

$\angle CAB = \angle ABO = H$ and hence, $\sin(H) = \dfrac{\omega}{\cos(\delta)} = \dfrac{y_s}{\cos(\delta)}$

$$\therefore y_s = \sin(H) \cdot \cos(\delta)$$

So the coordinates of the Sun can be represented via the Hour Angle $(H)$ and the Solar declination Angle $(\delta)$:

$$\boxed{\therefore \vec{S} = \begin{pmatrix} \cos(H) \cdot \cos(\delta) \\ \sin(H) \cdot \cos(\delta) \\ \sin(\delta) \end{pmatrix}}$$

This Unit vector formula only works if I the viewer am observing the sun from the equatorial plane (Axial plane $KLMN$) where $\phi = 0$ . But I am observing it at a non zero latitude $\phi$. I don't consider the longitude because the it is solely dependent upon the hour angle $(H)$. This means as shown in Fig 9 points $A$ and $B$ have parallel horizons. The Axial plane will have to be rotated by $(90 - \phi)$ in order to align the $Z$ and $X$ axes to the observer's $U$ and $NS$ axes. Alignment in here means making the axes parallel to the axes of the observer on the surface of Earth; the distance between the observer in the centre of the earth and an observer on the surface of the

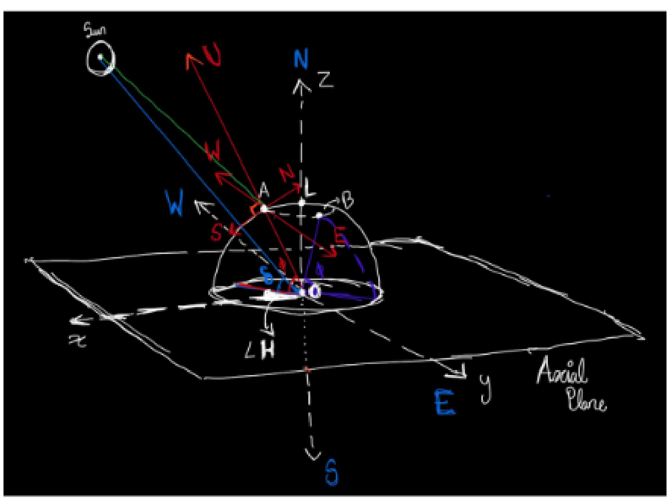

***Fig 10:*** Rotational Matrix is used to rotate the observer from O to A.

earth is irrelevant as 1 AU (Astronomical Unit≈ $149.6 \cdot 10^6$km) is significantly larger than the radius of our planet (6378.1 km) making the parallax error in the solar angles negligible for our calculations.

I cannot physically move the observer as this will break the unit vector between the Sun and the observer. Instead I rotate the whole coordinate system counterclockwise to overlap $L$ to $A$ . To do so the standard clockwise matrix rotational matrix[11] is used and then represented in the ENU format[12] as shown below:

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \text{Clockwise matrix rotation around the y axis}$$

$$R_y(90-\phi) = \begin{bmatrix} \cos(90-\phi) & 0 & -\sin(90-\phi) \\ 0 & 1 & 0 \\ \sin(90-\phi) & 0 & \cos(90-\phi) \end{bmatrix}, where \begin{matrix} \cos(90-\phi) = \sin(\phi) \\ \sin(90-\phi) = \cos(\phi) \end{matrix}$$

$$\therefore R_y(90-\phi) = \begin{bmatrix} \sin(\phi) & 0 & -\cos(\phi) \\ 0 & 1 & 0 \\ \cos(\phi) & 0 & \sin(\phi) \end{bmatrix}$$

So to get the new Solar vector $\left(\vec{S}_{ENU}\right)$ that matches the local horizon

we need to multiply $\vec{S}$ and $R_y(90-\phi)$. Because its a Matrix Multiplication order matters :

$$\vec{S}_{ENU} = R_y(90-\phi) \cdot \vec{S} = \begin{bmatrix} \sin(\phi) & 0 & -\cos(\phi) \\ 0 & 1 & 0 \\ \cos(\phi) & 0 & \sin(\phi) \end{bmatrix} \cdot \begin{bmatrix} \cos\delta\cos H \\ cos\delta\, sinH \\ \sin\delta \end{bmatrix}$$

Each coloum multiples to every component of the vector so,

$$\vec{S}_{ENU} = \begin{bmatrix} (\cos\delta\cos H)\cdot\sin(\phi) + (cos\delta\,\sin H)\cdot 0 + \sin\delta \cdot -\cos(\phi) \\ (\cos\delta\cos H)\cdot 0 + (cos\delta\,\sin H)\cdot 1 + \sin\delta\cdot 0 \\ (\cos\delta\cos H)\cdot\cos(\phi) + (cos\delta\,\sin H)\cdot 0 + \sin\delta\cdot\sin(\phi) \end{bmatrix}$$

$$\therefore \vec{S}_{ENU} = \begin{bmatrix} (\cos\delta\cos H)\cdot\sin(\phi) - \sin\delta\cdot\cos(\phi) \\ (cos\delta\,\sin H) \\ \sin\delta\cdot\sin(\phi) + (\cos\delta\cos H)\cdot\cos(\phi) \end{bmatrix}, \text{XY plane is flipped with respect to EN}$$

[11] CueMath (n.d.). *Rotation matrix - definition, formula, derivation, examples*. [online] Cuemath. Available at: https://www.cuemath.com/algebra/rotation-matrix/.

[12] "***The ENU (East-North-Up) format is a local, Cartesian coordinate system used for representing a position relative to a specific origin point on Earth's surface.***" Wikipedia. (2022). *Local tangent plane coordinates*. [online] Available at: https://en.wikipedia.org/wiki/Local_tangent_plane_coordinates.

The Latitude rotation only align the $Z$ axis to the latitude. Now as all points on the same latitude experience the same Sun movement along their local horizons at their own local time zones like $A$ and $B$ in Fig 9. But while their U (Up) are the same due to earth's rotation, the $XY$ planes vary. Meaning that the East-West directions of point $A$ are different to that of point $B$. From Fig 9 you can see that A is on the South-East quadrant. That's why instead of overlapping L to A, I overlap it to B which is longitudinally opposite to A at in the North-East quadrant. This constitutes to the Clockwise rotation. Now that I am on the correct quadrant, I switch the directions so that the North-South directions are now components of the $X$ axis and East-West directions are now components of the $Y$ axis. So the $S_{ENU}$ for observer is :

$$\boxed{\therefore \vec{S}_{ENU} = \begin{bmatrix} (cos\delta \, \sin H) \\ (\cos\delta \cos H) \cdot \sin(\phi) \; - \; \sin\delta \, \cdot \, \cos(\phi) \\ \sin\delta \, \cdot \, \sin(\phi) + (\cos\delta \cos H) \cdot \cos(\phi) \end{bmatrix}}$$

Now that I have gotten the observer in terms under the local ENU format. I can use the simple definition that the Solar Azimuth is the bearing from the North. So,

$\angle A$ is the Solar Azimuth

$$\tan(\angle A) = \frac{S_E}{S_N} = \frac{\cos\delta \sin H}{(\cos\delta \cdot \cos H) \cdot \sin\phi - \sin\delta \cdot \cos\phi} = \frac{\sin H}{\cos H \cdot \sin\phi - \tan(\delta) \cdot \cos\phi}$$

$$\boxed{\therefore \angle A = \arctan\left(\frac{\sin H}{\cos H \cdot \sin\phi - \tan(\delta) \cdot \cos\phi}\right)} \qquad \text{Formula 1}$$

$$\text{Now in this expression} \begin{Bmatrix} H < 0^\circ \text{ if sun is east of local meridian} \\ H > 0^\circ \text{ if sun is west of local meridian} \end{Bmatrix}$$

The Solar altitude is expressed as a relationship due to the alignment of the coordinate system.

