The Acquaintance Problem Among Six People

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Introduction:

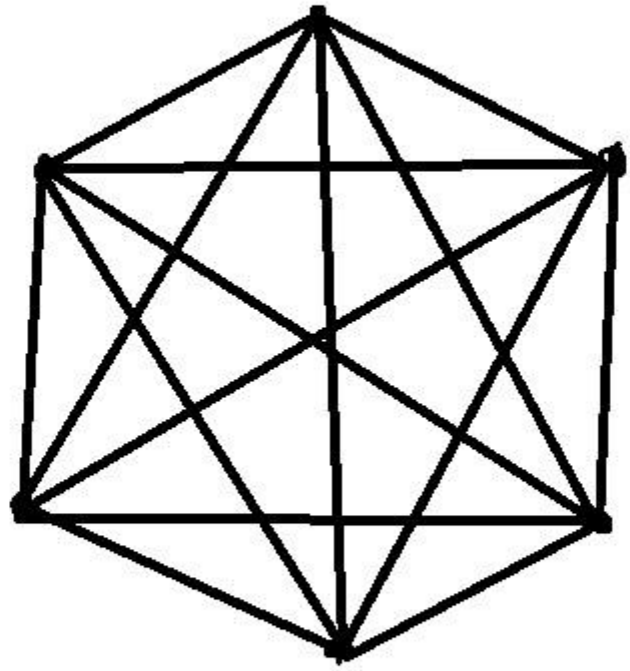
Let’s imagine a group that contains 6 people. Each pair of individuals either know each other or does not. Is it possible to avoid a situation in which there are three people who know each other in pairs, or three people who don’t know each other in pairs?

Surprisingly, the answer is **no**.

At first glance, it might seem that such a configuration can be avoided. Since each pair of people can either know each other or not, there are many possible cases. It may appear that finding one without three people who know each other in pairs, or three people who don’t know each other in pairs is possible. However, mathematics shows that this intuition is wrong.

Explanation:

To prove this, we need to turn a group of people into a graph, where an individual is a vertex and all of them are connected with edges.



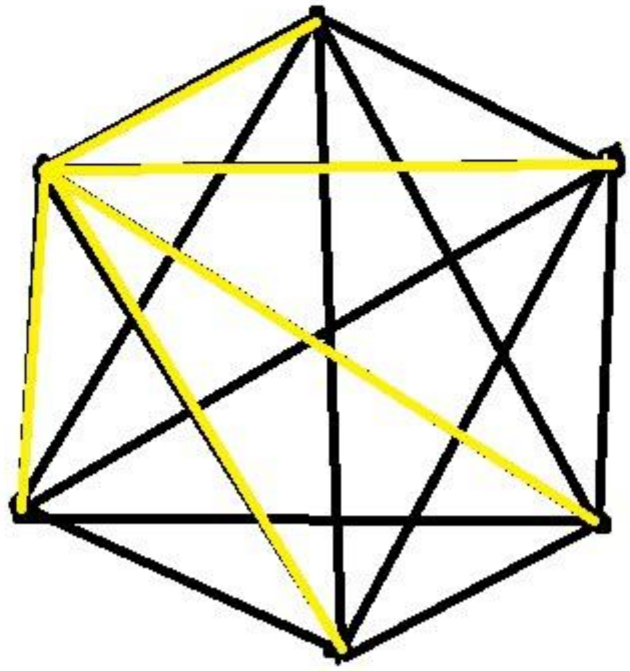
Each edge can be painted:

Blue – 2 people know each other.

Red – 2 people do not know each other.

We need to find a situation in which there are no red or blue triangles.

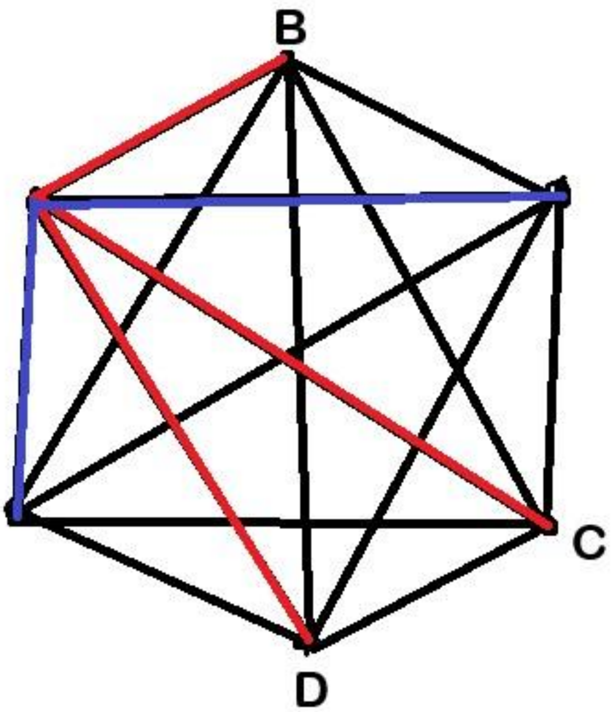
Let's take one vertex. It has 5 edges.



