Quantifying the Impact of Stochastic Income on UK Student Loan Repayments: A Monte Carlo Simulation Approach

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Abstract

The United Kingdom's income-contingent student loan system protects graduates from unaffordable repayments but transfers substantial fiscal risk to the government. Conventional policy evaluations rely on deterministic income projections, which ignore the profound uncertainty of graduate earnings trajectories. This paper develops a stochastic income model based on Geometric Brownian Motion and employs Monte Carlo simulation across 10,000 paths to quantify how income volatility affects lifetime repayments, the probability of full loan clearance, and the expected government write-off. Our baseline results reveal that income volatility systematically reduces expected repayments relative to the deterministic benchmark—from £56,476 to £38,833 in undiscounted terms—while increasing the expected government write-off from £33,170 to £51,534. This divergence is explained by Jensen's Inequality applied to the convex repayment function. We further conduct sensitivity analyses across volatility levels and repayment thresholds, and evaluate seven policy scenarios. The findings demonstrate that the fiscal cost of the student loan system is materially understated when income uncertainty is ignored, with important implications for the Resource Accounting and Budgeting (RAB) charge and the long-run sustainability of UK higher education finance.

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1. Introduction

The financing of higher education is one of the most consequential and contested areas of public policy in the United Kingdom. Since the introduction of income-contingent repayment (ICR) student loans for English undergraduates in 1998, and particularly following the expansion of tuition fees to £9,000 per year in 2012, the system has grown into a multi-billion-pound fiscal commitment. The outstanding stock of student loans in England exceeded £230 billion by 2024, and the government's own estimates suggest that between 30% and 57% of all loan outlay will ultimately be written off rather than repaid [1] [2].

At the heart of this system lies a fundamental tension: the income-contingent design provides valuable insurance to graduates against low earnings, but this insurance is not free. The government bears the residual risk that graduates will not earn enough to repay their loans within the statutory write-off period. The expected cost of this guarantee is captured by the Resource Accounting and Budgeting (RAB) charge, which the Office for Budget Responsibility (OBR) and Department for Education estimate annually. However, these estimates typically rely on deterministic or smoothed income projections that do not fully account for the stochastic nature of individual earnings over a 30- to 40-year horizon.

This paper addresses that gap. We argue that income volatility has a systematic and quantifiable effect on loan repayment outcomes, and that ignoring it leads to a material underestimate of the government's fiscal exposure. The mechanism is mathematical: the repayment function $R(Y) = 0.09 \max(Y - T, 0)$ is convex in income Y. By Jensen's Inequality, the expectation of a convex function of a random variable exceeds the function of the expectation. Consequently, for borrowers whose income fluctuates around the repayment threshold, periods of low income (zero repayments) are not symmetrically offset by periods of high income, and the net effect is lower expected lifetime repayments and higher expected write-offs.

The paper proceeds as follows. Section 2 provides an institutional overview of the UK student loan system. Section 3 develops the deterministic benchmark model. Section 4 introduces the stochastic income framework. Section 5 describes the Monte Carlo simulation methodology. Section 6 presents the results and sensitivity analysis. Section 7 assesses risk metrics and evaluates policy interventions. Section 8 discusses ethical and distributional implications. Section 9 concludes. Appendices provide formal mathematical derivations and Python simulation code.

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2. Institutional Background

2.1 The Plan 2 Student Loan System

Plan 2 student loans were issued to English undergraduates who began courses between the academic years 2012–13 and 2022–23. Students could borrow to cover tuition fees (up to £9,250 per year) and maintenance costs. The repayment terms are structured as follows [3]:

Feature	Plan 2 Details
Repayment rate	9% of income above the threshold
Repayment threshold	£28,470 (2024–25); frozen at £29,385 from April 2027 for three years
Interest rate	RPI to RPI + 3%, depending on income level
Write-off period	30 years after entering repayment
Default consequences	None — no credit risk to the borrower

This structure differs fundamentally from mortgage-style student loans used in many other countries. Because repayments depend solely on income rather than on the outstanding balance, borrowers are protected from default. However, this protection comes at a cost: a significant fraction of borrowers will never repay their loans in full, and the outstanding balance at the write-off date is absorbed by the government.

2.2 Plan 5 and Recent Reforms

For students commencing courses from 2023 onwards, a new Plan 5 system applies. Key differences include a lower repayment threshold of £25,000, a longer write-off period of 40 years, and an interest rate capped at RPI inflation (rather than RPI + 3%) [3]. The 2025 Autumn Budget further announced that the Plan 2 repayment threshold would be frozen at its April 2026 level (£29,385) for three years, rather than increasing with inflation, increasing average lifetime repayments for the 2022 entry cohort by approximately £3,000 in real terms [3].