$\angle h$ is the Solar Altitude angle between the Observers plane and the Sun

$$\boxed{\therefore \angle h = \arcsin(S_U) = \arcsin(\cos\delta \cos H \cos\phi \; + \; \sin\delta \sin\phi)} \qquad \text{Formula 2}$$

Before I calculate the Solar angles, I need to find the Solar declination (δ) for the 4 days.

To do so I use the approximate Solar declination formula[13] :

$$\delta = 23.44^\circ \times sin(\frac{360}{365} \times (n + 284))$$

| Date | Day of Year (n) | Solar Declination (δ) |
|---|---|---|
| May 1, 2025 | 121 | +14.94° |
| May 11, 2025 | 131 | +17.78° |
| May 21, 2025 | 141 | +20.17° |
| May 31, 2025 | 151 | +21.91° |

***Table 1:*** Solar declination Angle (δ)

Now that I have gotten δ, I use the derived formulas 1 and 2, to find the Solar azimuths (in degrees) and Solar altitudes (in degrees) of the Sun for the Secondary building for May 1st, 11th, 21st and 31st. To get the Hour Angle ($H$), the formula $H = 15° \cdot (T - 12)$ [14] is used, where $T$ is time represented as a decimal value. Like for time 13:30, $T = 13.5$.

| Local Time | Hour Angle | May 1st 2025 | | May 11th 2025 | | May 21st 2025 | | May 31st 2025 | |
|---|---|---|---|---|---|---|---|---|---|
| | | Solar Altitude | Azimuth | Solar Altitude | Azimuth | Solar Altitude | Azimuth | Solar Altitude | Azimuth |
| **8:30** | **-52.5°** | 24.75 | 97.98 | 27.09 | 96.16 | 28.88 | 94.43 | 30.09 | 92.88 |
| **9:00** | **-45.0°** | 29.23 | 104.38 | 31.60 | 102.53 | 33.41 | 100.74 | 34.63 | 99.13 |
| **9:30** | **-37.5°** | 33.59 | 111.17 | 36.00 | 109.31 | 37.85 | 107.49 | 39.10 | 105.82 |
| **10:00** | **-30.0°** | 37.74 | 118.50 | 40.21 | 116.66 | 42.11 | 114.82 | 43.40 | 113.10 |
| **10:30** | **-22.5°** | 41.60 | 126.48 | 44.14 | 124.72 | 46.11 | 122.90 | 47.47 | 121.17 |
| **11:00** | **-15.0°** | 45.06 | 135.25 | 47.69 | 133.65 | 49.75 | 131.92 | 51.18 | 130.22 |
| **11:30** | **-07.5°** | 48.00 | 144.89 | 50.71 | 143.57 | 52.87 | 142.04 | 54.40 | 140.45 |
| **12:00** | **00.0°** | 50.28 | 155.42 | 53.07 | 154.51 | 55.32 | 153.33 | 56.95 | 151.96 |
| **12:30** | **07.5°** | 51.77 | 166.72 | 54.61 | 166.36 | 56.92 | 165.65 | 58.64 | 164.65 |
| **13:00** | **15.0°** | 52.36 | 178.49 | 55.21 | 178.76 | 57.55 | 178.64 | 59.33 | 178.12 |
| **13:30** | **22.5°** | 52.02 | 190.31 | 54.81 | 191.22 | 57.15 | 191.70 | 58.94 | 191.69 |
| **14:00** | **30.0°** | 50.76 | 201.77 | 53.46 | 203.21 | 55.74 | 204.21 | 57.52 | 204.66 |
| **14:30** | **37.5°** | 48.68 | 212.52 | 51.27 | 214.36 | 53.46 | 215.73 | 55.19 | 216.52 |
| **15:00** | **45.0°** | 45.91 | 222.41 | 48.37 | 224.49 | 50.47 | 226.10 | 52.15 | 227.11 |
| **15:30** | **52.5°** | 42.58 | 231.40 | 44.92 | 233.62 | 46.94 | 235.35 | 48.56 | 236.47 |
| **16:00** | **60.0°** | 38.82 | 239.59 | 41.06 | 241.85 | 43.00 | 243.62 | 44.58 | 244.79 |
| **16:30** | **67.5°** | 34.74 | 247.07 | 36.91 | 249.33 | 38.79 | 251.10 | 40.33 | 252.27 |

***Table 2:*** Solar Azimuth, Solar Altitude and Hour angle for the 4 sample days in May 2025

[13] *I use "approximate" as earth's orbit is not perfect and is elliptical. The formula approximates the orbit as a perfect circular orbit with a cyclic pattern similar to a sine wave with a phase change of n+284 to match the March Equinoxes at n=81.* Susdesign.com. (2024). *Sustainable By Design :: Declination*. [online] Available at: https://www.susdesign.com/popups/sunangle/declination.php.
[14] This formula is derived from the Introduction section where each hour corresponds to a 15° change in the Sun's bearing.

The calculated Solar azimuth and Solar inclination angles are used with adjusted latitudinal and longitudinal for my Secondary building and are imputed into the simulation software. Fig 10 shows the Sun's trajectory for the 1st of May. The trajectory is slightly tilted towards the south which qualitatively supports our hypothesis. The time scales taken were beyond the school hours in order to better visual the trajectory. These were later adjusted to the duration of our average school day from 8:30-16:30.

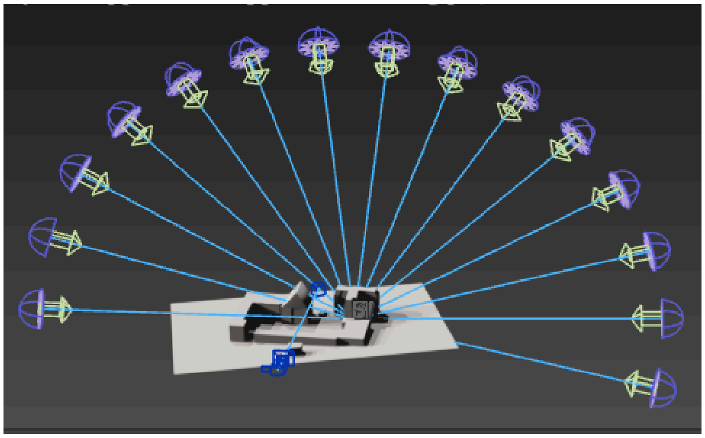

**Fig 11:** Hourly Sun Tracking for May 1st 2025 from 6:00-20:00

Fig 11 shows the shadow propagation of the whole school campus as the sun moves from the east (right hand side) to the west (left hand side). Because there is no shadow on the southern side of the building. Virtually no sunlight is directly exposed on the northern side of our building. Now that the Sunlight system is working I can start the Sunlight exposure simulation.

