Card Games & Stocks: Monte Carlo Methods

1 Introduction

How likely is it for a gambler to win? Is it ever possible to know that you'll win after a certain number of games?

For card games, where the amount of different starting layouts for the cards is so large, can you know how likely you are to win before looking at the cards in front of you?

In 1946, Stanislav Ulam was recovering from surgery and was playing Canfield Solitaire (a difficult variation with lower odds of winning). Ulam (a mathematician) became intrigued in the odds of a solitaire hand with 52 cards being winnable.

He realised that solving this problem analytically would be too difficult because of how a deck of cards can be arranged in 52! different ways (as a side note, 52 factorial is greater than the number of atoms in the world).

Instead, he decided to play several hands and record the amount that he won to work a relative probability of winning. By playing many hands of solitaire, Ulam reasoned that he could predict the probability of success with greater accuracy.

Ulam shared this idea with John Von Neumann at Los Alamos to help develop the hydrogen bomb. Neutron diffusion was modelled with the chance of a neutron causing nuclear fission in a nucleus as a random variable and solving to see if the fission reaction could cause a chain reaction leading to the bomb detonating. The method received the name 'Monte Carlo' from Ulam's uncle, who borrowed money from him to gamble in Monte Carlo.

Beyond its uses in physics, the Monte Carlo method has been applied in mathematics and finance, from methods for estimating π to predicting the future prices of stocks.

2 How does it work?

In basic versions of this method, an action is repeated over many simulations (often in the thousands or millions) and the frequency of the outcomes are used to determine their probabilities.

By the Law of Large Numbers, as the size of our sample increases our sample mean tends towards the mean of the population. So as the number of simulations increases, the average result will converge towards the expected value.

A simple example would be trying to work out the probability of rolling a 5 on a fair 6-sided dice.

Using python, we can create a piece of code that allows us to simulate throwing dice for large amounts of times. This means we can calculate the probability to high levels of accuracy without having to actually throw a die 10,000 times.

```
result=random.randint(1,6)

total= z
count=0

for x in range(total):
   result= random.randint(1,6)
   if result == 5:
      count= count + 1
```

P = count / total

Varying the total amount of dice thrown (z) gives us estimated probabilities with increasing levels of accuracy:

- When $z = 10$, $P = 0.2$
- When $z = 100$, $P = 0.18$
- When $z = 10000$, $P = 0.1641$

We know the probability of rolling a 5 on a fair dice to be 1/6 or 0.167 so we can see how the accuracy of our estimated frequency increases with the number of trials towards the actual probability.

Applying Monte Carlo methods is useful for games of chance like roulette where the probability of wining after a certain number of games is harder to calculate.

But how can we use this method for more complicated scenarios outside of casino games?

3 The Stock Market

Monte Carlo methods have found uses in finance also, namely predicting the future prices of stocks.

It may seem impossible to predict the price of stocks, but prices depend on variables like drift and volatility. Drift is the long-term movement of a stock price over time, whilst volatility is the short-term fluctuations of price.

The continuous growth rate for a stock can be modelled with the equation:

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t}$$

By avoiding Brownian Motion (W_t) for simplicity, we can change our model to:

$$S_t = S_0 e^{\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma z}$$

Where:
- S_t is the stock price after *t* amount of time
- S_0 is the current stock price
- Our drift is $\mu - 0.5 \times \sigma^2$ (we can simplify as our *t* value is 1)
- Volatility is σz (where z is a random value)

We can use historical data for our stock to find our mean price increase (μ) and the variance of our data (σ^2).

Therefore, drift can be calculated as:

Mean daily price increase – (Variance / 2)

As we already know today's value of the stock, we now only need to find our volatility (σz). We can use Monte Carlo to estimate the volatility of the stock for today by using a random number generator for our value of z.

We can rearrange our simplified equation to find the future stock price as:

$S_t = S_0 \times e^{(Drift + Volatility)}$

The iShares Physical Gold ETC is a stock that tracks the spot price of gold. We can use past data that is available online to calculate our mean rate of return and variance to be 16.2% and 374% respectively.

Using these values in our previous equations for drift, volatility, and stock price along with a random number generator for volatility, we can estimate that £100 invested in our stock will grow to £124.73

We can improve the accuracy of our estimate by repeating our monte carlo simulation over millions of times to find better values for our random variable z.

However, this value is flawed as our model assumes that the stock market is both perfectly efficient and not influenced by external factors such as the discovery of more gold.

Advanced versions of Monte Carlo methods (like the Markov Chain Monte Carlo) are used in finance for the pricing of stock options with the help of other factors like interest rates or expiration times of the option.

4 Conclusion

Monte Carlo methods have grown from a way to see the probability of winning at cards to one of the most useful tools for solving problems in fields from physics to finance. They provide a method for tackling problems that are too complicated to be solved with formulas by estimating probabilities over millions of simulations. They provide a distribution of possible outcomes from extremes cases to the average outcome.

5 References

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