2.3 The RAB Charge and Fiscal Cost

The Resource Accounting and Budgeting (RAB) charge is the government's estimate of the proportion of total loan outlay that will not be recovered, expressed in net present value terms. It is recorded as expenditure at the time the loan is made, reflecting the expected subsidy embedded in the ICR design [2]. The RAB charge for Plan 2 loans stood at approximately 34% in 2024–25, meaning that roughly one-third of every pound lent is not expected to be repaid [4]. For Plan 5, the charge is approximately 30%, reflecting the longer repayment period and lower threshold [4].

The OBR's March 2022 analysis showed that the 2022 reforms reduced the RAB charge from 57% to 37% for Plan 2 loans, primarily through the extension of the repayment period and the reduction in the repayment threshold [2]. These large swings in the RAB charge illustrate how sensitive the government's fiscal position is to the parameters of the repayment system — and, by extension, to the underlying income dynamics of graduates.

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3. Deterministic Benchmark Model

3.1 Income Trajectory

In the deterministic framework, a graduate's income Y_t at time t (years after graduation) grows at a constant continuous rate g:

$$Y_t = Y_0\, e^{gt}$$

where Y_0 is the initial income upon graduation. This specification implies a smooth, predictable earnings profile with no uncertainty.

3.2 Repayment Mechanics

The annual repayment R_t is defined as 9% of income in excess of the repayment threshold T:

$$R_t = 0.09 \max(Y_t - T,\ 0)$$

The outstanding loan balance L_t evolves according to the interest rate r and the repayment R_t:

$$L_{t+1} = \max\big(L_t(1+r) - R_t,\ 0\big)$$

Repayments cease either when the balance reaches zero (full repayment) or when the write-off period W is reached, at which point any remaining balance L_W is forgiven.

3.3 Present Value of Repayments

The present value (PV) of lifetime repayments, discounted at rate δ, is:

$$PV = \sum_{t=1}^{W} \frac{R_t}{(1+\delta)^t}$$

Baseline Deterministic Results. Under the calibrated parameters ($Y_0 =$ £$30,000$, $g = 3\%$, $L_0 =$ £$45,000$, $T =$ £$28,470$, $r = 3\%$, $\delta = 3\%$, $W = 30$ years), the deterministic model yields:

Metric	Value
PV of repayments	£31,334
Total undiscounted repayments	£56,476
Balance written off after 30 years	£33,170
Full repayment achieved	No

The deterministic model reveals that even a graduate starting at £30,000 and growing at a steady 3% per year does not repay their loan in full within 30 years. This is because the interest rate on the loan (3%) approximately matches the income growth rate, so the real burden of the loan does not diminish rapidly. The write-off of £33,170 represents a substantial government subsidy even in this optimistic scenario.

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4. Stochastic Income Model

4.1 Geometric Brownian Motion

To introduce income uncertainty, we model the graduate's income Y_t as a Geometric Brownian Motion (GBM). GBM is widely used in financial mathematics to model asset prices and income processes because it ensures non-negativity, produces log-normally distributed outcomes, and has well-understood statistical properties [5] [6]. The stochastic differential equation (SDE) is:

$$dY_t = \mu\, Y_t\, dt + \sigma\, Y_t\, dW_t$$

where μ is the expected drift (annual income growth rate), σ is the volatility of income, and W_t is a standard Wiener process (Brownian motion).

4.2 Solution via Itô's Lemma

Applying Itô's Lemma to $f(Y_t) = \ln Y_t$, we obtain:

$$d(\ln Y_t) = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma\, dW_t$$

Integrating from 0 to t:

$$Y_t = Y_0 \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]$$

This shows that Y_t is log-normally distributed with:

$$\ln Y_t \sim \mathcal{N}\left(\ln Y_0 + \left(\mu - \frac{\sigma^2}{2}\right) t,\ \sigma^2 t\right)$$

The expected income and variance are:

$$E[Y_t] = Y_0\, e^{\mu t}, \qquad \mathrm{Var}(Y_t) = Y_0^2\, e^{2\mu t}\left(e^{\sigma^2 t} - 1\right)$$

The term $-\sigma^2/2$ in the exponent is the Itô correction, which reflects the fact that the expected log-income grows more slowly than the drift μ due to the convexity of the exponential function. This correction is important: it means that higher volatility reduces the median income path even while leaving the mean unchanged.