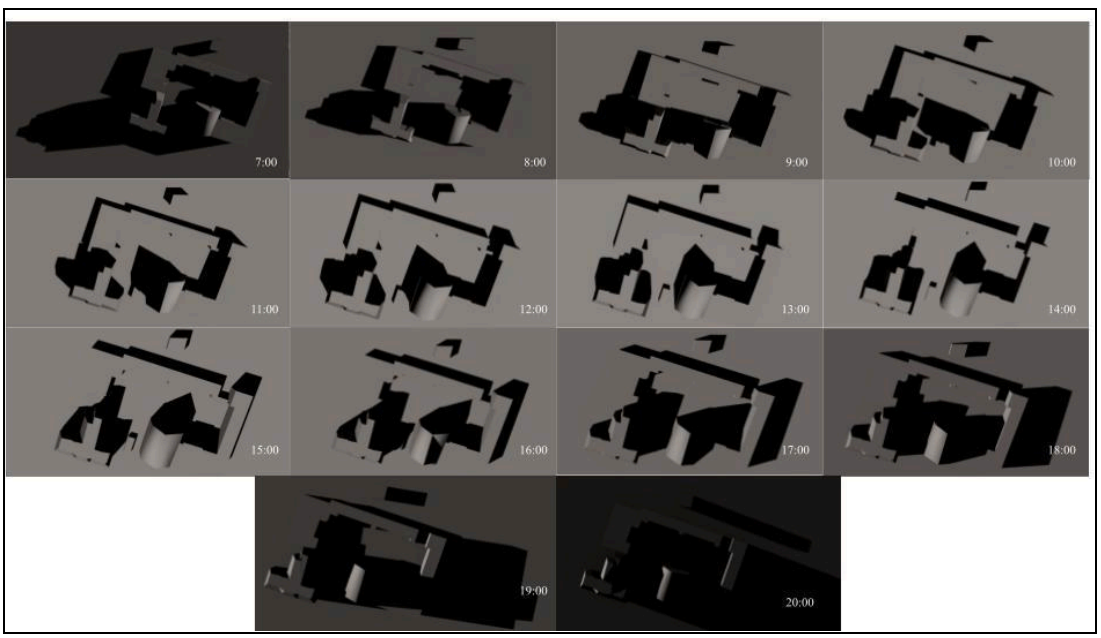


**Fig 12:** Shadow Study for May 1st 2025 for the School Building

I set up three virtual cameras as shown in Fig 12, in order to have uniform frames for which the percentage exposure will be calculated for each facade: South, East and West. As established

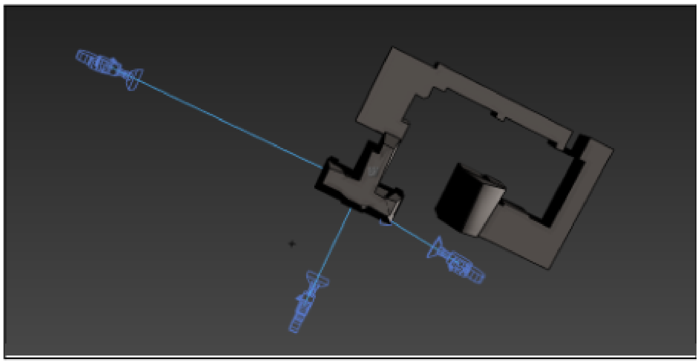

*Fig 13:* 3 virtual camera (in blue) set up for uniform frame analysis

before the northern face relieves virtually no direct sunlight and shall hence be excluded from this investigation.

I change my simulation environment to a Pseudocolour mode which puts the exposure from a greyscale to a multicolour scale on the colour spectrum as shown in Fig 14, 15 & 16.

