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TITLE PAGE

ODD VALUES OF THE RIEMANN ZETA FUNCTION

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ABSTRACT

In this paper, we focus our investigation on the Riemann zeta function that is identified by a sum formula, denoted by $\zeta(s) = \Sigma_{n \geq 1} 1/n^s = 1 + 1/2^s + 1/3^s + 1/4^s \ldots$ for $n \in \mathbb{N} \mid Re(s)>1$, and a product formula, denoted by $\Pi_{p \text{ prime}} (p^s /(p^s -1) = 2^s 3^s 5^s \ldots/(2^s-1)(3^s-1)(5^s-1)\ldots$. In particular, we investigate the odd values of the Riemann zeta function, denoted by $\zeta(2s+1)$. The Riemann zeta function is assigned a Riemann coprime manifold to reflect points of interest across a horizontal Cartesian (*x, y*) plane, denoted by $\{x, y \in \mathbb{R} \mid x>0\}$, and a vertical complex (*x, iy*) plane, denoted by $\{x, iy \in \mathbb{C} \mid i>0\}$. A key take away is that the Riemann coprime manifold generates a coprime multiplier and an inverse divisor from an infinite series of odd ones units that repeat in 5^{ths}. The even ones units (i.e., 2, 4, 6, 8) scale with the odd units (i.e., 1, 3, 5, 7, 9) in modulo 9 to generate an odd value boundary for the Riemann zeta function, denoted by $\zeta(smin) < \zeta(s) < \zeta(smax)$.

INTRODUCTION

The Riemann sum formula, denoted in Fig 1 below, and the Riemann zeta product (Euler prime) formula, denoted in Fig 2 below, are used in the mathematical field of number theory to evaluate the Riemann zeta function. Although the zeta sum and product formulas have different expressions they are integrated to always equal.

Fig 1

$$\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \cdots$$

Fig 2

$$\prod_{p \text{ prime}} \frac{1}{1-p^{-s}} = \frac{1}{1-2^{-s}} \cdot \frac{1}{1-3^{-s}} \cdot \frac{1}{1-5^{-s}} \cdot \frac{1}{1-7^{-s}} \cdots \frac{1}{1-p^{-s}} \cdots$$

The Riemann zeta function is assigned a Riemann coprime manifold R(A) to determine a series of odd boundary values that lie on the horizontal Cartesian (*x, y*) plane, in connection with crossing the vertical complex (*x, iy*) plane. A pair of Dirichlet polynomials, denoted as R(A) = $36k^2+36k+5 \equiv 5 \bmod 9$ and $36k^2-1 \equiv 8 \bmod 9$, form the Riemann coprime manifold R(A) whose 2x2 matrices have matrix subscripts (aii, aij, aji, ajj) form rows and columns in the form of an infinite coprime superset, denoted by gcd (*p, q*) = 1: $\{p \perp q \mid \in \mathbb{R}, p, q \text{ are coprime}\}$, together with an infinite prime subset *p*, denoted by $\{p \mid p \in \mathbb{R},\ p \text{ is prime}\}$ for $p, q \geq 5$. More precisely, the subscript rows horizontally form a twin-cousin coprime sequence whose ones units repeat in 5^{ths} to form a boundary multiplier at the 4^{th} and 5^{th} matrices. The subscript columns ones units diagonally cross to form a unity-sexy coprime sequence that vertically integrates with the rows.

The Riemann coprime manifold R(A) is illustrated in Fig' 3, 4, and 5 below:

Fig 3

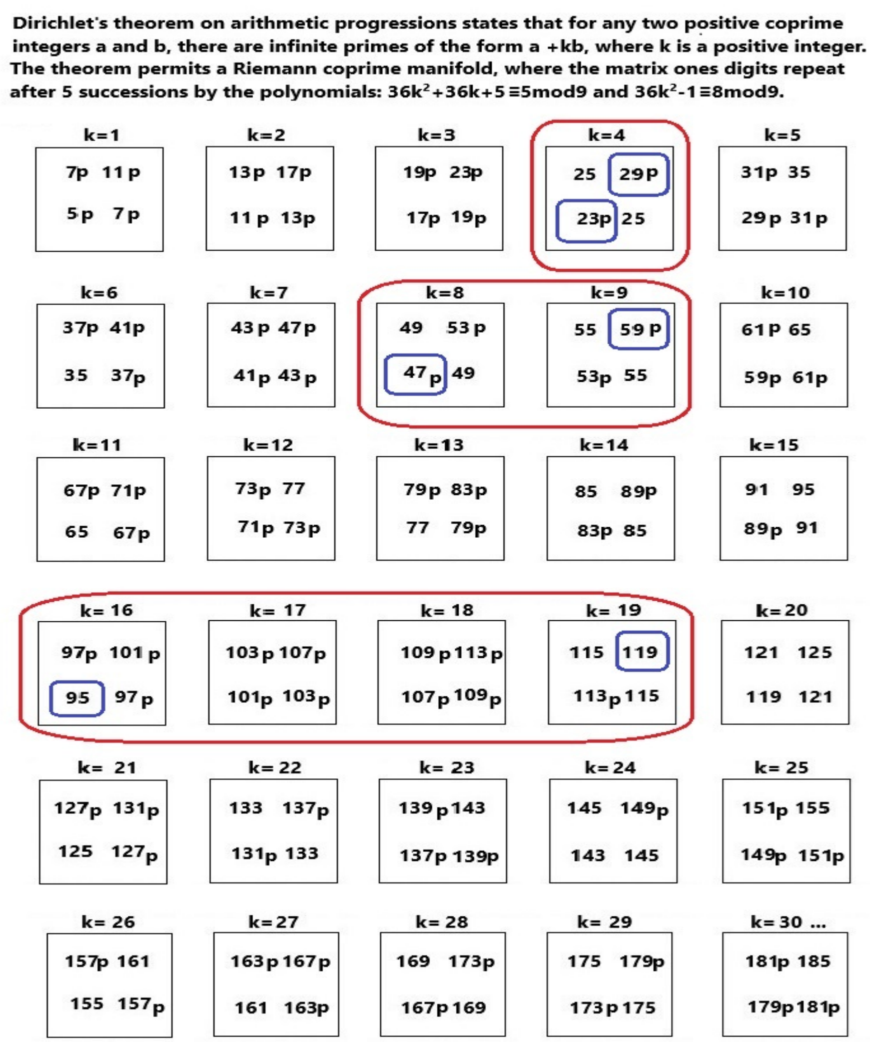


Fig 4

