The Legacy of Mathematics in India: From Brahmagupta to Ramanujan

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The trajectory of evolution of mathematics in India represents a remarkable intellectual journey encompassing a long and rich tradition in mathematics, spanning over a millennium. It is a narrative of advanced theoretical knowledge, logical rigor, and foundational discoveries that continue to influence modern mathematics and overall development of the subject.

Brahmagupta (7th century), one of the earliest great mathematicians of India, in his seminal work *Brahmasphutasiddhanta* (The Opening of the Universe), which was the pioneering treatise to clearly define operations involving zero, treating it not merely as a placeholder but as a number in its own right and negative numbers, establishing rules such as the product of two negative numbers being positive, conceptualizing negative numbers as “debts” and positive numbers as “fortunes”. Brahmagupta was the first to solve quadratic equations of the form $ax^2+bx+c=y$ and at the same time he also provided a formula for the area of a cyclic quadrilateral, known as Brahmagupta’s formula:

$$Area = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Where s= the semi-perimeter

Later, Bhaskara I and Bhaskara II (also known as Bhaskaracharya) further expanded and enriched the knowledge of Indian mathematics. Bhaskaracharya’s main treatise was *Siddhanta Siromaṇi* (Crown of Treatises) that covers arithmetic, algebra, and geometry in a poetic and accessible manner. It has 3 major parts: *Lilavati*, the first volume of the *Siddhanta Śiromaṇi* which contains thirteen chapters, 278 verses, mainly arithmetic and measurement. *Bijaganita*, the second volume of *Siddhanta Siromaṇi*, contains 213 verses and is dedicated to algebra. *Gaṇitadhyaya* and *Goladhyaya* of *Siddhanta Siromaṇi* are related to astronomy. He made early advances in calculus like ideas by solving indeterminate equations and anticipated ideas similar to differential calculus, particularly in his treatment of rates of change and maxima-minima problems, concepts of instantaneous motion and derivatives centuries before modern scientists and mathematicians like Newton and Leibniz propounded and formalized calculus to the modern world.

Another champion from the classical age of Indian mathematics was Aryabhata, the first of the major mathematician-astronomers, whose work slightly predates Brahmagupta but strongly influenced later scholars. His path-breaking work, *Aryabhatiya*, consists of 108 verses and 13 introductory verses, and is divided into four *pāda*s or chapters, namely:-

1. *Gitikapada*: (13 verses): It describes large units of time—*kalpa* (1 *kalpa* = 4.32 billion human years), *manvantara* (1 *Manvantara*= 306,720,000 years), and yuga (1 yuga= 12,000 years on Earth)
2. *Ganitapada* (33 verses): It deals with measurement (called *kṣetra vyāvahāra*), arithmetic and geometric progressions, simple, quadratic, simultaneous, and indeterminate equations (called *kuṭṭaka*)
3. *Kalakriyapada* (25 verses): It describes different units of time and a method for determining the positions of planets for any given day.
4. *Golapada* (50 verses): Geometric/trigonometric aspects of the celestial sphere.

Aryabhata was a pioneer in many ways.
For example, he was the first mathematician to approximate the value of pi (π). In his *Ganitpada*, Aryabhata writes- *"Add four to 100, multiply by eight, and then add 62,000. By this rule the circumference of a circle with a diameter of 20,000 can be approached."*
What he means is that for a circle of diameter 20000, circumference will be:
= 100+4=104
And,
104 x 8=832
Thus,
62000+832=62832
If we divide this circumference by diameter, i.e.:
62832/20000= 3.1416= pi (π) which is accurate to two parts in one million

As far as area of triangle is concerned, Aryabhata, in his *Ganitapada 6*, put forwarded the area of a triangle as- "*for a triangle, the result of a perpendicular with the half-side is the area."* Aryabhata also worked on the concept of *sine* by the name of *ardha-jya*, which literally means "half-chord". In fact, *Jya*, *koṭi-jya* and *utkrama-jya* are three trigonometric functions introduced by Indian mathematicians and astronomers. *Jya* and *koti-jya* are, in fact, closely related to the modern trigonometric functions of sine and cosine. Ironically some authors traced back the origins of the modern terms of "sine" and "cosine" to the Sanskrit words *jya* and *koti-jya*.