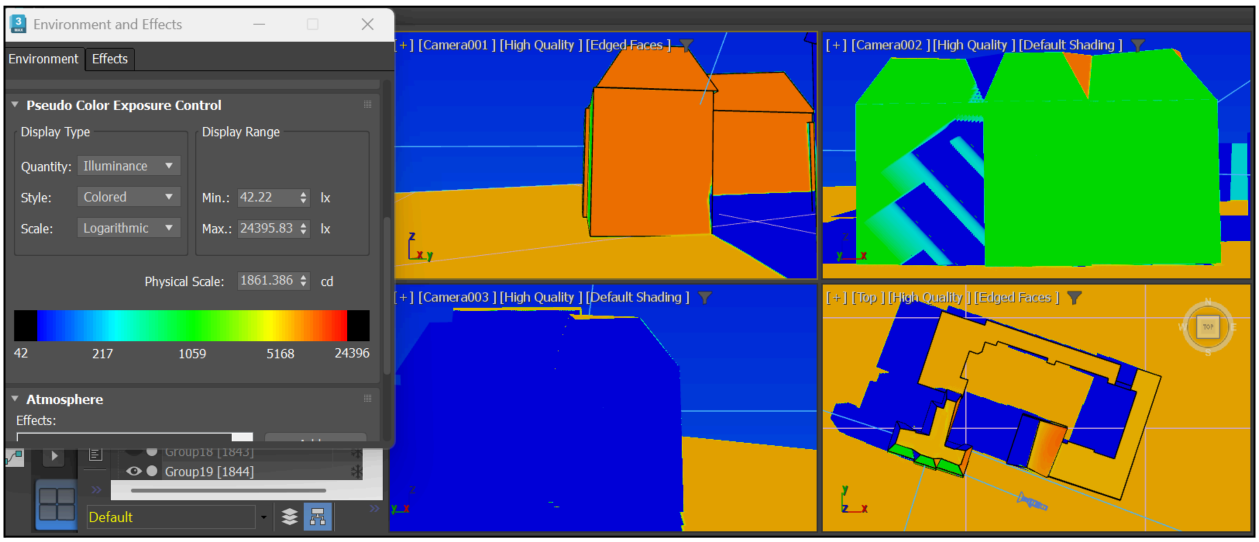


*Fig 14:* Pseudocolour Exposure for May 11th at 08:30

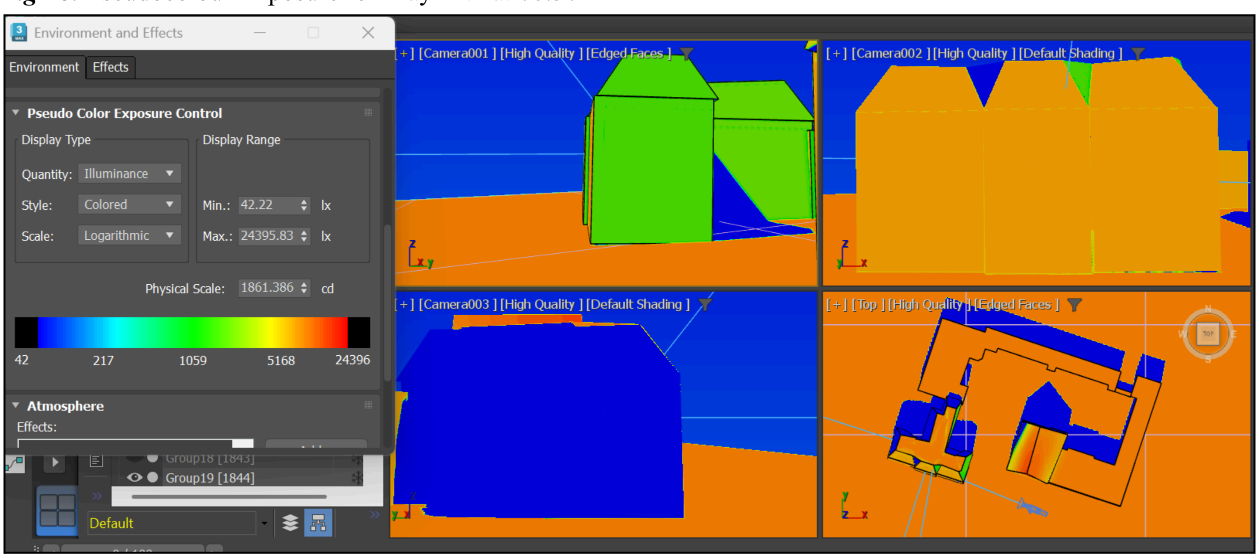


*Fig 15:* Pseudocolour Exposure for May 11th at 12:30

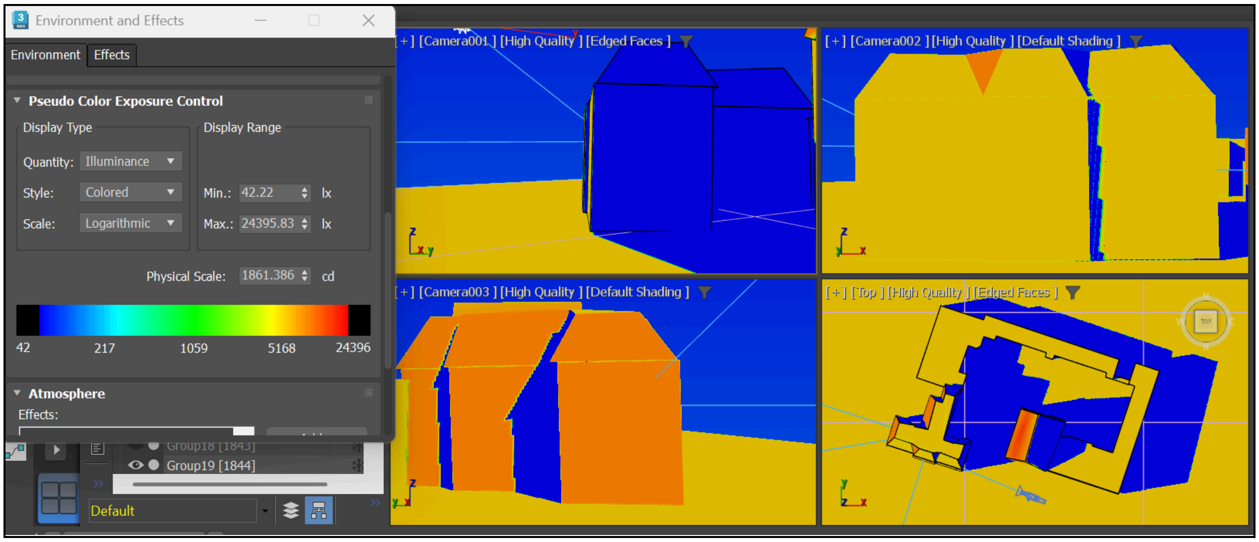


***Fig 16:*** Pseudocolour Exposure for May 11th at 16:30

These pseudocolour images show how much sunlight each facade of the building receives at a specific date and time, with warmer colours (yellow–red) representing higher illumination and cooler colours (blue–green) representing lower illumination. By comparing the four views, I can clearly see which sides of the building are exposed to direct sunlight and which remain in shadow, allowing me to visualise sunlight distribution.

The first step to the calculation of the percentage illuminance on the facade is to remove the ground. This is because the ground will interfere with the percentage colour and the true exposure will hence be inaccurate due to the extra exposure contribution from the ground. I use a website[15] to find the percentage of dominant colour in each view (South, East and West) multiply it by its respective lux ($lmm^{-2}$) using Fig 17, excluding the black colour as it is not part of the facade and divide the total by $(100\% - Black\%)$.

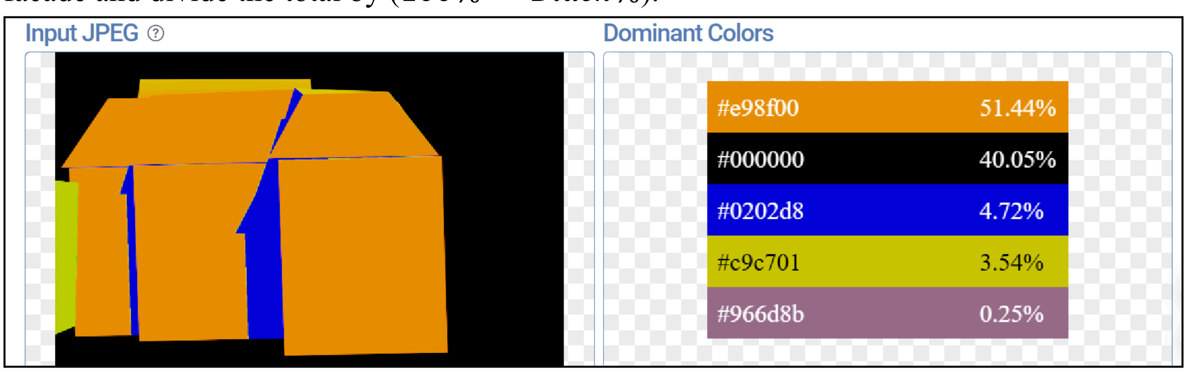


***Fig 17:*** Colour percentage extraction for West Facade (Camera 003) for May 11th at 14:30

[15] onlinejpgtools.com. (n.d.). *Find Dominant JPG Colors*. [online] Available at: https://onlinejpgtools.com/find-dominant-jpg-colors.

Fig 17 shows an example of how the colour percent is extracted.

