"The maths of meltdown: probability, correlation and the collapse of the 2008 housing market"

701-billion-dollar bailout, Lehman brother bankruptcy, AIG bailout and mass hysteria amongst the financial system, not primarily due to greed, or the lack of awareness, but due to multiple mathematical failures that would lead to a systemic collapse. Was this due to an over-reliance on models? Poor economic judgement? The inability to correctly price risk?

The 2008 financial crisis wasn't an improbable black swan but a mathematically predictable consequence of embedding Gaussian assumptions into systemically interconnected institutions- the inappropriate reduction of fat-tailed, correlated default risk to tractable normality did not merely cause catastrophe, it actively constructed the conditions for it.

The Gaussian assumption

So what is the normal distribution? According to Wikipedia: 'in probability theory and statistics, a normal distribution (also referred to as a Gaussian distribution) is a type of continuous probability distribution for a real-valued random variable. In finance, the normal distribution (bell curve) describes how returns cluster around an average with a predictable level of dispersion (standard deviation). Most observations fall near the centre, with fewer extreme outcomes as you move away from the mean- hence this allows for investors to define what an extreme/rare event would be; in our context this would be the financial crisis.

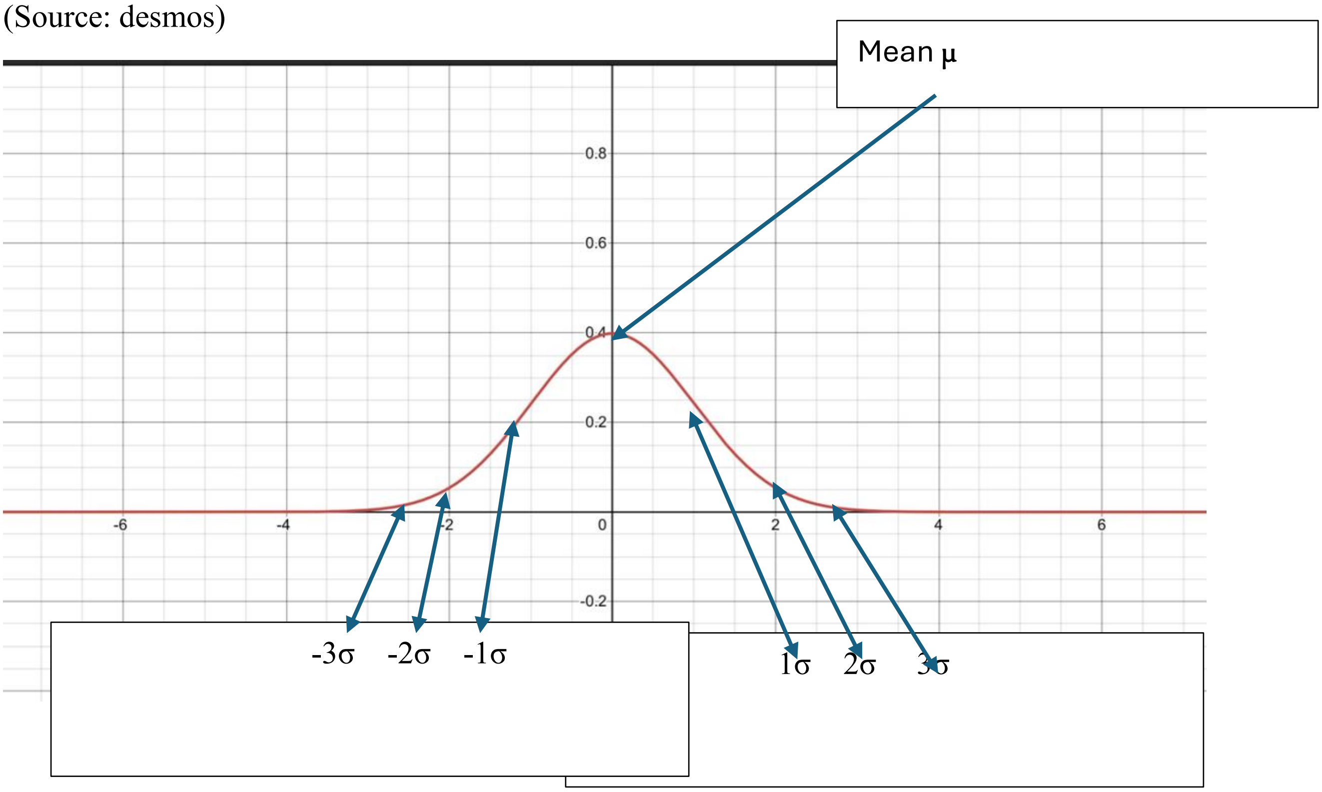
There are three key symbols/rules for the normal distribution:

The mean is represented by μ this is the highest point of a bell curve

The standard deviation is represented by sigma, which identically is the

Bell curve:

(Source: desmos)



As shown in the figure above the Gaussian distribution assigns negligible probability to extreme deviations beyond 3 standard deviations

Tails beyond 2 sigma are considered extremely low under Gaussian assumptions.

