Optimising Flow: Calculus In Skateboarding

Skateboarding and mathematics are not often mentioned in the same sentence. One is associated with creativity, movement and style, while the other is often thought of as numbers and equations. Yet when a skateboarder chooses a line through a skatepark, they are oblivious to the fact that they are solving a mathematical problem involving geometry and optimisation. But can the concept of "flow" in skateboarding be explained through mathematical modelling and optimisation?

Many professional skateboarders choose efficient 'lines' through a skatepark. Professional skateboarders frequently discuss the importance of choosing the right line through a park, particularly during competitions. For example, the iconic vert skateboarder, Tony Hawk has often described planning lines in advance during competitions so that tricks connect smoothly while maintaining speed. A line in skateboarding is a sequence of tricks performed one after another, usually planned in advance. This can be between obstacles such as ledges and stair sets. Through doing this, skateboarders aim to maintain a 'flow' through the skatepark. However, this can sometimes be difficult, which is where the mathematics comes in. This is the optimisation problem. The goal is to minimise time, distance, and energy lost while turning. Overall the aim is to model skatepark movement using calculus to determine the optimal paths through the skatepark.

It is possible to model a skatepark using coordinates by placing it onto a coordinate plane. Ledges, ramps and other objects are modelled as points, curves and regions in this plane. Or they can simply be represented as points. For example, a rail could be a line segment, a ramp the arc of a circle, and a bowl a curved surface. You can use the distance formula,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{when the points are } (x_1, y_1) \text{ and } (x_2, y_2)$$

To find the distance between points, which are the ramps and rails in this case. Which allows potential skate lines to be compared mathematically. Showing that a skatepark can be represented as a geometric system of functions and points.

Skate lines can also be represented using mathematical functions. This line can be modelled mathematically as a curve such as $y = f(x)$ or as a parametric path. For instance, a straight approach toward an obstacle can be represented by a linear function, which shows a constant direction and slope. In contrast, when a skateboarder turns smoothly, the motion can be modelled as part of a circular arc or another curved function. In many cases, parametric equations such as $x(t)$ and $y(t)$ are particularly useful because they describe the skateboarder's position in terms of time as they move along the line. This allows both direction and speed to be considered in the model. As a result, different mathematical curves can represent different styles of movement in skateboarding, helping to show how riders link tricks together through smooth and controlled paths.

Path optimisation can be linked to calculus through derivatives. For example to find the best line, we minimise some quantity. For example the total distance or the amount of turning. Derivatives can be used here, which is the idea of dy/dx (the gradient function), in order to help identify the minimum values. Smooth curves require continuous derivatives. The aim is to produce the optimal line, by balancing short distance with a smooth curvature.

Sharp turns can be minimised through using curvature. These sharp turns slow skaters down because they introduce high levels of friction and resistance that go against the forward momentum. When you turn sharply, the wheels are forced to rub against the ground rather than roll freely, converting kinetic energy into heat and noise, which decreases the speed of the skateboard. Therefore, the optimal path will minimise curvature to maintain the starting speed, often resulting in a wider, more fluid line. Sharp turns force the skateboarder to fight against intense, outward centrifugal force rather than using that energy for forward momentum, showing that it is transferred to other stores of energy. Curvature uses the formula,

$$K = \frac{|y|}{(1+y^2)^{3/2}}$$

Where a larger curvature leads to a ‘tighter’ turn and a smaller curvature leads to a ‘smoother’ flow. This links mathematics directly to the technique of skateboarding.

The model can be applied to a simple skatepark example. Imagine a simple skatepark with two ramps and a rail. The ramps are at the coordinates (1,1) and (2,3), and the rail is on (3,5). Rather than simply connecting these points with straight lines, a more realistic skate line can be modelled as a smooth curve passing through or near these locations. In mathematical terms, this path could be represented using a parametric curve x(t),y(t), allowing the motion of the skateboarder through the park to be described continuously. Using calculus, the total length of this path can be calculated using the arc length formula,

$$L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

In this simple skatepark example, calculus can be used to model and predict the most efficient path a skateboarder might take between obstacles. Different possible paths between the obstacles can be represented as mathematical curves rather than simply straight lines. For example, one potential route may be a straight line between two obstacles, while another may be a smooth curved path that follows the natural shape of a ramp or bowl. Using calculus, these paths can be analysed by comparing quantities such as their arc length and curvature. The arc length allows the total distance travelled along each curve to be calculated, while analysing curvature helps determine whether the turns are smooth or excessively sharp. By comparing these properties for different possible curves, it becomes possible to identify the route that minimises total distance while avoiding unnecessary sharp turns. In this way, the intuitive idea of maintaining “flow” in skateboarding can be interpreted mathematically as selecting a path that minimises both total path length and curvature, demonstrating how optimisation and calculus can be used to predict efficient movement through a skatepark.

EXAMPLE:

The distance between the ramp (1,1) and rail (3,5):

$$d = \sqrt{(3-1)^2+(5-1)^2} = 4.47213595\ldots$$
$$\approx 4.5 \text{ units}$$

the quadratic function the points lie on

$$y = x^2 - x + 1$$

$$L = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = 2x - 1$$

$$L = \int_1^3 \sqrt{1+(2x-1)^2}\, dx$$

$$L = \int_1^3 \sqrt{4x^2-4x+2}\, dx$$

$$L = 5.0 \text{ units}$$

Therefore from the first ramp to the rail, the distance travelled along the curve is approximately 5 units. By comparing this with the length of other possible paths (for example a straight-line route between points), calculus allows us to determine which route through the skatepark is shorter or smoother.

In conclusion, although skateboarding is often seen as a creative and expressive activity in a different way to mathematics, the movement of a skateboarder through a skatepark can be analysed using mathematical concepts. By modelling a skatepark on a coordinate plane, the paths taken by skateboarders can be represented as mathematical curves. Calculus allows these paths to be analysed and compared by measuring quantities such as total distance and curvature, helping to determine which routes are the most efficient. Geometry and optimisation therefore provide a framework for explaining why certain lines through a skatepark feel smoother and maintain better momentum than others. Ultimately, the concept of ‘flow’ in skateboarding is not only an artistic or stylistic idea, but also one that can be understood through mathematical modelling and analysis.