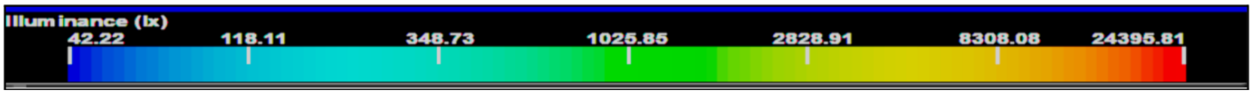


***Fig 18:*** Illuminance Spectrum Used to compare the HEX codes

I do an example calculation using percentages from Fig 17.

$$I_{avg} = \frac{(51.44\% \cdot 15891.29)+(4.72\% \cdot 42.22)+(3.54\% \cdot 2629)+(0.25\% \cdot 113)}{(100\%-40.05\%)} = 13794.53lx \approx 13790lx$$

Using the same calculation process I calculate the average illuminance for all three facades for times (8:30-16:30) for the 4 days. All values are measured in lux (lx). Table 3 below, shows that:

| Local time | May 1st 2025 | | | May 11th 2025 | | | May 21st 2025 | | | May 31st 2025 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | East | South | West | East | South | West | East | South | West | East | South | West |
| 8:30 | 17530 | 924 | 68 | 18180 | 786 | 64 | 18330 | 795 | 58 | 18550 | 800 | 49 |
| 9:00 | 17380 | 2029 | 157 | 17490 | 2155 | 149 | 17570 | 1868 | 99 | 17790 | 1881 | 139 |
| 9:30 | 16790 | 3458 | 1778 | 16900 | 3144 | 1744 | 17020 | 3149 | 1384 | 17300 | 3171 | 529 |
| 10:00 | 15630 | 8132 | 2904 | 16020 | 6510 | 2194 | 16480 | 6854 | 1292 | 16720 | 6883 | 1246 |
| 10:30 | 14900 | 9024 | 3126 | 15140 | 8523 | 3322 | 15650 | 9476 | 2985 | 15980 | 9503 | 4124 |
| 11:00 | 10860 | 10910 | 3780 | 12350 | 11570 | 3646 | 12870 | 12380 | 3157 | 13100 | 12420 | 7890 |
| 11:30 | 7334 | 12270 | 4201 | 10560 | 11980 | 3726 | 11010 | 12690 | 3248 | 11220 | 12710 | 12020 |
| 12:00 | 6722 | 12900 | 4954 | 8932 | 13040 | 3886 | 10390 | 13790 | 3479 | 10620 | 13820 | 15330 |
| 12:30 | 5809 | 18720 | 10230 | 7144 | 17840 | 3974 | 8983 | 18260 | 3507 | 9236 | 18300 | 17290 |
| 13:00 | 1502 | 18830 | 11550 | 1831 | 18330 | 8266 | 2312 | 19050 | 7754 | 2370 | 19090 | 14510 |
| 13:30 | 298 | 17890 | 12230 | 302 | 18010 | 1067 | 994 | 18120 | 9643 | 1011 | 18150 | 11020 |
| 14:00 | 297 | 16570 | 13450 | 294 | 16530 | 12280 | 318 | 17090 | 11840 | 326 | 17130 | 12630 |
| 14:30 | 289 | 16430 | 14230 | 293 | 16140 | 13790 | 308 | 16440 | 13340 | 315 | 16490 | 14200 |
| 15:00 | 286 | 15680 | 15840 | 294 | 15750 | 15060 | 306 | 16270 | 14760 | 309 | 16320 | 15870 |
| 15:30 | 278 | 13890 | 16210 | 282 | 14890 | 15880 | 305 | 15350 | 15750 | 307 | 15410 | 16520 |
| 16:00 | 277 | 13360 | 17300 | 281 | 13990 | 16990 | 311 | 14770 | 16850 | 312 | 14810 | 17050 |
| 16:30 | 277 | 12840 | 18120 | 281 | 13260 | 17910 | 306 | 13540 | 17630 | 311 | 13580 | 17350 |

***Table 3:*** Average Lumex for Each Facade for the 4 days

My dependent variable is **the illuminance (lx) on each facade** which is dependent on the variable of **time (mins)**. Therefore for my graphs, I plot Time (mins) vs. Illuminance (lx).

Now I graph these points using Geogebra[16]. Geogebra does not limit our polynomial fit, but

[16] www.geogebra.org. (n.d.). *GeoGebra Classic - GeoGebra*. [online] Available at: https://www.geogebra.org/classic/.

Polynomials with $x^{10}$ have their correlation coefficient between $0.985 < R^2 < 0.99$ which is reasonably perfect for the extent of this exploration while keeping the integration doable.

While this is a limitation as all the points are not taken into account it enables for a doable Integration as having a higher polynomial would have made the process of integration much more tedious and unnecessary for the accuracy of this exploration. All the Polynomial functions in degree 10 are presented in the form:

$$f(x) = \sum_{k=0}^{10} a_k x^k \quad where\ a_k \in \mathbb{R},\ and\ a_k \neq 0$$

Now all the coefficient and the y-intercept will only be represented in 3 significant figures to keep the calculations clean and simple. The equations are presented later in table 4. I will only integrate the Polynomial functions for the time frame I provided and find the area under the curve or total illuminance for that school day (8:30-16:30). The graphs of the 4 days are shown below with the 3 facades. **Colour grey represents the eastern facade, green for the southern facade and red for the western facade.** Each facade is seen to have a local maximum within the constrained timeframe. I converted the time to minutes from (Hr:Min) because this enables for the time to be in a single unit and hence the polynomial fit is easier to be derived by Geogebra.

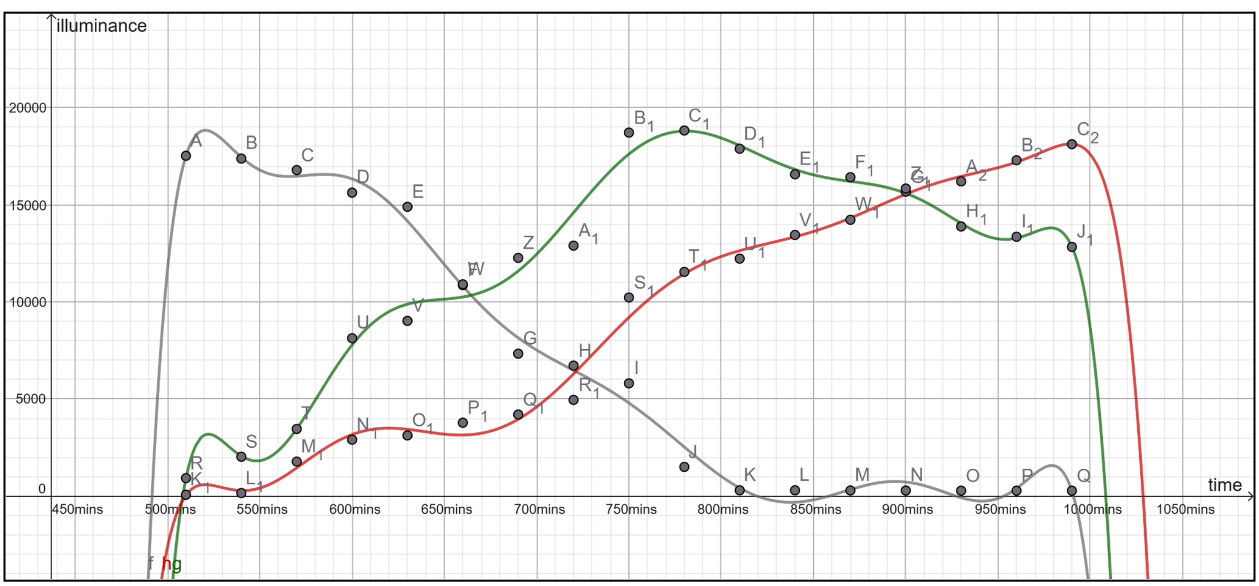


***Fig 19:*** Time (mins) vs Illuminance (lx) for 1st of May 2025

The Southern facade's exposure intersects with both the Eastern and Western facades, indicating moments when their solar exposure is equal, but the Eastern and Western facades intersect at a much lower exposure level, showing their peak exposures occur at completely different times of the day.