A central assumption underpinned modern financial risk models is that asset returns follow a normal distribution- this undermines the likelihood of extreme events (much like the financial crisis) and results in these events to be essentially unconsidered when making such mathematical models. Under this framework, outcomes are symmetrically distributed around the mean, with the probability of extreme deviations declining exponentially. This implies that 'tail events' such as sever market crashes, are exceedingly rare. For instance, a movement several standard deviations from the mean would be considered virtually impossible within a Gaussian framework. However, empirical evidence contradicts this assumption, as extreme events occur far more frequently than the model predicts- thus the Gaussian model does not merely underestimate risk- it fundamentally misrepresents the 'normal' distribution of financial returns

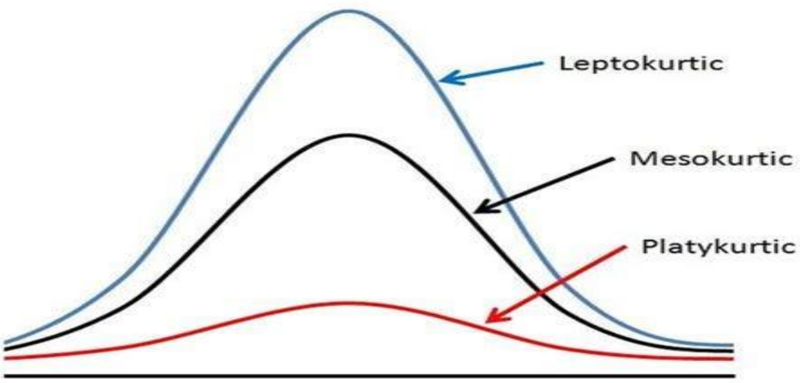
Fat Tails

In contests to the normal distribution, the financial markets exhibit 'fat-tailed' behaviour, where extreme events have significantly higher probability. This means that events deemed highly improbable under Gaussian assumptions occur surprising regularity. Consequently, models based on normality. This implies that events considered virtually impossible under normal assumptions-such as widespread mortgage defaults- are, in fact, features of the system. The failure to account for fat tails meant that risk models severely underestimated the probability of an extreme financial crisis occurring. Consequently, institutions were exposed to levels of risk far exceeding their expectations. This disconnected between theoretical distributions and observed behaviour highlights a critical flaw: the prioritisation of mathematical tractability over empirical accuracy. From this perspective, the crisis can be seen as an inevitable consequence of the underestimation of of extreme events occurring and using inappropriate statistical assumptions to determine complex financial probabilities

Correlation and dependance

Beyond individual risk misestimation, the crisis was amplified by flawed models such as the Gaussian copula model. This model assumed that correlations between mortgage default probabilities was low, hence this led to financial firms engaging in risky activities which would lead to high levels of diversification within the financial systems. However, in periods of economic stress correlations tend to increase sharply as defaults are driven by common macroeconomic factors e,g falling house prices. This shows how risk transitioned from being isolated to a systematic issue. As a result, the diversification failed precisely when it was most needed, transforming nations rose defaults into a systemic collapse. The underestimation of correlation therefore did not merely misprint risk-it structurally embedded fragility within the financial system.

Kurtosis



A mesokurtic distribution(normal) has moderate tails, meaning extreme events are rare. In contrast, a leptokurtic distribution has heavier tails and a sharper peak, indicating a higher probability of extreme outcomes. This makes leptokurtic distributions more suitable for modelling financial returns, where large deviations occur more frequently than predicted by normal distribution.

This figure shows the increasing kurtosis leads to heavier tails, highlighting the higher likelihood of extreme deviations overlooked by Gaussian assumption

The limitations of Gaussian modelling can be more precisely understood through the concept of kurtosis, which measures the weight of a distribution's tails relative to its variance. A normal distribution has a kurtosis of three, implying relatively thin tails and a low probability of extreme deviations. However, financial return distributions consistently exhibit excess kurtosis, indicating a significantly higher likelihood of rare but severe events. This distinction is crucial, as standard risk metrics such as variance fail to capture the impact of tail behaviour. By assuming low kurtosis, pre-crisis models systematically underestimated the probability and magnitude of large losses, effectively compressing extreme risk into negligible probabilities. Consequently, institutions were not merely unaware of tail risk—they mathematically excluded it from their models. This mischaracterisation played a central role in the build-up to the 2008 crisis, as it allowed highly leveraged positions to appear statistically safe despite their exposure to catastrophic outcomes.

Evaluation

While the misuse of Gaussian assumptions, correlation models, and the neglect of higher moments such as kurtosis were central to the mispricing of risk, it would be reductive to attribute the crisis solely to mathematical failure. Crucially, the limitations of these models were not entirely unknown; both fat-tailed behaviour and correlation instability had been extensively documented in academic literature prior to the crisis. The persistence of these models therefore reflects not ignorance, but selective reliance driven by institutional incentives and the demand for tractable, implementable frameworks. In particular, measures such as Value at Risk prioritised computational simplicity over statistical robustness, effectively sidelining tail risk despite its known significance. Moreover, the assumption of stable correlations and low kurtosis was not an inherent necessity, but a modelling choice that facilitated the expansion of leverage and the proliferation of complex securities. Thus, mathematics did not fail in isolation; rather, it was the interaction between elegant but simplifying assumptions and human overconfidence that proved destabilising. In this sense, the crisis can be understood not as a failure of mathematics itself, but as a failure to respect its limitations when applied to complex, adaptive systems.

Conclusion

In conclusion, the economic crisis was not an unpredictable event, but the product of systematic mathematical underestimation. The reliance on Gaussian distributions imposed thin-tailed assumptions on inherently fat-tailed financial data, while the neglect of kurtosis obscured the true likelihood of extreme events. Simultaneously, flawed correlation models transformed what appeared to be diversified assets to be assets that contained high levels of risk and were responsible for the losses occurring within financial firms. However, these mathematical flaws were amplified by their uncritical application within financial institutions, where simplicity and profitability often took precedence over accuracy and financial safety. Ultimately, the crisis illustrates a broader lesson: mathematical models do not merely describe reality—they shape decisions within it. When their assumptions are misaligned with the systems they represent, they can lead to systematic failure. As such, the collapse was not simply a failure of markets or models, but a failure to recognise the limits of abstraction in the face of real-world complexity.