The Chaotic Nature of the Sausage Roll

Aisha Faisal

April 2026

1 Introduction

Could the process of making a sausage roll be considered chaotic?

If you've never tackled puff pastry before, attempting to make a sausage roll could certainly result in chaos and unpredictability. Or perhaps you could consider it chaotic in a strictly mathematical sense instead. Does the repeated stretching and folding seem familiar? Suppose you even found yourself wondering, *"where exactly does a singular particle of flour I added to my pastry end up in the sausage roll?"*

You might think that, with sufficient modelling, figuring this out would be theoretically possible. However, you would be wrong.

You may have heard the well known question:

> *"Does the flap of a butterfly's wings in Brazil set off a tornado in Texas?"*

While this is more poetic than realistic, it is less about the butterfly causing the tornado itself, but about the unpredictable impact tiny, microscopic changes can have on a system. Can we truly predict whether a storm will occur in the future given that we don't know the exact value of every impacting condition? Similarly, can we predict where two particles of flour will end up in the pastry of a sausage roll if we don't know the exact location of each particle? In order to understand the relevance of these situations, or even what a chaotic system is, we have to look into how Edward Lorenz came across the concept in 1961.

2 Discovery of the Butterfly Effect

In 1961, MIT meteorologist Edward Lorenz was using a system of differential equations on a simple computer to simulate different weather conditions, with variables representing things like wind speed and temperature. Wanting to see a sequence again that he had previously observed, he decided to save time by starting the simulation in the middle and inserting values he had received mid simulation earlier on. Returning from a coffee break he found that the calm weather conditions the system had displayed before had changed into a harsh storm instead. He came to find that the values had been changed as inputted them rounded to 3 digit numbers, one of them being 0.506127 written as 0.506. This small change was equal to around one part in a thousand, and the consensus at the time was that such a change should not have an impact, but it did. So, can we precisely predict long-term weather events if we are not aware of every detail that might make an impact? As Lorenz himself said:

> *"If there is any error whatever in observing the present state—and in any real system such errors seem inevitable—an acceptable prediction of an instantaneous state in the distant future may well be impossible"*

He was right about this. To this day, reliable weather forecasts are still speculative and worthless for longer than a week. While the small decimal difference may not be relevant for short term forecasts, the system exponentially stretches that small difference over time until it becomes a significant change.

It might seem to you that the events that led to the change in weather were entirely random. However, when Lorenz created a new system with three non-linear differential equations, a reduced model of the former, he found that rather than randomness, these displayed infinite complexity, staying within bounds but never repeating. It formed a double-spiral shape, similar to a butterfly with its two wings. This system was called the "Lorenz Attractor", and can be understood further by looking at the equations themselves.

3 The Lorenz Attractor

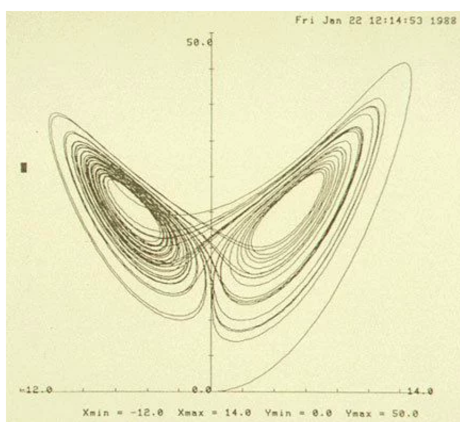

Figure 1: *The Lorenz Attractor, which famously resembles the wings of a butterfly*

The Lorenz system, as you can see in **Figure 1**, is made up of these 3 differential equations:

$$dx/dt = \sigma(y - x) \tag{1}$$

$$dy/dt = x(\rho - z) - y \tag{2}$$

$$dz/dt = xy - \beta z \tag{3}$$

Lorenz found this system was chaotic (non-periodic trajectory and sensitive dependence on initial conditions), and took on the well known butterfly shape, for values

$$\sigma = 10 \quad \beta = 8/3 \quad \rho = 28 \tag{4}$$

The actual mathematics behind the shape of the system is quite complex, so instead we can manipulate the equations with a bit of extra knowledge in such a way so that we can understand why the system takes on this shape and how this makes it unpredictable.

One thing we can easily find using differential equations is the fixed (or as you may know it, stationary) points of the system. These are relevant as a fixed point in a 3D system can either be stable or unstable.

By setting the equations to zero, we get the fixed point equations:

$$x = y \quad \rho x - z - xy \quad xy = \beta z \tag{5}$$

Which can be further simplified to form:

$$x(-x^2/\beta + \rho - 1) = 0 \tag{6}$$

So from here we can see that the origin (0,0,0) is a fixed point. The other two fixed points we found are:

$$x = \pm\sqrt{\beta(\rho - 1)} \tag{7}$$

The other two fixed points only exist when $\rho > 1$ (as you cannot take the square root of a negative number), and there is a reason for why that is the case, which will make a bit more sense later on.