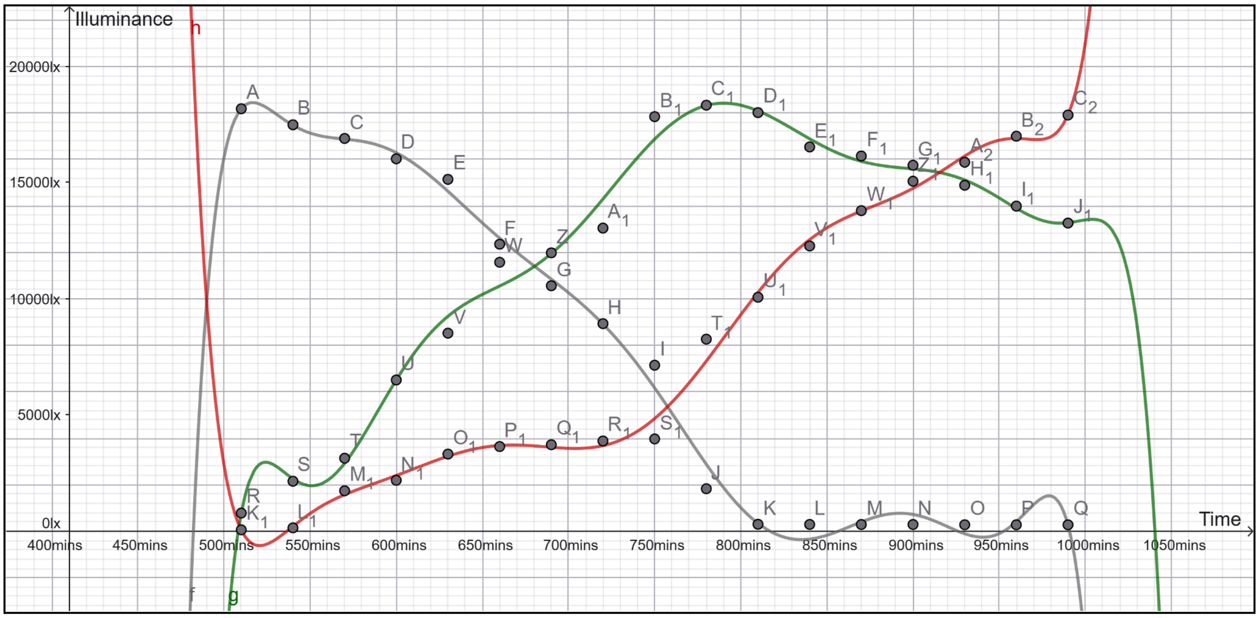


***Fig 20:*** Time (mins) vs Illuminance (lx) for 11th of  May 2025

The overall duration of light is slightly extended compared to the 1st of May, though the time of maximum intensity remains unchanged. The steady mid day period shown by the green curve is less prominent than the one observed on May 1st.

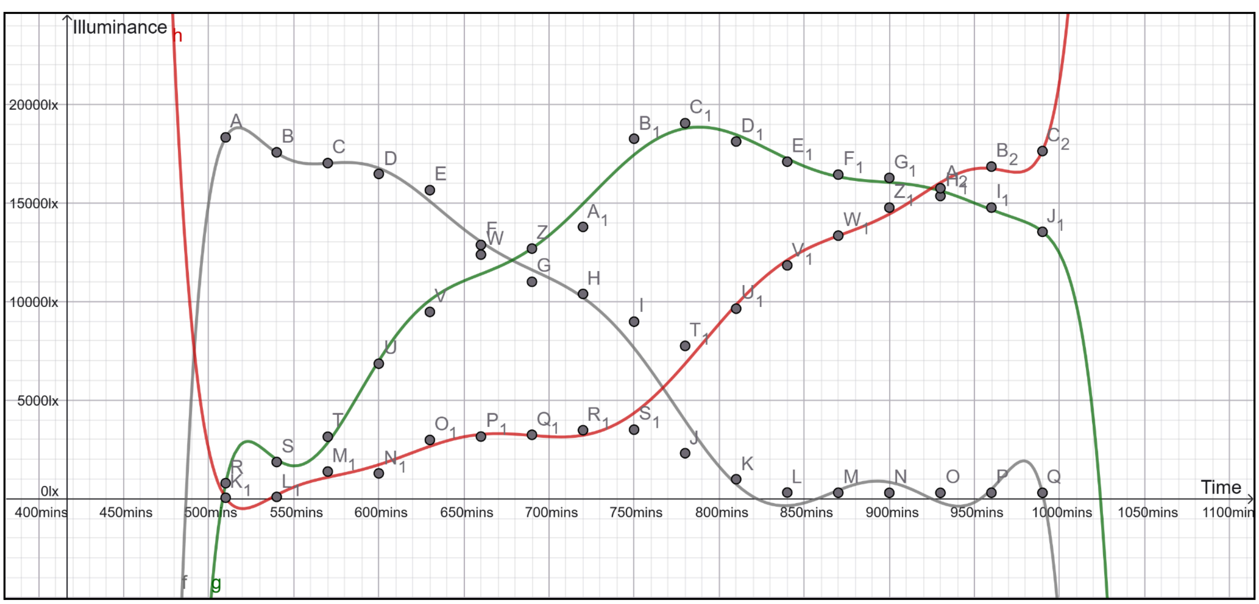


***Fig 21:*** Time (mins) vs Illuminance (lx) for 21st of  May 2025

The duration of the light period is consistent with the middle of the month, showing no further significant change in daylight hours.

