The Kelly Criterion

The fine balance between maximising growth without risking ruin

By Finlay Cowen

If you had a 60% chance to double your money would you risk everything? To most, the answer feels obvious. A clear majority chance of winning, combined with the possibility of doubling your wealth seems an opportunity too good to refuse. Without mathematical reasoning, this proves to be not just a good bet, but an irresistible one. However, John Larry Kelly Jr thought differently. Originally, in an attempt to solve signal noise problems on long distance telecommunications in 1956, John proved otherwise.

At first glance, the logic is compelling. If you stake £1: with probability, 0.6 you gain £1 and with probability 0.4, you lose £1. The expected profit is:

$E = 0.6(1) + 0.4(-1) = 0.2$

So on average you gain 20 pence per pound bet. This is a positive expected value meaning the bet is favourable. Many would conclude a rational response may be to stake as much as possible and potentially even everything. This intuition however, rests on a flawed assumption: that maximising expected value is the same as maximising success. So the question Kelly set out to answer was: when faced with a favourable bet, how much should we stake while managing risk and our growth in wealth?

To solve this Kelly developed the formula:

$$f^* = \frac{bp - q}{b}$$

Where:

f^* - is the optimal fraction of wealth to bet
b - is the net odds (profit per £1 staked)
p - is the probability of winning
$q = 1-p$ (the probability of losing)

Here, rather than maximising expected profit, Kelly maximises the expected logarithmic growth of wealth. The formula determines the fraction of wealth that maximises the long-term growth through compounding. It does this by balancing the ideas of exploiting an edge, and the danger of overexposure to loss. By representing wealth multiplicatively, where losses and wins scale the total capital, Kelly's formula accounts for the asymmetry that large losses are far more damaging than equivalent gains are beneficial. When calculating, the resulting percentage represents the exact point at which increasing the stake no longer improves growth, but instead begins to reduce it. Therefore, The Kelly Criterion reframes rational decision making: it demonstrates that aggressive and large stakes aren't optimal even when the odds are favourable. Kelly aims to calibrate risk with precise allocation of capital to maximise sustained growth. Using our earlier probabilities, we can find an optimal value of fraction of wealth to gamble. We take "b" as 1 as we expect to double our money. This calculation is computed below:

$$f^* = \frac{(1)(0.6) - 0.4}{1} = 0.2$$

Here we can see, using the Kelly Criterion we get a value for “f*” that equals 0.2. This means to maximise our wealth growth in the long run we must wager 20% of our current capital rather than all of it or even a significant amount. The result is surprising, yet despite being favourable the bet is cautious.

To understand why this is, we must look closer at how Kelly modelled the growth of wealth.

The key insight behind The Kelly Criterion is that wealth grows multiplicatively rather than additively. Kelly considers a question of growth rather than profit. Over many repetitions the relevant quantity is the rate at which wealth compounds rather than the average profit. To model this we use Kelly’s Growth Function for wealth.

Returning to our original scenario now, using Kelly’s growth function:

$$G(f) = p\log(1 + bf) + (1 - p)\log(1 - f)$$

This function measures the expected growth rate of wealth. The logarithms are crucial because it shows the asymmetry of gains and losses that we mentioned earlier. For example, losing 50% requires a 100% gain just to break even. The logarithmic framework naturally accounts for this imbalance. And when inserting our earlier probabilities, and follow Kelly betting f=0.2 (where [1+bf] is your wealth after a win and [1-f] is your wealth after a loss) we see this:

$$G(0.2) = 0.6\log(1.2) + 0.4\log(0.8)$$

$$G(0.2) = 0.6(0.1823) + 0.4(-0.2231)$$

$$G(0.2) = 0.1094 - 0.0892 = 0.0202$$

Our value here is positive meaning wealth accumulates overtime. However when we follow rationale and stake all of our wealth on good odds (when f=1) we see this:

$$G(1) = 0.6\log(2) + 0.4\log(0)$$
$$G(1) = -\infty$$

This means that over repeated bets: the strategy leads to inevitable complete loss.

The Kelly Formula we used earlier to find that f=0.2 for our example was no mistake. It is not just an arbitrary rule but rather an optimisation of the growth function. By first taking the general formula for the growth function we differentiate with respect to f. By differentiating term by term, we get:

$$\frac{d}{df}[p\log(1+bf)] = \frac{pb}{1+bf}$$
$$\frac{d}{df}[(1-p)\log(1-f)] = -\frac{1-p}{1-f}$$