At least 3 of them have the same color according to the *pigeonhole principle*.

> *The Pigeonhole principle* states that if we have n items and m containers, with n>m, then at least one container has more than one item. In our case n = 5 (edges), m = 2 (colors). If we distribute them evenly we will have this situation:
>
> > 2 red edges
> >
> > 2 blue edges
> >
> > And 1 edge left. It needs to be colored either blue or red, so there will be at least 3 of one color.

Let it be:

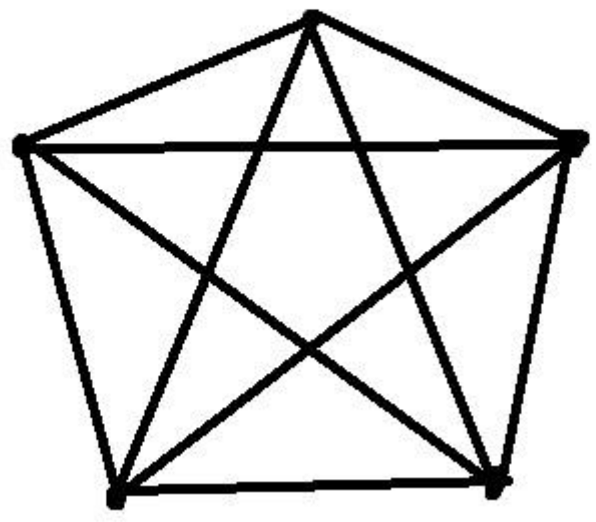


Now, we have a triangle BCD. Neither BC, DC, nor BD can be painted red, because a red triangle would appear. This means that BCD is a triangle colored blue.

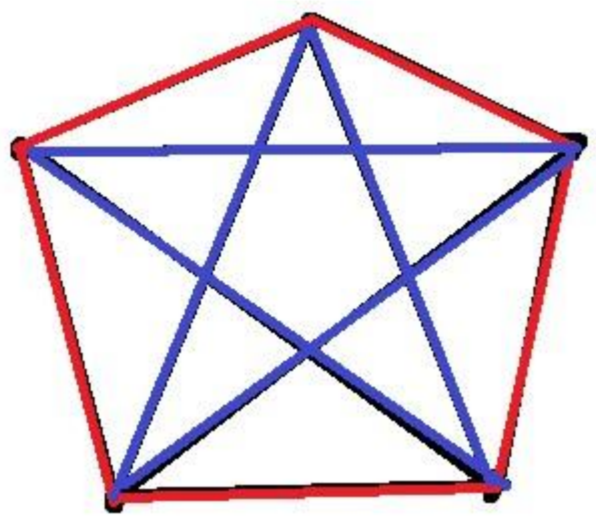
To sum up, a triangle exists anyway.

Other cases:

Now let's take a group of 5:



This graph could be painted like this:



This shows that the statement does not hold for 5 vertices, since a counterexample exists.

Therefore, the minimum number of vertices for which the property stated in the problem holds is 6.

Generalization:

This problem is an example of *Ramsey Theory*, and it could be written as $R(3 , 3) = 6$. This means that the minimum number of vertices for a red triangle or a blue triangle to appear is six. More generally, $R(m , n)$ is the smallest number of vertices needed to guarantee either a group of *n* vertices all connected in one color or a group of *m* vertices all connected in the other color.

Conclusion:

The result in the acquaintance problem among six people is no coincidence. It relates to Ramsey’s theory, which studies the minimum number of objects required to guarantee the emergence of a certain structure. In our case, this means that as few as six people are enough to ensure that a group of three people - some of whom know each other and some of whom do not - will inevitably form.

Thus, Ramsey’s theory asserts that in sufficiently large systems, complete chaos is impossible: order inevitably emerges regardless of the distribution of connections.