We can determine whether the fixed point is stable or not by finding the eigenvalues for the fixed point (0, 0, 0) We can write out the system as a 3 x 3 matrix like so:

$$\begin{bmatrix} -\sigma & \sigma & 0 \\ \rho - z & -1 & -x \\ y & x & -\beta \end{bmatrix}$$

If we put in the fixed point (0, 0, 0) and consider what might happen when ρ varies (keeping σ, β constant) we are looking at this matrix:

$$\begin{bmatrix} -\sigma & \sigma & 0 \\ \rho & -1 & 0 \\ 0 & 0 & -\beta \end{bmatrix}$$

The eigenvalue of a matrix is found by solving the characteristic equation,

$$det(A - \lambda I) = 0 \tag{8}$$

$$\begin{vmatrix} -\sigma-\lambda & \sigma & 0 \\ \rho & -1-\lambda & 0 \\ 0 & 0 & -\beta-\lambda \end{vmatrix} = 0$$

We can expand along the third column

$$-\beta - \lambda_3 = 0 \tag{9}$$

$$\lambda_3 = -\beta \tag{10}$$

As β is positive, this eigenvalue (for z) is negative. When an eigenvalue is positive, it means that the system is unstable, whereas when it is negative, it is stable. This eigenvalue indicates stability in the z direction, which sort of acts as a flattening force (sort of like rolling out pastry dough before the fold). As for the other two values, they can be determined by solving the formula, which would lead to:

$$\begin{aligned} (-\sigma-\lambda)(-1-\lambda) - \sigma\rho &= 0 \\ \lambda^2 + \lambda + \sigma\lambda + \sigma - \sigma\rho &= 0 \\ \lambda^2 + (\sigma+1)\lambda + (1-\rho)\sigma &= 0 \end{aligned} \tag{11}$$

Here we know, in any quadratic equation, if the constant c is negative, one root must be positive. This means one eigenvalue must be positive. Our constant is $c = (1-\rho)\sigma$, which is negative for $\rho > 1$. The system is only unstable for $\rho > 1$. (Now we can make sense of why the other root did not exist for $\rho < 1$!!!) When the system is stable, it is pulled back into the origin, meaning the other points will not exist. When $\rho = 1$, the origin loses its stability and two new points are now existing (known as pitchfork bifurcation).

Due to this positive eigenvalue, when $\rho > 1$ (which it is for the standard values used by Lorenz), it becomes unstable and solutions grow exponentially, moving quickly away from the origin. The positive value acts as a multiplier, essentially stretching the system, (like pastry dough!) while the non-linear terms (like xy or xz) fold the trajectories back towards its attractor, creating complex layers (does this remind you of a specific baked good?).

Because of this stretching motion, a small initial error can become significant and impact the long term behavior of the system, known as the **Butterfly Effect**. This explains why we cannot predict weather far into the future, and why the system has sensitive dependence on initial conditions.

You can see how the graph takes on this fascinating shape and how this instability causes unpredictability. Our next example is more literal, and much easier to understand, so it can help you visualise the process of increasing error more easily. That is, by once again modelling the next strange attractor as a puff pastry.

(yay!)

4 The Bakers Map

This is the perfect attractor for us to visualise chaos clearly, hence it is the main focus of the essay, despite it being a shorter section.

It is called the bakers map because of the way it 'stretches', 'cuts' and 'stacks' the system, similar to the way a baker might make the puff pastry of a sausage roll. While two points might seem to be the same, the lack of accuracy means you cannot guess the position of a particle of flour after approximately *3.3n* folds, given that the values you have are to n decimal places. A very simplified visual demonstration can be seen below.

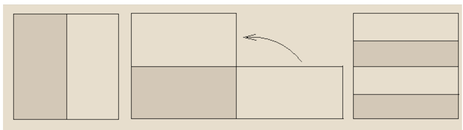

Figure 2: *The stretching and folding of the 'dough'.*

The equations behind this function can be seen below:

$$x_{n+1} = 2x_n, \quad (\text{mod } 1) \tag{12}$$

$$y_{n+1} = \begin{cases} 1/2y_n & \text{if } x_n < 1/2 \\ 1/2y_n + 1/2 & \text{if } x_n \geq 1/2 \end{cases}$$

The x equation is the 'stretching' of the dough. The mod function literally cuts off the integer part of the number, essentially cutting the dough like one might do to puff pastry and stacking it back on top of the other piece, forming the intricate lamination.

The y equation ensures conservation of area. It makes sure as the dough gets longer (when stretched) it also gets thinner, so area remains constant. This is important, if you aren't conserving area you are making dough out of thin air.

The exponential part of this attractor is the 'stretching' of the dough. For one stretch and stack, the number of layers doubles and the thickness halves. After 10 stretches, you get $2^{10} = 1024$ layers of puff pastry for your sausage roll.

We can observe the exponential increase in error by laying it out on a table.

Fold	Actual Value	Predicted value	Error
0	0.3000	0.3001	0.0001 (initial error)
1	0.6000	0.6002	0.0002
2	0.2000	0.2004	0.0004
5	0.6000	0.6032	0.0032
10	0.2000	0.3024	0.1024 (around 10% of map)
12	0.8000	0.2096	0.5904 (opposite side of map)

Can you now see how quickly the error grows? What seems like a negligible error in the beginning becomes very significant by 12 folds, making long term predictions quite literally impossible, like Lorenz said. We never know where exactly we added the flour, and therefore can never know exactly where it ended up due to topological mixing (caused by the stretching and folding).

5 Conclusion

We began this essay asking whether predicting the position of a particle of flour in a pastry is possible, and we have now understood why it is not. However, no matter how unpredictable a strange attractor is, it is never truly random. Even if we can't keep track of one singular particle, we still can figure out what the final pastry will look like.

This is what makes chaos theory so important, it helps show that the universe isn't just a mess of random accidents. It is deterministic, which means the rules are fixed, yet its sensitivity to initial conditions makes it hard to track a singular particle. The stretching and folding of nature is what has made infinite layers of complexity in a finite space, our universe, which essentially means our universe is just one big sausage roll.

Figure 3: *A sausage roll (for reference)*

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