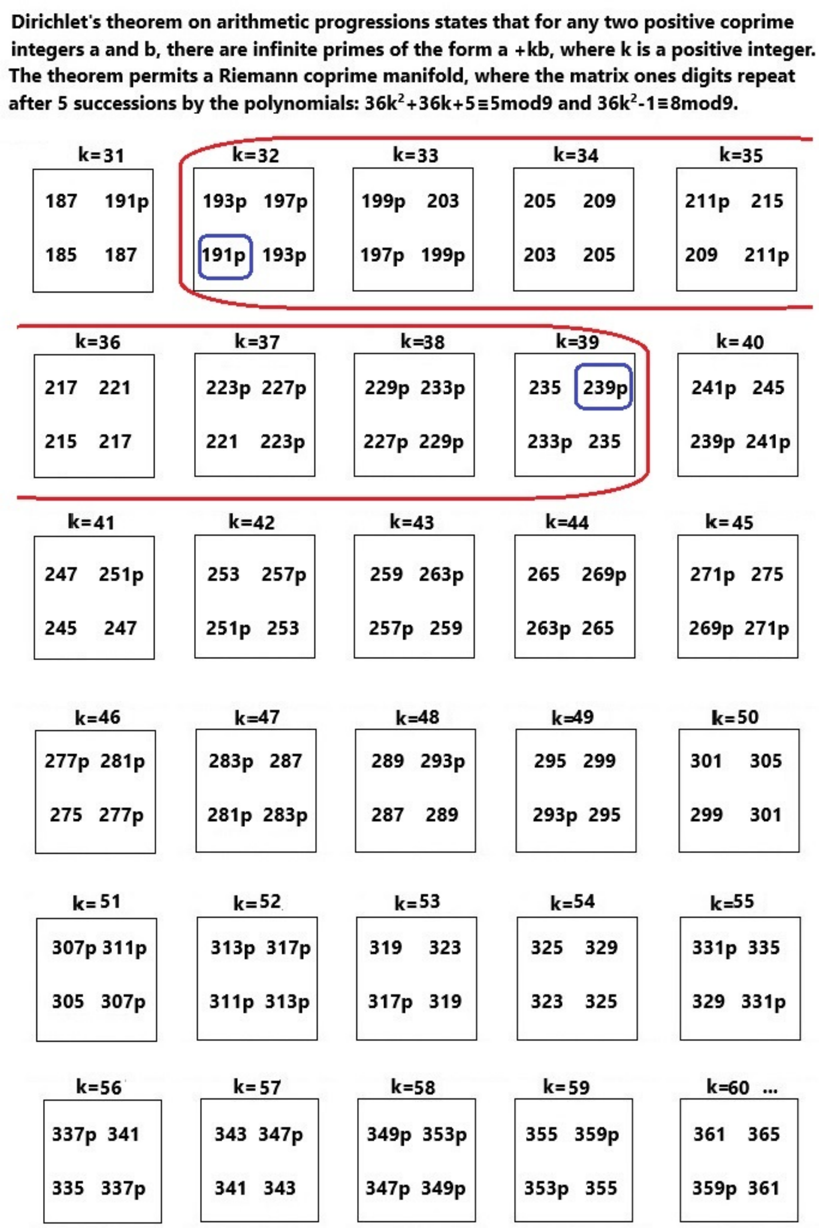
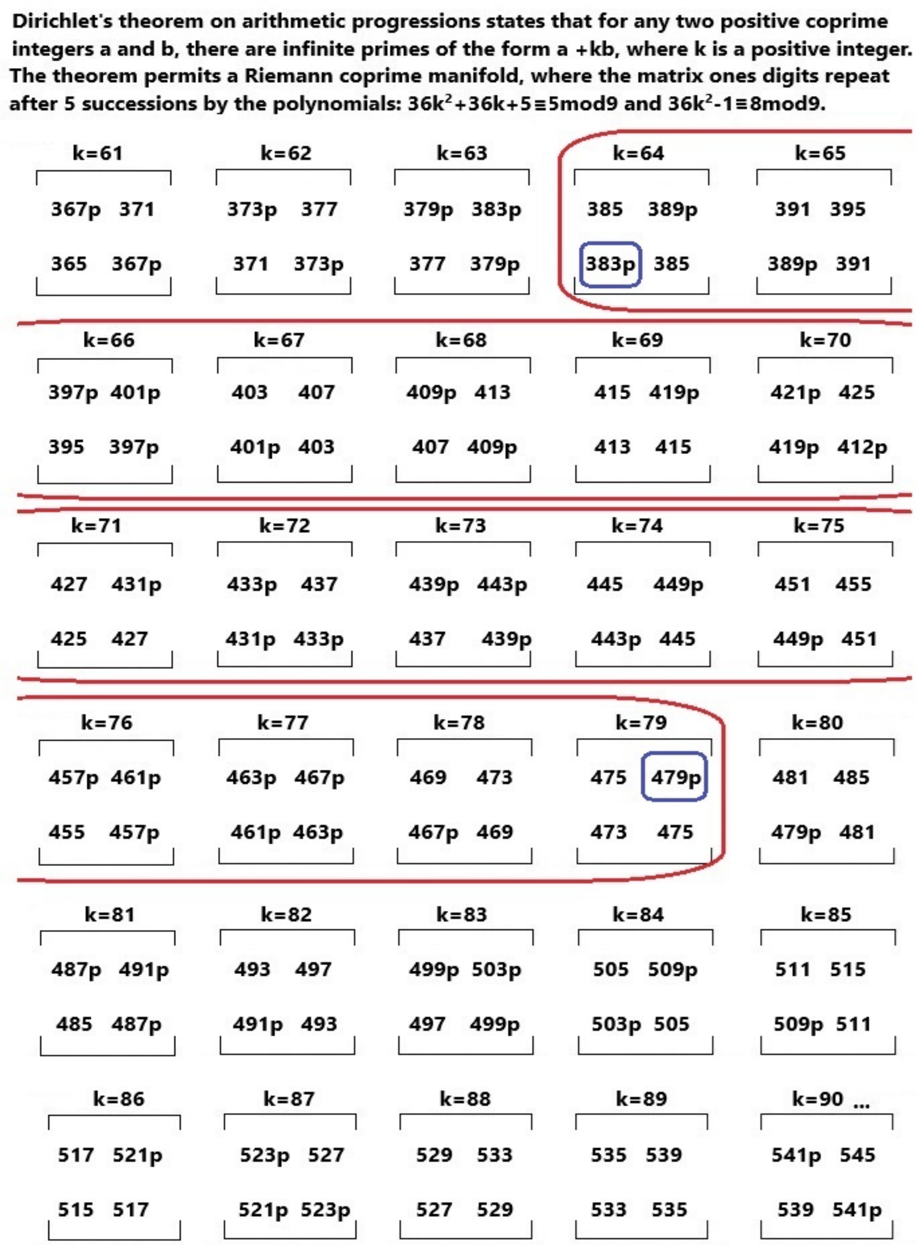