4.3 The Lifetime Repayment Functional

Under the stochastic model, the lifetime present value of repayments is a random variable:

$$PV = \sum_{t=1}^{W} \frac{0.09 \max(Y_t - T,\ 0)}{(1+\delta)^t}$$

This functional is path-dependent: the total repayment depends on the entire trajectory of Y_t, not just its terminal value. Moreover, the loan balance evolves stochastically, and repayments cease if the balance is cleared before the write-off date. No closed-form analytical solution exists for $E[PV]$ under these boundary conditions, which motivates the use of Monte Carlo simulation.

4.4 Jensen's Inequality and the Convexity Effect

The repayment function $R(Y) = 0.09 \max(Y - T, 0)$ is convex in Y: it is flat (zero) for $Y \leq T$ and linear for $Y > T$, with a kink at the threshold. By Jensen's Inequality, for any random variable Y and convex function R:

$$E[R(Y)] \geq R(E[Y])$$

This inequality has a direct economic interpretation. Consider a graduate whose expected income is £35,000 (above the threshold). In the deterministic case, they always repay $0.09 \times (35,000 - 28,470) = £587$ per year. In the stochastic case, their income fluctuates: in some years it falls below the threshold (zero repayment), and in others it rises above it. The convexity of the repayment function means that the gains from high-income years are not symmetric with the losses from low-income years. However, the critical insight is that when income falls below the threshold, repayments drop to zero — creating a floor that benefits the borrower but harms the government.

For a borrower near the threshold, the net effect of volatility is to reduce expected repayments (because the zero floor is frequently binding) and increase the expected write-off. For a borrower well above the threshold, the effect is more complex: volatility increases the probability of dipping below the threshold in some years, again reducing expected repayments. Figure 7 illustrates this convexity gap.

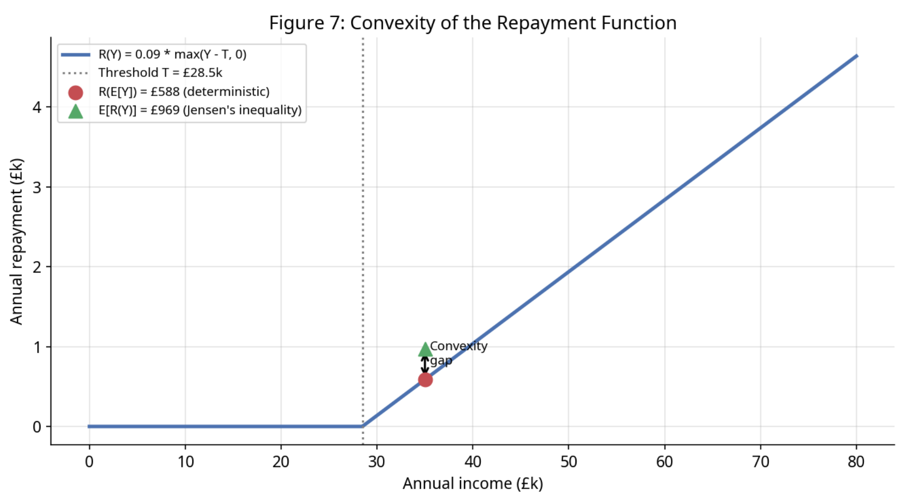

Figure 7: Convexity of the Repayment Function

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5. Monte Carlo Simulation Methodology

5.1 Discretization

We discretize the GBM using the Euler-Maruyama scheme with annual time steps ($\Delta t = 1$):

$$Y_{t+1} = Y_t \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\, Z_t\right], \quad Z_t \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0,1)$$

This exact discretization (valid for GBM) avoids the approximation error of the standard Euler scheme and ensures that Y_t remains strictly positive.

5.2 Simulation Algorithm

For each of $N = 10,000$ paths:

1. Draw $Z_t \sim \mathcal{N}(0,1)$ for $t = 1, \dots, W$.
2. Compute the income path $\{Y_t\}_{t=1}^{W}$ using the discretization above.
3. Simulate the loan balance evolution: at each year t, compute the repayment $R_t = 0.09\max(Y_t - T, 0)$, capped at the outstanding balance; apply interest; update the balance.
4. Record the present value of repayments, total undiscounted repayments, the write-off amount (if any), and whether the loan was fully repaid.