Madhava of Sangamagrama, considered being the founder of the *Kerala school of astronomy and mathematics,* was an Indian mathematician and astronomer. Madhava was among the first authors to have developed infinite series expansions for trigonometric functions like sine and cosine, anticipating concepts of calculus centuries before Newton and Leibniz. The transition from finite series of algebra to the infinite series is often attributed to Madhava. Modern reference to such series was given in Europe, when the first such series were developed by James Gregory in 1667. Since Madhava understood the concept and calculations of estimate of an error term (or correction term), it can be implied that he understood the limit nature of the infinite series. One of Madhava's series is known from the treatise *Yuktibhaṣa*, written by Jyeṣṭhadeva (who was an astronomer-mathematician of the Kerala school of astronomy and mathematics founded by Madhava of Sangamagrama) which contains the derivation and proof of the power series for inverse tangent, discovered by Madhava, which states that-

"The first term is the product of the given sine and radius of the desired arc divided by the cosine of the arc. The succeeding terms are obtained by a process of iteration when the first term is repeatedly multiplied by the square of the sine and divided by the square of the cosine. All the terms are then divided by the odd numbers 1, 3, 5,... The arc is obtained by adding and subtracting respectively the terms of odd rank and those of even rank. It is laid down that the sine of the arc or that of its complement whichever is the smaller should be taken here as the given sine. Otherwise the terms obtained by this above iteration will not tend to the vanishing magnitude."

This ultimately leads to:-

$$r\theta = \frac{r\sin\theta}{\cos\theta} - (1/3)\,r\,\frac{(\sin\theta)^3}{(\cos\theta)^3} + (1/5)\,r\,\frac{(\sin\theta)^5}{(\cos\theta)^5} - (1/7)\,r\,\frac{(\sin\theta)^7}{(\cos\theta)^7} + \cdots$$

That is equivalent to:

$$\theta = \tan\theta - \frac{\tan^3\theta}{3} + \frac{\tan^5\theta}{5} - \frac{\tan^7\theta}{7} + \cdots$$

This series was earlier called as Gregory's series (named after James Gregory, who rediscovered it three centuries after Madhava). Today, it is referred to as the Madhava-Gregory-Leibniz series.

At the same time, Madhava's through his *Mahajyānayana prakāra* ("Methods for the great sines") worked on the value of the mathematical constant Pi and led to the following infinite series expansion of π, which now a days is known as the Madhava-Leibniz series-

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1},$$

Many texts (like *Sadratnamala*) and many authors (like J. J. O'Connor, E. F. Robertson, R Gupta etc.) suggest that Madhava must have given the accurate value of $\pi = 3.14159265358979324$ (correct to 17 decimal places).

Considering the historical yet factual findings of Madhava, in recent literature the Gregory series and the Leibniz series are rechristened as the Madhava–Newton series, Madhava–Gregory series, or Madhava–Leibniz series to pay him respect. This is important while considering that according to American mathematician, David Bressoud, “there is no evidence that the Indian work of series was known beyond India, or even outside of Kerala, until the nineteenth century”.

Yuktibhaṣa, written by Jyeṣṭhadeva (astronomer-mathematician of the *Kerala school of astronomy and mathematics*), describe the planetary theory which is similar to that later adopted by Danish astronomer Tycho Brahe.