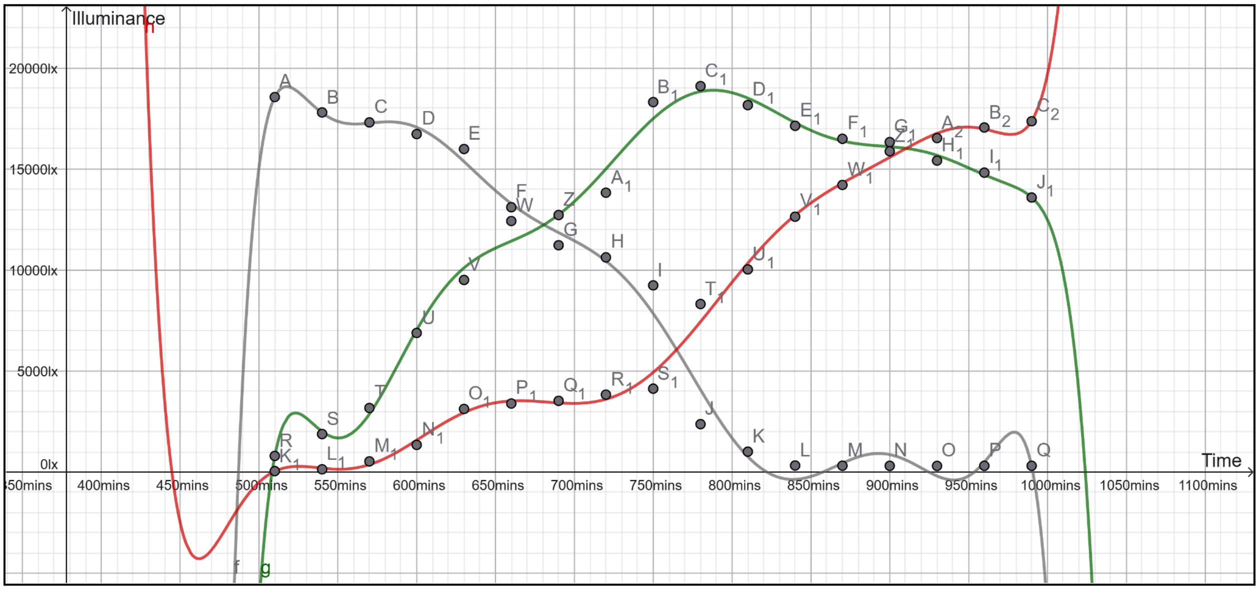


***Fig 22:*** Time (mins) vs Illuminance (lx) for 31st of  May 2025

This final graph maintains the consistent domain and the invariant time of the global maximum observed in late May. Now the area under the curve will determine the aggregate illuminance received for each facade and how it varies for the month of May 2025.

Before I jump into finding the area under the curve I want to analyse the duration of exposure. From qualitatively analysing the 4 graphs, I can see each facade has its own local maxima at the three extremes of the time frame. This logically makes sense as the sun rising hits the east facade with direct sunlight for the morning hours. Because the sun's trajectory is at a southern declination, the Southern facade receives the highest exposure amounts. The western facade is a reflection of the eastern facade's exposure roughly at midday $x$=750 (12:30). The eastern facade seems to only receive  measurable sunlight exposure until $x$=810 (13:30) after which it starts to plateau at around 310-280 lx. The Western facade only receives direct sunlight from about $x$=780 (13:00) which fits in with the previous time as the sun cannot expose the east and west simultaneously.

| Day | Facade | Polynomial fit |
|---|---|---|
| | East | $M_{E1}(x)$ =$-6.08\times10^{-19}x^{10} - 4.56\times10^{-15}x^{9} - 1.53\times10^{-11}x^{8} - 3.03\times10^{-8}x^{7} - 3.91\times10^{-5}x^{6} + 0.0344x^{5} - 20.9x^{4} + 8660x^{3} - 2.34\times10^{6}x^{2} + 3.72\times10^{8}x - 2.64\times10^{10}$ |
| | West | $M_{W1}(x)$ =$-2.77\times10^{-19}x^{10} - 2.11\times10^{-15}x^{9} - 7.18\times10^{-12}x^{8} - 1.44\times10^{-8}x^{7} - 1.89\times10^{-5}x^{6} + 0.0168x^{5} - 10.3x^{4} + 4322x^{3} - 1.18\times10^{6}x^{2} + 1.89\times10^{8}x - 1.36\times10^{10}$ |
| May 1st | South | $M_{S1}(x)$ =$-7.07\times10^{-19}x^{10} + 5.36\times10^{-15}x^{9} - 1.82\times10^{-11}x^{8} + 3.63\times10^{-8}x^{7} - 4.73\times10^{-5}x^{6} + 0.0420x^{5} - 25.8x^{4} + 1.08\times10^{4}x^{3} - 2.93\times10^{6}x^{2} + 4.70\times10^{8}x - 3.37\times10^{10}$ |
| | East | $M_{E11}(x)$ =$-4.26\times10^{-19}x^{10} - 3.16\times10^{-15}x^{9} - 1.05\times10^{-11}x^{8} - 2.05\times10^{-8}x^{7} - 2.62\times10^{-5}x^{6} + 0.0228x^{5} - 13.7x^{4} + 5584x^{3} - 1.49\times10^{6}x^{2} + 2.34\times10^{8}x - 1.64\times10^{10}$ |
| | West | $M_{W11}(x)$ =$-2.97\times10^{-19}x^{10} - 2.22\times10^{-15}x^{9} + 7.41\times10^{-12}x^{8} - 1.46\times10^{-8}x^{7} + 1.87\times10^{-5}x^{6} - 0.0164x^{5} + 9.91x^{4} - 4078x^{3} + 1.09\times10^{6}x^{2} - 1.73\times10^{8}x + 1.22\times10^{10}$ |
| May 11th | South | $M_{S11}(x)$ =$-3.14\times10^{-19}x^{10} - 2.43\times10^{-15}x^{9} - 8.39\times10^{-12}x^{8} - 1.71\times10^{-8}x^{7} - 2.27\times10^{-5}x^{6} + 0.0205x^{5} - 12.8x^{4} + 5428x^{3} - 1.50\times10^{6}x^{2} + 2.45\times10^{8}x - 1.78\times10^{10}$ |
| | East | $M_{E21}(x)$ =$-6.04\times10^{-19}x^{10} - 4.50\times10^{-15}x^{9} - 1.50\times10^{-11}x^{8} - 2.93\times10^{-8}x^{7} - 3.75\times10^{-5}x^{6} + 0.0327x^{5} - 19.7x^{4} + 8066x^{3} - 2.16\times10^{6}x^{2} + 3.39\times10^{8}x - 2.39\times10^{10}$ |
| | West | $M_{W21}(x)$ =$-3.30\times10^{-19}x^{10} - 2.46\times10^{-15}x^{9} + 8.21\times10^{-12}x^{8} - 1.61\times10^{-8}x^{7} + 2.07\times10^{-5}x^{6} - 0.0180x^{5} + 10.9x^{4} - 4464x^{3} + 1.20\times10^{6}x^{2} - 1.88\times10^{8}x + 1.33\times10^{10}$ |
| May 21st | South | $M_{S21}(x)$ =$-3.74\times10^{-19}x^{10} - 2.88\times10^{-15}x^{9} - 9.92\times10^{-12}x^{8} - 2.01\times10^{-8}x^{7} - 2.66\times10^{-5}x^{6} + 0.0239x^{5} - 14.9x^{4} + 6303x^{3} - 1.74\times10^{6}x^{2} - 2.83\times10^{8}x - 2.06\times10^{10}$ |
| | East | $M_{E31}(x)$ =$6.25\times10^{-19}x^{10} - 4.66\times10^{-15}x^{9} - 1.55\times10^{-11}x^{8} - 3.04\times10^{-8}x^{7} - 3.89\times10^{-5}x^{6} + 0.0339x^{5} - 20.4x^{4} + 8359x^{3} - 2.23\times10^{6}x^{2} + 3.52\times10^{8}x - 2.48\times10^{10}$ |
| | West | $M_{W31}(x)$ =$-1.48\times10^{-19}x^{10} - 1.08\times10^{-15}x^{9} + 3.54\times10^{-12}x^{8} - 6.79\times10^{-9}x^{7} + 8.48\times10^{-6}x^{6} - 0.00721x^{5} + 4.21x^{4} - 1675x^{3} + 4.33\times10^{5}x^{2} - 6.57\times10^{7}x + 4.45\times10^{9}$ |
| May 31st | South | $M_{S31}(x)$ =$-3.76\times10^{-19}x^{10} - 2.90\times10^{-15}x^{9} - 9.97\times10^{-12}x^{8} - 2.02\times10^{-8}x^{7} - 2.67\times10^{-5}x^{6} + 0.0241x^{5} - 14.96x^{4} + 6334x^{3} - 1.75\times10^{6}x^{2} - 2.84\times10^{8}x - 2.06\times10^{10}$ |