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6. Results and Sensitivity Analysis

6.1 Baseline Stochastic Results

The Monte Carlo simulation yields the following baseline results, compared with the deterministic benchmark:

Metric	Deterministic	Stochastic (Mean)	Difference
PV of repayments	£31,334	£23,769	-£7,565 (-24.1%)
Total undiscounted repayments	£56,476	£38,833	-£17,643 (-31.2%)
Write-off amount	£33,170	£51,534	+£18,364 (+55.4%)
P(full repayment)	0%	0.0%	—
P(govt incurs write-off)	100%	100.0%	—
95th percentile write-off	—	£108,652	—
Expected shortfall (top 5%)	—	£109,101	—

The results are striking. The introduction of income volatility reduces expected lifetime repayments by approximately 31% in undiscounted terms, and increases the expected government write-off by 55%. The deterministic model substantially overestimates the government's expected recovery. This divergence arises precisely because the deterministic model assumes the graduate always earns above the threshold, while the stochastic model allows for years of below-threshold income where no repayments are made.

The 95th percentile write-off of £108,652 and the expected shortfall of £109,101 reveal the tail risk embedded in the system. In the worst 5% of outcomes, the government writes off more than twice the initial loan balance — a consequence of compound interest accumulating on an unpaid balance over 30 years.

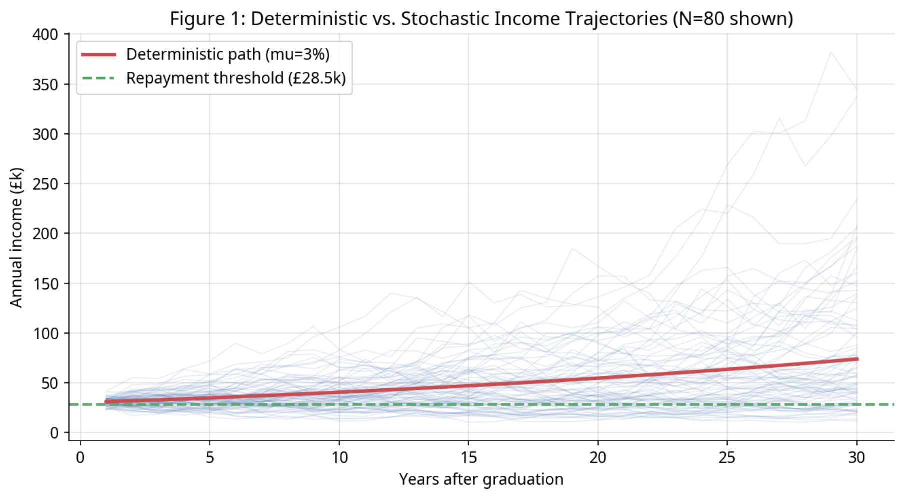

Figure 1: Deterministic vs. Stochastic Income Trajectories

Figure 1 shows 80 simulated income paths (blue) alongside the deterministic trajectory (red). The repayment threshold is shown as a green dashed line. Many paths spend extended periods below the threshold, generating zero repayments.

Figure 2 shows the distribution of total undiscounted repayments across 10,000 simulated paths. The distribution is right-skewed, with a long tail of high repayments from paths where income grows strongly. The stochastic mean (green) falls well below the deterministic value (red), illustrating the systematic downward bias of deterministic models.

6.2 Sensitivity to Income Volatility

We vary σ from 5% to 30% while holding all other parameters constant. The results are presented in Table 1.

Table 1: Sensitivity of Outcomes to Income Volatility (σ)