Dirichlet's theorem on arithmetic progressions states that for any two positive coprime integers a and b, there are infinite primes of the form a +kb, where k is a positive integer. The theorem permits a Riemann coprime manifold, where the matrix ones digits repeat after 5 successions by the polynomials: 36k²+36k+5≡5mod9 and 36k²-1≡8mod9.
k=31
187 191p
185 187
k=32
193p 197p
191p 193p
k=33
199p 203
197p 199p
k=34
205 209
203 205
k=35
211p 215
209 211p
k=36
217 221
215 217
k=37
223p 227p
221 223p
k=38
229p 233p
227p 229p
k=39
235 239p
233p 235
k=40
241p 245
239p 241p
k=41
247 251p
245 247
k=42
253 257p
251p 253
k=43
259 263p
257p 259
k=44
265 269p
263p 265
k=45
271p 275
269p 271p
k=46
277p 281p
275 277p
k=47
283p 287
281p 283p
k=48
289 293p
287 289
k=49
295 299
293p 295
k=50
301 305
299 301
k=51
307p 311p
305 307p
k=52
313p 317p
311p 313p
k=53
319 323
317p 319
k=54
325 329
323 325
k=55
331p 335
329 331p
k=56
337p 341
335 337p
k=57
343 347p
341 343
k=58
349p 353p
347p 349p
k=59
355 359p
353p 355
k=60 ...
361 365
359p 361

Fig. 5


Dirichlet's theorem on arithmetic progressions states that for any two positive coprime integers a and b, there are infinite primes of the form a +kb, where k is a positive integer. The theorem permits a Riemann coprime manifold, where the matrix ones digits repeat after 5 successions by the polynomials: $36k^2+36k+5 \equiv 5 \bmod 9$ and $36k^2-1 \equiv 8 \bmod 9$.
k=61
367p 371
365 367p
k=62
373p 377
371 373p
k=63
379p 383p
377 379p
k=64
385 389p
383p 385
k=65
391 395
389p 391
k=66
397p 401p
395 397p
k=67
403 407
401p 403
k=68
409p 413
407 409p
k=69
415 419p
413 415
k=70
421p 425
419p 412p
k=71
427 431p
425 427
k=72
433p 437
431p 433p
k=73
439p 443p
437 439p
k=74
445 449p
443p 445
k=75
451 455
449p 451
k=76
457p 461p
455 457p
k=77
463p 467p
461p 463p
k=78
469 473
467p 469
k=79
475 479p
473 475
k=80
481 485
479p 481
k=81
487p 491p
485 487p
k=82
493 497
491p 493
k=83
499p 503p
497 499p
k=84
505 509p
503p 505
k=85
511 515
509p 511
k=86
517 521p
515 517
k=87
523p 527
521p 523p
k=88
529 533
527 529
k=89
535 539
533 535
k=90 ...
541p 545
539 541p