Besides these eminent ancient Indian mathematicians and astronomers, there were many other proponents of mathematics and astronomy during ancient India. For example, Devacharya from Kerala, India was the first astronomer to give a rule for finding the precession of equinoxes. Similarly, Udayadivakara who was an Indian astronomer and mathematician and authored book called *Sundari*, which highlights *cakravala* method (which is a cyclic algorithm to solve indeterminate quadratic equations, including Pell's equation or Pell–Fermat equation, is any Diophantine equation of the form $x^2-ny^2=1$ where n is a given positive non-square integer, and integer solutions are sought for x and y) for solving indeterminate integral equations of the form $Nx^2+1=y^2$. At the same time the Sundari has given solution of the indeterminate equation of the form $Nx^2+C=y^2$ where C can be positive or negative. Parameshvara Nambudiri, another ancient Indian mathematician, worked upon the mean value type formula for the inverse interpolation of the sine and he is often regarded as the first mathematician to give a formula for the radius of the circle circumscribing a cyclic quadrilateral. Nilakantha Somayaji, ancient Kerala astronomer and mathematician, developed a computational system for a partially heliocentric planetary model in which Mercury, Venus, Mars, Jupiter and Saturn orbit the Sun, which in turn orbits the Earth. This was similar to the Tychonic system later proposed by Danish astronomer Tycho Brahe. Chitrabhanu, composer of *karaṇa* (a concise astronomical manual), gave integer solutions to 21 types of systems of two simultaneous Diophantine equations in two unknowns.

The Fibonacci numbers or sequence (sequence in which each element is the sum of the two elements that precede it), named after the Italian mathematician Leonardo of Pisa, also known as Fibonacci, who introduced the sequence to Western European mathematics in his 1202 book Liber Abaci, were first described in Indian mathematics as early as 200 BC in work by Pingala on enumerating possible patterns of Sanskrit poetry formed from syllables of two lengths. Pingala is sometimes also credited with the first use of zero, as he used the Sanskrit word *śhunya* for zero. Virahanka numbers are an ancient Indian sequence, equivalent to the modern Fibonacci sequence (0,1,1,2,3,5,13 …..), where each number is the sum of the two preceding ones. Described by the Indian scholar Virahanka, another ancient Indian mathematician, is often regarded as the first mathematician in the world to propose the Fibonacci sequence when he used this pattern to count rhythmic patterns in Sanskrit poetry, long before Fibonacci. This list is endless; there are many unsung heroes in ancient India who contributed in some or other manner to the field of mathematics and astronomy.

Srinivasa Ramanujan: The Mathematical Genius

"An equation means nothing to me, unless it expresses a thought of God."

-*Srinivasa Ramanujan*

Srinivasa Ramanujan was an Indian mathematician who worked during the early 20th century. The then Western mathematician came to know about this mathematical genius through the mail correspondence which Ramanujan began in 1913 with the English mathematician G. H. Hardy at the University of Cambridge. Ramanujan is associated with nearly 3,900 results (mostly identities and equations) in his life. Ramanujan is associated with:

1. Ramanujan prime, partition function p(n), the Ramanujan conjecture,
2. Ramanujan theta function,
3. Mock theta function (which have found tremendous applications in modern string theory and black hole physics), etc.
4. He also discovered the Hardy–Ramanujan number 1729 which is the smallest number expressible as the sum of two cubes in two different ways.

His list of contributions in the field of mathematics is so vast that his birthday, 22 December is celebrated as National Mathematics Day in India.

Before concluding my essay, I would like to mention two distinct personalities in the field of mathematics in India. First is D.R. Kaprekar who discovered what is called as *Kaprekar's constant*, 6174, which is a number that acts as a "black hole" for a specific 4-digit number routine. Any 4-digit number with at least two distinct digits, when subjected to arranging its digits in descending then ascending order and subtracting, will reach 6174 in 7 steps or less.

The second mathematical genius is Shakuntala Devi, popularly known as the "Human Computer". On 18 June 1980, she demonstrated the multiplication of two 13-digit numbers – 7,686,369,774,870 × 2,465,099,745,779, these numbers being randomly picked by the Department of Computing at Imperial College London. She correctly answered 18,947,668,177,995,426,462,773,730 in 28 seconds. She has many such feats to her name.

The contribution of Indian mathematicians has been incredible from time immortal. In fact, India is the land which produced the *Bakhshali* manuscript which is regarded as "the oldest extant manuscript in Indian mathematics". Hopefully, Indians will keep contributing to the field of mathematics in the future as well.

And with that said, I conclude with the wise words of John D. Barrow:

> The Indian system of counting is probably the most successful intellectual innovation ever devised by human beings. It has been universally adopted. ...It is the nearest thing we have to a universal language.

John D. Barrow, The Book of Nothing (2009)