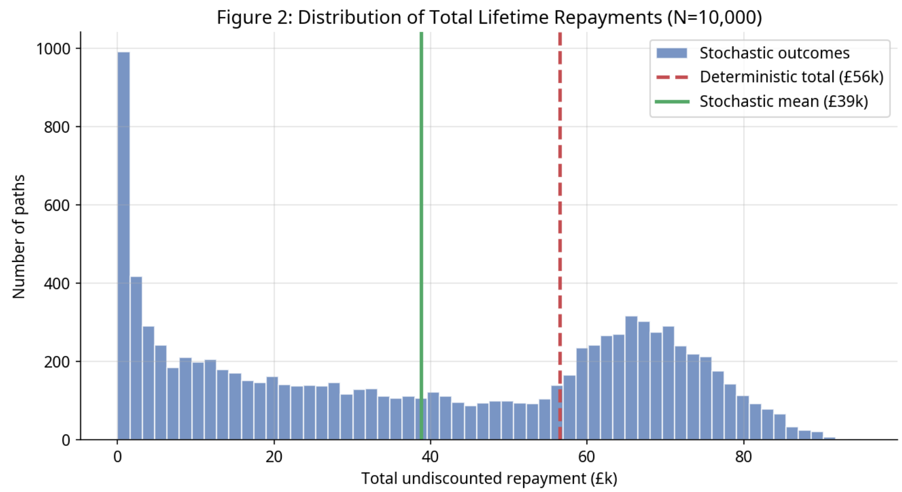

Figure 2: Distribution of Total Lifetime Repayments

Volatility (σ)	E[PV] (£k)	E[Total Repaid] (£k)	E[Write-off] (£k)	P(Full Repayment) (%)	P(Govt Loss) (%)
5%	24.5	42.8	44.7	0.0	100.0
10%	24.5	41.1	46.6	0.0	100.0
15%	23.8	38.8	51.7	0.0	100.0
20%	22.6	36.1	55.9	0.0	100.0
25%	21.5	33.6	59.7	0.0	100.0
30%	20.4	31.3	63.7	0.0	100.0

The results confirm the theoretical prediction: higher income volatility systematically reduces expected repayments and increases expected write-offs. Moving from $\sigma = 5\%$ to $\sigma = 30\%$ reduces expected total repayments by approximately 27% (from £42.8k to £31.3k) and increases the expected write-off by 42% (from £44.7k to £63.7k). This is the convexity effect of Jensen's Inequality in action.

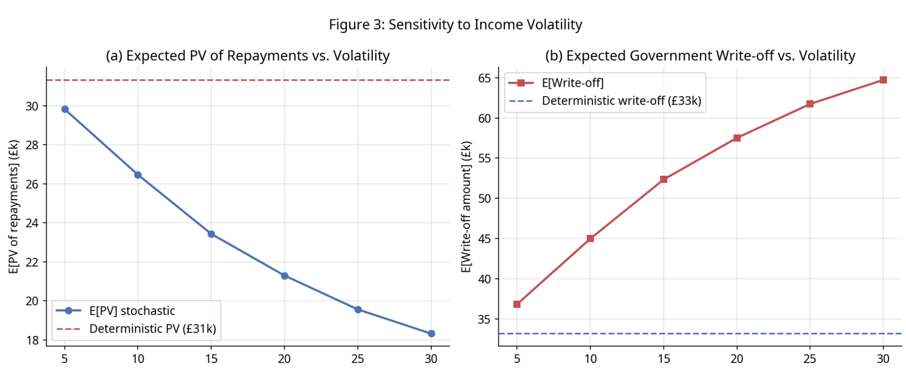

Figure 3: Sensitivity to Income Volatility

Figure 3 illustrates the monotone relationship between volatility and the expected write-off (panel b), and the declining expected PV of repayments (panel a). The red dashed line in panel (a) shows the deterministic PV, which is consistently above the stochastic mean.

Threshold (£k)	E[PV] (£k)	E[Total Repaid] (£k)	E[Write-off] (£k)	P(Full Repayment) (%)
£20,000	27.1	44.2	39.8	0.0
£25,000	25.4	41.5	44.6	0.0
£28,470	23.8	38.8	51.7	0.0
£30,000	22.8	37.2	54.2	0.0
£35,000	19.7	32.5	62.7	0.0
£40,000	16.4	27.4	71.5	0.0

As the threshold rises, fewer graduates earn above it in any given year, reducing repayments and increasing the expected write-off. The relationship is approximately linear: each £5,000 increase in the threshold reduces expected total repayments by roughly £3–4k and increases the expected write-off by a similar amount. This has direct implications for policy: threshold freezes (which effectively lower the real threshold over time) increase repayments and reduce the RAB charge, while threshold increases have the opposite effect.