The Dirichlet theorem states that for any two coprime integers *a* & *d* there exists infinitely many primes of the form $a + nd$, where *n* is a positive integer. The Riemann coprime manifold R(A) interchanges the *a* term with *x*, *n* with *i,* and *d* with *y* to formulate a series of boundaries that lie on a vertical complex (*x, iy*) plane and a horizontal Cartesian (*x ,y*) plane. The boundary includes a minima bond multiplier, *denoted by bmin,* and a maxima bond multiplier *denoted by bmax,* that factor by an inverse divisor, denoted by $1/(1^s+2^s+3^s+4^s+5^s+6^s)$.

The *bmin* value is generated by a constant of 5 based on an infinite series of odd ones: 1,3,5,7,9. The *cmax* value is generated by a constant of 4 based on an infinite series of even ones 2, 4, 6, or 8. And, an inverse divisor factors, $\Sigma\ 1/n^s{}_{zeta} = 1/(1^s+2^s+3^s+4^s+5^s+6^s)$ to form a boundary for each odd value of the Riemann zeta function in R^2 space, denoted by $\zeta(s)min < \zeta(s) < \zeta(s)max$.

For $\zeta(1)$, the odd value of the Riemann zeta function forms a harmonic series that corresponds to a singularity pole at s=1 that diverges to infinity. For $\zeta(2s)$, the even values of the Riemann zeta function have already been evaluated as precise factors of π. For $\zeta(2s+1)$, the odd values of the Riemann zeta function are bound by an optimal pair of minimal '$\zeta(s)$min' and maximal '$\zeta(s)$max' real numbers $\mathbb{R}^2$.

For the ζ(3) value, commonly referred to as the Apery constant, proximate to a 1.202056903 constant, the odd value of the Riemann zeta function corresponds to the Riemann coprime manifold R(A) when evaluated at 2^0 or 1 (one) matrix at the k = 2^2 or the 4^{th} matrix, denoted by ζ(3)min: {k = 2^2, 4, n = 23} and ζ(3)max: {k =2^2, n = 29}. The *bmin* multiplier value is given by 5(23) to be factored by an inverse divisor, denoted: $\Sigma\ 1|n^3_{zeta} = 1/(1^3+2^3+3^3+4^3+5^3+6^3)$ proximate to 1.19029163 to yield a minimum boundary value of 115/1.19029163 proximate to a 96.614978297 constant. The *bmax* multiplier value is given by 4(29) to be factored by the inverse divisor to yield a maximum boundary value of 116/1.19029163 proximate to a 97.4551085511 constant. The constants are rounded to a natural number of 96 to yield a real boundary in $\mathbb{R}^2$ space for approximating ζ(3), denoted by ζ(5)min: 5(23)/96 ≈1.197916667 < ζ(3): ≈1.202056903 < ζ(3)max: 4(29)/96 ≈1.208333333.

For the ζ(5) constant proximate to a 1.036927755 constant, the odd value of the Riemann zeta function corresponds to the Riemann coprime manifold R(A) when evaluated at 2^1 or 2 (two) matrices, beginning at the k=2^3 (8^{th}) matrix and ending at the 9^{th} matrix, denoted by ζ(5)min: {k = {2^3, 8), n = 47}, and ending ζ(5)max: {k = 9, n = 59}. The *bmin* multiplier value is given by 5(47) to be factored by an inverse divisor, denoted by $\Sigma\ 1|n^5 = 1/(1^5+2^5+3^5+4^5+5^5+6^5)$, to yield a minimum boundary value of 235/1.036790394 proximate to a 226.6610506 constant. The *bmax* multiplier is given by 4(59) to be factored by the inverse divisor to yield a maximum boundary value of 236/1.036790394 proximate to a 227.6255658 constant. The constants are rounded to a natural number of 227 to yield a real boundary in $\mathbb{R}^2$ space for approximating ζ(5), denoted by ζ(5)min: 5(47)/227 ≈1.035242291 < ζ(5): ≈1.036927755 < ζ(5)min: 4(59)/227 ≈1.039647577.