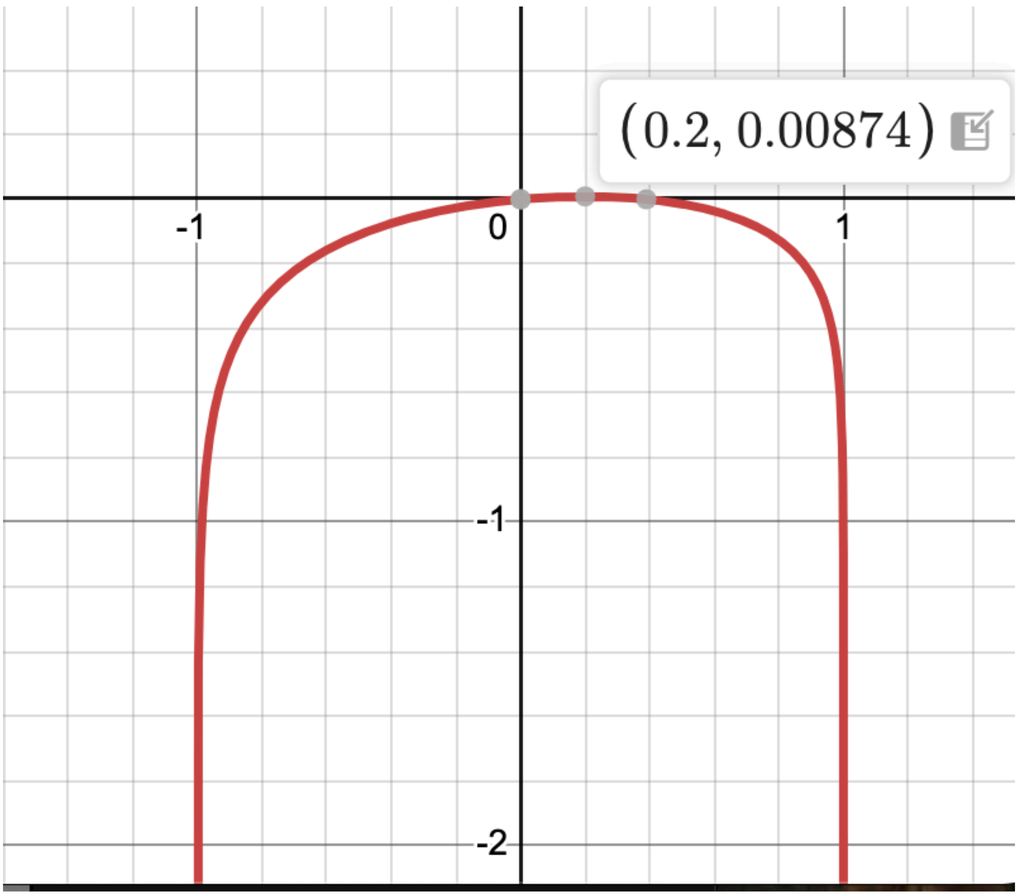
And therefore by finding a whole expression of the differential of G(f) we get:

$$G'(f) = \frac{pb}{1+bf} - \frac{1-p}{1-f}$$

Now, to find the point of most optimised growth, we set our new function G’(f) = 0. This equates to finding the turning point of the graph of the growth function where gradient = 0 as we have already found a gradient function by differentiating. And by solving this and setting it equal to “f” to get a formula for the optimum fraction of wealth to wager to maximise growth, we get:

$$f = \frac{pb - (1 - p)}{b}$$

This function to find the optimum point correlates to our earlier equation for the Kelly Criterion (remember [1-p] = p). Here I show the graph of the growth function with our probabilities from the original example:



Here, when our earlier probabilities are applied to the general formula, we are shown a visual function for our value of “f” which is indicated by then gradient. The value of x shows the fraction of our wealth we wager. When wagering all of our wealth or non of our wealth at 1 and -1, we see a gradient of negative infinity. This means that at -1, no growth will occur and at 1, after repeated bets a complete loss of wealth is inevitable. However, not staking enough results in slow sub-optimal growth. When the gradient is 0 at our turning point we find the optimum value of “f”. With our earlier probabilities displayed graphically, we see a value of 0.2. This matches the value computed using The Kelly Formula at the beginning.

The derivation reveals the intricacy of The Kelly Criterion, it shows that optimal strategy when staking wealth isn’t intuition nor by maximising expected gains, but rather finding the balance where increased risk no longer increases growth. Beyond this point, increasing the stake leads to diminishing returns and eventually negative growth.

The Kelly Criterion proves as an efficient tool of the finance industry in areas such as Hedge Funds and Quantitive Trading. In modern finance, the central challenge is not simply identifying profitable opportunities, but determining how much capital to commit to them. Investors are constantly faced by decisions with uncertainty: a stock may appear undervalued, a trading strategy may show a statistical edge, or a market signal may suggest a likely price movement. Yet acting on these opportunities without restraint can be as dangerous as ignoring them entirely. The Kelly Criterion provides a framework for resolving this tension by linking probability directly to position sizing. While Kelly’s model doesn’t always hold up in financial markets (due to uncertain probabilities, flaws in human behaviour that affect markets and correlated outcomes), Fractional Kelly strategies are often used that scale back the values for “f*” to account for loss and uncertainty in markets. A Fractional Kelly strategy shifts the aim of the model from what is mathematically optimal to what is robust under uncertainty. This idea is particularly relevant in professional investing, where overconfidence and excessive risk-taking have historically led to catastrophic failures. Financial crises and hedge fund collapses often share a common feature, overexposure to positions believed to be safe. An infamous example of this occurred in 1998. Long Term Capital Management (LTCM) a hedge fund, had $4.7 Billion in equity but had borrowed roughly $124 Billion, giving it a leverage of approximately 25-1. After losses accumulated, its effective leverage became far higher. This meant a private sector rescue was needed from the Fed to prevent disorderly liquidation. It shows that the LTCM’s trades weren’t irrational in isolation however, enormous leverage was used in order to magnify returns when the edge was small. Kelly’s model warns against this as even though there was a positive edge, it did not justify huge stakes. Once losses hit and capital shrank, leverage rose further, forcing liquidation at the worst possible moment. Kelly’s general principle is proven here as when edges are small and uncertainty is real, the optimal fraction of wealth that should be at stake is also small. Extreme leverage on small positive margins makes ruin much more likely.

What began as an apparently obvious truth, a 60% chance to double ones money, reveals under mathematical scrutiny a far more nuanced truth. The Kelly Criterion shows that success is not determined by the attractiveness of a single bet but rather how risk can be managed across time. Therefore by maximising logarithmic growth rather than expected value, we maximise returns. In the real world this theory holds, showing how restraint, by sustaining growth without collapse creates a true accumulation of wealth. Ultimately The Kelly Criterion reframes rational decision making with mathematical scrutiny. It is not about maximising gains in moments of opportunity but about sustaining growth without risking collapse.

References

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