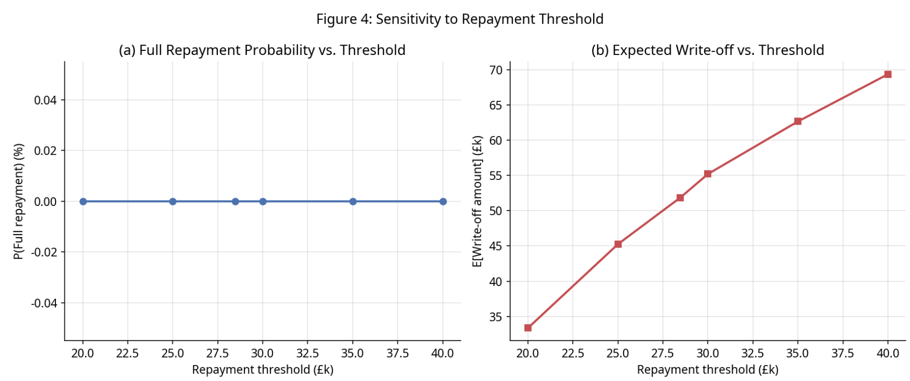

Figure 4: Sensitivity to Repayment Threshold

Figure 4 shows the monotone decline in full repayment probability (panel a) and the monotone increase in expected write-off (panel b) as the threshold rises.

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7. Risk Assessment and Policy Solutions

7.1 Government Risk Metrics

The stochastic model allows us to quantify the government's fiscal risk in a rigorous manner. Three key risk metrics are computed from the baseline simulation:

Risk Metric	Value
Probability of any write-off (P(govt loss))	100.0%
Expected write-off	£51,534
95th percentile write-off	£108,652
Expected Shortfall (top 5% of losses)	£109,101

The probability of any write-off being 100% reflects the fact that, under baseline parameters, the loan balance grows faster than repayments for most income trajectories. The expected shortfall of £109,101 — more than twice the initial loan balance — represents the average loss in the worst 5% of scenarios. This tail risk is driven by compound interest accumulating on unpaid balances over the full 30-year period.

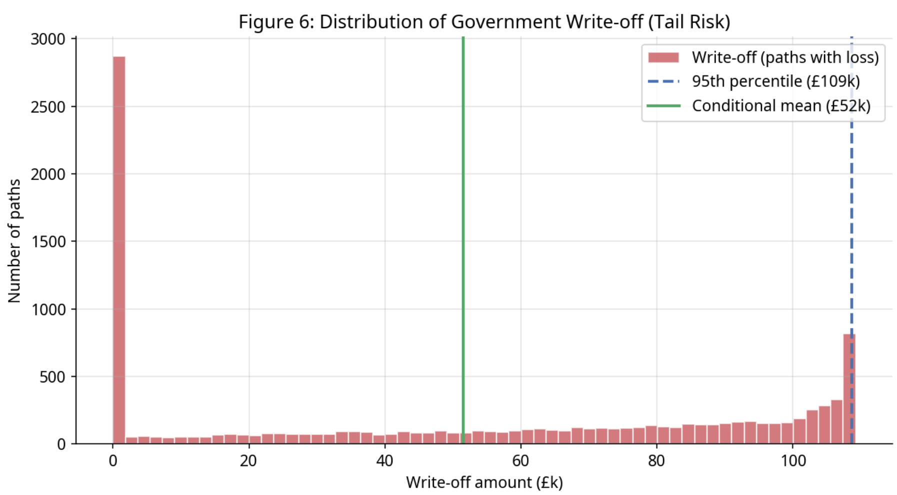

Figure 6: Distribution of Government Write-off (Tail Risk)

Figure 6 shows the distribution of write-off amounts conditional on a write-off occurring. The distribution is right-skewed, with a long tail driven by paths where income remains persistently low. The 95th percentile (blue dashed line) and conditional mean (green) are marked.

7.2 Policy Scenario Analysis

We simulate seven policy scenarios to evaluate the impact of adjusting key parameters on both borrower outcomes and government fiscal exposure. The results are summarized in Table 3.

Table 3: Policy Scenario Comparison

Scenario	E[PV] (£k)	E[Total Repaid] (£k)	E[Write-off] (£k)	P(Full Repayment) (%)	P(Govt Loss) (%)
Baseline (T=£28,470, W=30y, r=3%)	23.7	39.0	51.7	0.0	100.0
Lower threshold (T=£25,000)	26.7	42.7	44.4	0.0	100.0
Higher threshold (T=£35,000)	19.2	32.9	62.7	0.0	100.0
Longer write-off (W=40 years)	28.2	51.4	55.0	0.0	100.0
Shorter write-off (W=25 years)	20.6	31.7	51.0	0.0	100.0
Lower interest rate (r=1%)	20.2	31.9	24.0	0.0	100.0
Higher interest rate (r=5%)	26.7	45.3	110.2	0.0	100.0