For the $\zeta(7)$ value proximate to a 1.008349277 constant, the odd value of the Riemann zeta function corresponds to the Riemann coprime manifold when evaluated at 2^2 or 4 (four) matrices beginning at the $k = 2^4$ (16^{th}) matrix and ending at the $k = 19^{th}$ matrix, denoted by $\zeta(7)$min: $\{k = (2^4, 16), n = 95\}$ and $\zeta(7)$max: $\{k = 19, n = 119\}$. The *bmin* multiplier is given by 5(95) to be divided by an inverse divisor, denoted by $\Sigma\ 1|n^7 = 1/(1^7+2^7+3^7+4^7+5^7+6^7)$, to yield a minimum boundary value of 475/1.008347155 proximate to a 471.067923031 constant. The *bmax* multiplier is given by 4(119) to be factored by the inverse divisor to yield a maximum boundary value of 476/1.008347155 proximate to a 472.059644974 constant. The constants are rounded to a natural number of 472 to yield a real boundary in $\mathbb{R}^2$ space for approximating $\zeta(7)$, denoted by $\zeta(7\text{min})$: $5(95)/472 \approx 1.006355932 < \zeta(7)$: $\approx 1.008349277 < \zeta(7\text{max})$: $4(119)/472 \approx 1.008474576$.

For the $\zeta(9)$ value proximate to a 1.002008392 constant, the odd values of the Riemann zeta function correspond to the Riemann coprime manifold R(A) when evaluating at 2^3 or 8 (eight) matrices beginning at the k= 2^5 (32^{nd}) matrix and ending at the 39^{th} matrix, denoted by $\zeta(9)$min: {k= (2^5, 32), n=191 {k = 39, n = 239}. The *bmin* multiplier is given by 5(191) and divided by an inverse divisor, denoted by $\Sigma\ 1|n^9 = 1/(1^9+2^9+3^9+4^9+5^9+6^9)$, to yield a minimum boundary value of 955/1.002008357 proximate to a 953.085863335 constant. The *bmax* multiplier is given by 4(239) and factored by the inverse divisor to yield a 956/1.002008357 value proximate to a 954.083859003 constant. The constants are rounded to a natural number of 954 to yield a real boundary in $\mathbb{R}^2$ space for $\zeta(9)$, denoted by $\zeta(9)$min: $5(191)/954 \approx 1.001048218 < \zeta(9): \approx 1.002008392 < \zeta(9)$max: $4(239)/954 \approx 1.002096436$.

For the ζ(11) constant proximate to 1.000494189, the odd values of the Riemann zeta function correspond to a real number boundary when the Riemann coprime manifold R(A) is evaluated at 16 matrices from the 64^{th} matrix to the 79^{th} matrix, denoted by ζ(11)min: {k= (2^6, 64), n=383 {k = 79, n = 479}. More precisely, a minima multiplier is given by 5(383) and divided by an inverse divisor, denoted by $\Sigma\ 1|n^{11} = 1/(1^{11}+2^{11}+3^{11}+4^{11}+5^{11}+6^{11})$, to yield a 1915/1.000494188 value proximate to 1914.05409743. A maxima multiplier is given by 4(479) and factored by the inverse divisor to yield a 1916/1.000494188 value proximate to 1915.053603. The divisor is rounded to natural numbers to yield an optimal real number boundary solution for ζ(11), denoted by ζ(11)min: 5(383)/1915 ≈1.000100001 < ζ(11): ≈1.000494189< ζ(9)max: 4(479)/1915 ≈1.000522193.

For ζ(s) large integers, the odd values of the Riemann zeta function have a microscopic boundary near s=1 to suitably estimate with a short formula: $\zeta(s) = 1 + 2^{-S}$. For example, the short formula for ζ(17) is proximate to 1.0000076294 and the long version is 1.0000076303.

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