***Table 4:*** Line of best fit for each Facade for the 4 days

Each facade has a line of best fit with polynomials of power 10. If I take the general structure, integrate that and use the same process for all the 12 polynomial equations.

$$\int \left( \sum_{k=0}^{10} a_k x^k \right) \cdot dx = F(x)$$

where $F(x) = \sum_{k=0}^{10} \frac{a_k x^{k+1}}{k+1} + C$

I need a definite integral between points 510 mins and 990 mins. I use the simple rule:

$$\int_a^b f(x)dx = F(b) - F(a)$$

$f(x)$ is the polynomial function fit
where $a$ is the lower bound at 510 (8:30)
and $b$ is the upper bound at 990 (16:30)

$$\boxed{\therefore \int_{510}^{990} f(x)dx = F(990) - F(510)}$$

Using this rule, I get the total exposure for each facade between 8:30 and 16:30. Below is a sample calculation for the eastern facade for May 1st (Shown graphically in Fig 23):

$$\text{Here } f(x) = M_{E1}(x)$$

$$\int_{510}^{990} f(x)dx = F(990) - F(510) = \sum_{k=0}^{10} \frac{a_k(990)^{k+1}}{k+1} - \sum_{k=0}^{10} \frac{a_k(510)^{k+1}}{k+1} = -1698261189776 - (-1698264454096)$$

$$= 3264320\, lx \text{ mins} \approx \boxed{3264000\, lx \text{ mins}}$$

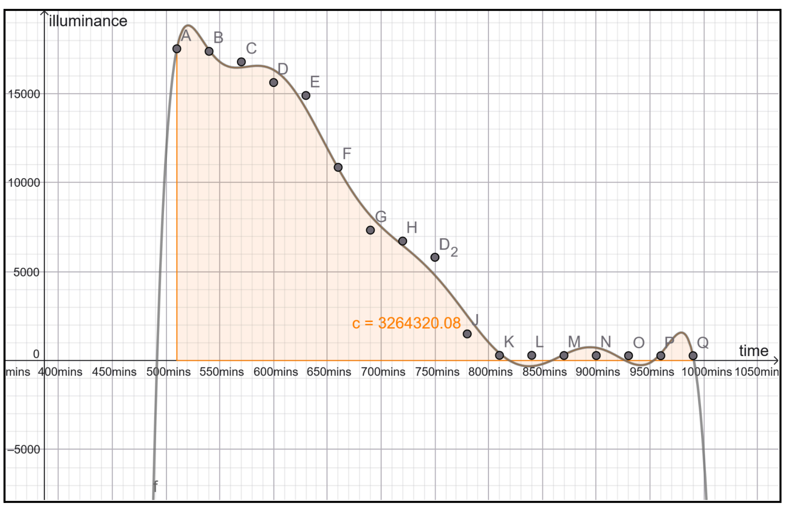


*Fig 23:* Total illuminance for May 1st Eastern Facade

The unit of lx min is used to find the total exposure of illuminance in each facade. Exposure in photography is the total amount of light that reaches a photographic film or sensor[17]. It depends on both the brightness of the light (lux) and the duration of exposure. In my exploration, this concept applies to sunlight exposure, where total light received by a facade over time can be measured in lux·minutes to compare how much sunlight different facades receive.

[17] Wikipedia Contributors (2025). *Exposure (photography)*. [online] Wikipedia. Available at:https://en.wikipedia.org/wiki/Exposure_(photography)#Photometric_and_radiometric_exposure

In the same process I find the total exposure for all the other facades. The total Illuminance Exposure (lx min) is presented in the table below rounded to 4 significant figures:

| Day | Facade | Total Illuminance (lx min) | % of total Illuminance in a Day | Total Illuminance for the Day (lx min) |
|---|---|---|---|---|
| May 1st | East | 3264000 | 24.26% | ***13450000*** |
| | West | 4239000 | 31.50% | |
| | South | 5952000 | 44.24% | |
| May 11th | East | 3543000 | 26.99% | ***13130000*** |
| | West | 3698000 | 28.17% | |
| | South | 5888000 | 44.84% | |
| May 21st | East | 3755000 | 28.64% | ***13030000*** |
| | West | 3158000 | 24.76% | |
| | South | 6112000 | 46.60% | |
| May 31st | East | 3823000 | 28.01% | ***13650000*** |
| | West | 3694000 | 27.07% | |
| | South | 6129000 | 44.92% | |
| Total | East | ***14390000*** | ***27.01%*** | ***53250000*** |
| | West | ***14790000*** | ***27.77%*** | |
| | South | ***24080000*** | ***45.22%*** | |

***Table 5:*** Total illuminance (lx min) for each facade for all 4 days

This brings me to the end of my calculations. Now that I have calculated the total sunlight exposure for all the 3 facades. I will discuss the outcome of this exploration in the final section.

**3 Conclusion:**

In conclusion, the Southern facade receives the most sunlight for the month of May with a total of over **24.08 million lx mins** in the span of 4 days (May 1st, 11th, 21st and 31st). The western and eastern facade were expected to have similar exposure levels to one another, however lower than that of the southern facade. This is reflected on my values with respective exposures of **14.79 million** and **14.39 million lx mins**. The Southern facade on average accounted for **45.22%**

of the total exposure making all classrooms with windows along this facade the most prone to the bright sun during the month of May in our Secondary Building.

The method used to determine the Sunlight exposure was really sophisticated which took into account the natural axial tilt of the earth at 23.44 degrees as well as the latitude and longitude of my school. These were used to find the Solar azimuth and altitude which were inputted into the simulation software to simulate the exposure on each facade. The results obtained from **Table 5** agree with my theory and the trajectory of the Sun during the month of May.

However there are also certain anomalies in this dataset such as the decreasing trend in total Illuminance per day is not consistent for 31st of May. It is also important to note that the simulation simulated the Sun with no atmosphere, cloudcover or weather data for the month of May. This could suggest that the values obtained are much higher than the true extent to which each facade's sunlight exposure.

This investigation can be extended in multiple different directions. One could take into account the area covered by windows for each facade. It might be that even though the southern facade receives the highest sunlight exposure, due to small or less windows, the true exposure inside classrooms is much lower. Another extension could be to use this data to study Solar panel placements on our Secondary Building roofs and determine which placements yield the most energy.

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