Several findings merit particular attention. First, **lowering the interest rate to 1%** dramatically reduces the expected write-off from £51.7k to £24.0k — a reduction of 54% — while also reducing total repayments. This is because lower interest means the loan balance grows more slowly, making it easier for repayments to keep pace. This scenario most closely approximates the Plan 5 design (where interest is capped at RPI). Second, **raising the interest rate to 5%** more than doubles the expected write-off to £110.2k, illustrating how high interest rates create a debt trap: repayments cannot keep pace with interest accrual, and the balance spirals upward. Third, **extending the write-off period to 40 years** increases total repayments substantially (from £39.0k to £51.4k) but also increases the expected write-off, because the longer period allows more interest to accumulate on unpaid balances. Fourth, **lowering the repayment threshold** is the most effective lever for reducing the government's write-off: moving from £28,470 to £25,000 reduces the expected write-off by

14%.

Figure 5: Policy Scenario Comparison

Figure 5 compares expected total repayments (blue) and expected write-offs (red) across the seven policy scenarios. The interest rate is the most powerful driver of the write-off amount, while the threshold has the largest impact on total repayments.

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8. Mathematical Depth: Formal Derivations

8.1 Itô's Lemma Applied to the Income Process

Let $f(Y) = \ln Y$. By Itô's Lemma for a function of a diffusion process:

$$df = \frac{\partial f}{\partial Y} dY + \frac{1}{2} \frac{\partial^2 f}{\partial Y^2} (dY)^2$$

Substituting $dY = \mu Y\, dt + \sigma Y\, dW_t$ and using $(dW_t)^2 = dt$:

$$d(\ln Y_t) = \frac{1}{Y_t}(\mu Y_t\, dt + \sigma Y_t\, dW_t) + \frac{1}{2}\left(-\frac{1}{Y_t^2}\right)\sigma^2 Y_t^2\, dt$$

$$= \mu\, dt + \sigma\, dW_t - \frac{\sigma^2}{2}\, dt = \left(\mu - \frac{\sigma^2}{2}\right) dt + \sigma\, dW_t$$

Integrating from 0 to t gives the closed-form solution stated in Section 4.2.

8.2 Jensen's Inequality and the Convexity Argument

The repayment function $R(Y) = 0.09 \max(Y - T, 0)$ is convex (it is the composition of a linear function and the max operator). For any random variable Y_t and convex function R, Jensen's Inequality states:

$$E[R(Y_t)] \geq R(E[Y_t])$$

with equality if and only if Y_t is deterministic (i.e., $\sigma = 0$). This implies that the expected annual repayment under the stochastic model is at least as large as the repayment under the deterministic model with the same expected income.

However, this result applies to each individual year in isolation. The path-dependent structure of the loan — where the balance is reduced by repayments and the loan terminates at the write-off date — introduces a more complex interaction. In particular, years of zero repayment (when $Y_t < T$) allow the balance to grow through interest accrual, increasing the write-off at the end of the period. The net effect, as demonstrated by our simulations, is that higher volatility reduces total lifetime repayments and increases the expected write-off, even though Jensen's Inequality suggests higher expected annual repayments in isolation.

The resolution of this apparent paradox lies in the loan balance dynamics. When $Y_t < T$, the repayment is zero, the balance grows, and the borrower is further from full repayment. When $Y_t > T$, the repayment is positive but may be insufficient to offset the interest accrual on a large balance. The asymmetry is compounded over 30 years, leading to a systematic increase in the expected write-off as volatility rises.

8.3 Discretization Error

The Euler-Maruyama discretization used in this paper is exact for GBM (i.e., it produces no discretization error) because the solution to the GBM SDE is known in closed form. The discretized scheme:

$$Y_{t+1} = Y_t \exp\left[\left(\mu - \frac{\sigma^2}{2}\right) + \sigma Z_t\right]$$

is equivalent to sampling directly from the log-normal distribution of Y_{t+1} conditional on Y_t. This ensures that the simulation is free from the numerical artifacts that affect Euler discretizations of more complex SDEs.

9. Ethical and Distributional Implications

9.1 Who Bears the Risk?

The stochastic analysis reveals a fundamental asymmetry in the distribution of risk between borrowers and the government. Borrowers benefit from the zero floor on repayments: in years of low income, they make no payments, and the shortfall is absorbed by the government through interest accrual and eventual write-off. This insurance is valuable, particularly for graduates who experience career breaks, periods of unemployment, or work in lower-paid sectors.

However, the insurance is not uniformly distributed. Graduates who consistently earn above the threshold — typically those in high-paying professions — repay more in total and may even repay more than they borrowed in real terms due to interest accrual. Graduates who consistently earn below the threshold — often women (due to career breaks and the gender pay gap), those in lower-paid sectors, or those from disadvantaged backgrounds — repay very little and have large write-offs. The system is thus progressive in the sense that high earners pay more, but it also means that the government subsidy is disproportionately concentrated among lower earners.

9.2 Does Volatility Benefit Borrowers or the Government?

Our results show that higher income volatility benefits borrowers at the expense of the government. When income is volatile, borrowers spend more time below the threshold (zero repayments), accumulating interest on their balances. This increases the write-off and reduces the government's recovery. From the borrower's perspective, the insurance value of the ICR system is higher when income is more volatile, because the zero floor is more frequently binding.

This has an important implication for policy design: the government's fiscal exposure is not just a function of the average income of graduates, but of the distribution of income outcomes. Policies that increase income inequality or volatility — such as labor market deregulation or the expansion of precarious employment — will increase the government's expected write-off, even if average graduate earnings remain unchanged.

9.3 Equity of Policy Alternatives

The policy scenarios analyzed in Section 7 have markedly different distributional implications. Lowering the repayment threshold increases repayments for all graduates earning above the new threshold, with the largest absolute impact on those with incomes just above the threshold. Extending the write-off period benefits lower earners (who are more likely to have balances written off) but increases the total repayment burden for higher earners who would have repaid in full under a shorter period. Reducing the interest rate is the most equitable intervention: it reduces the debt burden for all borrowers, particularly those who would otherwise see their balances spiral upward, and reduces the expected write-off for the government.

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10. Conclusion

This paper has developed a stochastic framework for analyzing UK student loan repayments under income uncertainty. The key findings are as follows.

First, the deterministic model substantially overestimates expected lifetime repayments and underestimates the expected government write-off. Under baseline parameters, introducing income volatility reduces expected total repayments by 31% (from £56,476 to £38,833) and increases the expected write-off by 55% (from £33,170 to £51,534). This divergence is explained by the convexity of the repayment function and Jensen's Inequality.

Second, income volatility is a first-order determinant of the government's fiscal exposure. The expected write-off increases monotonically with σ, from £44.7k at $\sigma = 5\%$ to £63.7k at $\sigma = 30\%$. This suggests that the RAB charge — the government's headline measure of the cost of the student loan system — is sensitive to assumptions about income volatility, and that using deterministic or smoothed income projections will systematically understate the true fiscal cost.

Third, the interest rate is the most powerful policy lever for reducing the government's tail risk. Reducing the interest rate from 3% to 1% cuts the expected write-off by 54%, from £51.7k to £24.0k. This reflects the fact that high interest rates cause unpaid balances to compound rapidly, creating a debt spiral that is difficult for repayments to overcome.

Fourth, the system embeds a significant insurance value for borrowers, particularly those with volatile or low incomes. This insurance is not free: it is funded by the government through the RAB charge. Policies that increase income inequality or volatility will increase this cost, even if average graduate earnings are unchanged.

The framework developed here is applicable beyond the UK context. Any income-contingent loan or tax system — including graduate taxes, income-share agreements, and social insurance contributions — involves a repayment function that is convex in income. The stochastic analysis presented here provides a general methodology for quantifying the fiscal cost of such systems under income uncertainty.

Future work should extend the model to incorporate heterogeneous initial incomes (reflecting the distribution of graduate earnings), mean-reverting income processes (to capture the tendency of earnings to revert to a long-run average), and correlated income shocks across cohorts (to model macroeconomic risk). Incorporating empirical income distributions from administrative data — as in

Britton, Shephard, and van der Erve (2019) [7] — would further improve the calibration and policy relevance of the analysis.

Appendix A: Baseline Parameter Calibration

Parameter	Symbol	Value	Source/Rationale
Initial loan balance	L_0	£45,000	Approximate average debt for 2022 cohort [3]
Initial income	Y_0	£30,000	Approximate median graduate starting salary
Repayment threshold	T	£28,470	Plan 2 threshold (2024–25) [3]
Income drift	μ	3%	Approximate real earnings growth for graduates
Income volatility	σ	15%	Calibrated to UK earnings volatility literature
Loan interest rate	r	3%	Simplified constant (approximates RPI)
Discount rate	δ	3%	Standard real discount rate
Write-off period	W	30 years	Plan 2 statutory write-off [3]
Simulation paths	N	10,000	Sufficient for stable distributional estimates

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